Recall:

Def: The graph of all quadratic equations is the U-shaped curve called a **parabola**.

Def: The **Vertex** of any parabola is the so called “turn around point”. It is the place where the parabola reaches its maximum or minimum value.

## The Graph of $f\left(x\right)=a\left(x-h\right)^{2}+k$

Let us call the form $f\left(x\right)=a\left(x-h\right)^{2}+k$, **Vertex Form** of a quadratic function, since it readily shows us that the vertex will always be at $(h,k)$.

Now recall that if we replace the $x$ in $f\left(x\right)=x^{2}$ with $x-h$ this will correspond with a horizontal shift to the right $h$ units. If we add a number $k$ to that function, this will correspond with a vertical shift up or down by $k$units.

So consider the functions $f\left(x\right)=\left(x+2\right)^{2}-1 \& g\left(x\right)=x^{2}+4x+4-1 \& h\left(x\right)=x^{2}+4x+3$

These are all the same graphs, however $f(x)$ is easiest to sketch than the others because it readily shows off its horizontal and vertical shifts.

Can you think of a way to make the function $h(x)$ appear in Vertex Form? What processes must we do to accomplish this form?

Ex:

Find the vertex form of the function $p\left(x\right)=x^{2}+6x-10$. Then find the vertex, then sketch it.

$$p\left(x\right)=x^{2}+6x-10$$

$$p\left(x\right)=x^{2}+6x+\left(\frac{6}{2}\right)^{2}-\left(\frac{6}{2}\right)^{2}-10=\left(x+3\right)^{2}-9-10=\left(x+3\right)^{2}-19$$

So the vertex is $(-3,19)$

or use vertex form $\left(-\frac{b}{2a},f\left(-\frac{b}{2a}\right)\right)=\left(-\frac{6}{2\left(1\right)},p\left(-\frac{6}{2}\right)\right)=(-\frac{3}{2},19)$

Ex:

Find the vertex form of the function $p\left(x\right)=x^{2}+2x-1$. Then find the vertex, then sketch it.

$$p\left(x\right)=x^{2}+2x-1$$

$$p\left(x\right)=x^{2}+2x+\left(\frac{2}{2}\right)^{2}-\left(\frac{2}{2}\right)^{2}-1=\left(x+1\right)^{2}-1-1=\left(x+1\right)^{2}-2$$

So the vertex is $(-1,-2)$

Or use vertex formula $\left(-\frac{2}{2\*1},p\left(-\frac{2}{2}\right)\right)=\left(-1,p\left(-1\right)\right)=(-1,-2)$

Ex:

Find the vertex form of the function $p\left(x\right)=-2x^{2}+6x-3$. Then find the vertex, then sketch it.

$$p\left(x\right)=-2x^{2}+6x-3=-2\left(x^{2}-3x\right)-3$$

$$p\left(x\right)=-2\left(x^{2}-3x\right)-3=-2\left(x^{2}-3x+\left(\frac{3}{2}\right)^{2}-\left(\frac{3}{2}\right)^{2}\right)-3$$

$$=-2\left(x-\frac{3}{2}\right)^{2}-3+\frac{18}{4}=-2\left(x-\frac{3}{2}\right)^{2}-\frac{12}{4}+\frac{18}{4}=-2\left(x-\frac{3}{2}\right)^{2}+\frac{3}{2}$$

So the vertex is $\left(\frac{3}{2},\frac{3}{2}\right)$

Or use vertex formula $\left(-\frac{6}{2\left(-2\right)},p\left(-\frac{6}{2\left(-2\right)}\right)\right)=\left(-\frac{3}{2},p\left(-\frac{3}{2}\right)\right)=\left(-\frac{3}{2},-\frac{3}{2}\right)$

Ex:

Find the vertex form of the function $p\left(x\right)=-2x^{2}+10x-7$. Then find the vertex, then sketch it.

$$p\left(x\right)=-2x^{2}+10x-7=-2\left(x^{2}-5x\right)-7$$

$$p\left(x\right)=-2\left(x^{2}-5x\right)-7=-2\left(x^{2}-5x+\left(\frac{5}{2}\right)^{2}-\left(\frac{5}{2}\right)^{2}\right)-7$$

$$=-2\left(x-\frac{5}{2}\right)^{2}-7+\frac{50}{4}=-2\left(x-\frac{5}{2}\right)^{2}-\frac{28}{4}+\frac{50}{4}=-2\left(x-\frac{5}{2}\right)^{2}+\frac{11}{2}$$

So the vertex is $\left(\frac{5}{2},\frac{11}{2}\right)$

Or use vertex formula $\left(-\frac{10}{2\left(-2\right)},p\left(-\frac{10}{2\left(-2\right)}\right)\right)=\left(\frac{5}{2},\frac{11}{2}\right)$

The Vertex of a Parabola

The vertex of a parabola given by $f\left(x\right)=ax^{2}+bx+c$ is

$$\left(-\frac{b}{2a},f\left(-\frac{b}{2a}\right)\right) or \left(-\frac{b}{2a},\frac{4ac-b^{2}}{4a}\right)$$

* The $x$-coordinate of the vertex is $–\frac{b}{2a}$
* The $y$-coordinate of the vertex is$ f\left(-\frac{b}{2a}\right)$
* The axis of symmetry is $x=-\frac{b}{2a}$

## Finding x and y intercepts

Recall that an **x-intercept** is the point where a curve or graph intercepts the x axis.

Also that a **y-intercept** is the point where a curve or graph intercepts the y axis.

They all have a y value of $y=0$

What do all x-intercepts have in common? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

They all have a x value of $x=0$

What do all y-intercepts have in common? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Set $y=0$ and find x.

So what do we do to find the x-intercepts? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Set $x=0$ and find $y$.

And what do we do to find the y-intercepts? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Ex: Find the x and y intercepts of the function $f\left(x\right)=3x^{2}+6x-9$

X intercepts: Let y=0:

$$0=3\left(x^{2}+2x-3\right)=3\left(x+3\right)(x-1)$$

$$\rightarrow x=1,-3$$

So the x intercepts are $\left(1,0\right)\&(-3,0)$

Y intercepts: Let x=0:

$$f\left(0\right)=3\left(0\right)^{2}+6\left(0\right)-9=-9$$

So the y intercept is $(0,-9)$