*Synthetic division* is an efficient process that minimizes the “clutter” (variables and some numbers) when dividing. However, it can only be used when dividing by a binomial. If the leading coefficient of the binomial is not 1, you must do some adjustment. (Won’t have to in homework – see “To Think About”.)

Ex 1 Use synthetic division to divide.

\*Note: synthetic division only works when dividing by terms of the form (x-a)

1. $\frac{(x^{2}+17x+66)}{x+6}$ (no Remainder)
2. 5.3.4 $\left(3x^{3}+10x^{2}+6x-4\right)÷\left(x+2\right) $ $(no Remainder)$ Ans:$3x^{2}+4x-2$
3. 5.3.8 $(6x^{3}-4x^{2}-42x+7)÷(x-3)$ (Remainder) $Ans:6x^{2}+14x+\frac{7}{x+3}$
4. 5.3.10 $(2x^{3}+7x^{2}-5)÷(x+3)$ (Remainder) Ans:$2x^{2}+x-3+\frac{4}{x+3}$

(don’t forget to save the place of the x term)

Other problems

1. $2x^{3}+2x^{2}-44x+7$ divided by $x+4$ Ans: $2x^{2}-6x-20+\frac{87}{x+4}$
2. $\left(3x^{4}+8x^{3}-7x^{2}-9x+15\right)÷\left(x+3\right)$ Ans: $3x^{3}-x^{2}-4x+3+\frac{6}{x+3}$
3. $\left(2x^{4}-x+3\right)÷\left(x-2\right)$ Ans: $2x^{3}+4x^{2}+8x+15+\frac{33}{x+3}$

Challenge problem:

What about the case where we are not dividing by something of the form x-a?

Consider:

$$\frac{2x^{2}+7x+3}{2x+1}=\frac{\left[2x^{2}+7x+3\right]}{2x+1}=\frac{\frac{1}{2}\left[2x^{2}+7x+3\right]}{\frac{1}{2}\left(2x+1\right)}=\frac{\frac{1}{2}\left[2x^{2}+7x+3\right]}{(x+\frac{1}{2})}=\frac{1}{2}\left[\frac{\left[2x^{2}+7x+3\right]}{\left(x+\frac{1}{2}\right)}\right]$$

$\begin{matrix}-\frac{1}{2} \\\end{matrix}\begin{matrix}2&7&3\\\downright &-1&-3\\2&6&0\end{matrix} \rightarrow 2x+6 $

$$=\frac{1}{2}\left[\frac{\left[2x^{2}+7x+3\right]}{\left(x+\frac{1}{2}\right)}\right]=\frac{1}{2}\left[2x+6\right]=x+3 $$

5.3.25 $\left(4x^{3}-6x^{2}+6\right)÷\left(2x+3\right)=\frac{\left(4x^{3}-6x^{2}+6\right)}{\left(2x+3\right)}=\frac{\frac{1}{2}\left(4x^{3}-6x^{2}+6\right)}{\frac{1}{2}\left(2x+3\right)}=\frac{\frac{1}{2}\left(4x^{3}-6x^{2}+6\right)}{\left(1x+\frac{3}{2}\right)}=\frac{1}{2}\left[\frac{\left(4x^{3}-6x^{2}+6\right)}{\left(x+\frac{1}{2}\right)}\right]$ so carry out the synthetic division and don’t forget to multiply our result by ½.

## The Remainder Theorem

The remainder obtained by dividing $P\left(x\right)$ by $(x-r)$ is $P\left(r\right)$.

I.E. $P\left(x\right)=\left(x-r\right)Q\left(x\right)+R$ where $Q\left(x\right)=\frac{P\left(x\right)}{\left(x-r\right)}$

Ex: Given $P\left(x\right)=x^{3}+7x^{2}+3x+4$ then find $P(-5)$

$\begin{matrix}-5\\\\\end{matrix} \begin{matrix}1&7&3\\\downright &-5&-10\\1&2&-7\end{matrix} \begin{matrix}4\\35\\39\end{matrix}$ so $P\left(-5\right)=39$

To learn more about this, try problem 6.7.37

To Prove the remainder theorem, note that any polynomial $P(x)$ can be rewritten as $\left(x-r\right)∙Q\left(x\right)+R$, where $Q\left(x\right)=\frac{P\left(x\right)}{\left(x-r\right)}$is the quotient polynomial, and R is some constant number that is the remainder from the division.

1. How do we know that R must be a constant?
2. Show that $P\left(r\right)=R $ (this says that $P(r)$ is the remainder when $P(x)$ is divided by $(x-r))$.

