Goals:

* Necessary Definitions (divisible, divisor, factor, prime number, composite number)
* Prime Factorization
* Exponents
* Divisibility Tests
* Unique Factorization Theorem

# Necessary Definitions

Def: a number, lets call it a, is said to be **divisible** by another number, call it b, if and only if the remainder is zero when you divide a by b.

Example: the number 24 is divisible by 6, if and only if when you divide 24 by 6 there is a zero remainder, which it does.

$\frac{24}{6}=4$ (with no remainder)

So we say 24 is divisible by 6.

Def: any number(s) or expression(s) that are being multiplied are called factors.

Multiplication is the operation that separates factors.

Example:

$$6∙4=24$$

6 & 4 are called factors since they are being multiplied. Further, we can say that they are both factors of 24, their resulting product.

Example:

$$2∙3∙5=30$$

$2,3,\& 5$ are all factors since they are being multiplied, and we can say they are factors of 30 as well.

Example:

$$5∙x$$

Both $5 \& x$ are factors since they are being multiplied.

Example:

$$6+4=10$$

Neither 6 or 4 are factors since they are not being multiplied. They are being added, so they are called addends instead.

Def: A **prime number** is a whole number greater than 1 whose only factors are 1 & itself.

Or

Another definition is, any whole number greater than 1 whose only divisors are 1 & itself.

Example:

$2$ is prime (the only even prime actually) since 2 only has factors of 1 & 2.

Example:

17 is prime since its only factors are 1 &17.

Example:

6 is not prime, since yes it does have 1 & 6 as factors, BUT it ALSO has 2 & 3 as factors as well!!

A list of some prime numbers is:

$$2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43,47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97,…$$

Def: A **composite number** is any whole number that is not prime.

Or

A **composite number** is any whole number that can be divided by numbers other than one and itself.

Composite numbers are anti-primes.

# Prime Factorization

Every composite number can be factored down to a product of primes.

How do we find a numbers prime factorization?

## Factor Tree method:

## Ladder Method:

The ladder method just tries to break down a number by trying to divide out eavery prime number from 2 on. If the number attempted divides it we write that divisor in the left column and we write the quotient under the previous number. If a prime does not divide the number, then we write nothing down and we simply try the next prime number until the last number in the right hand column is 1, which indicates that the number is factored completely.

Example: Find the Prime factorization of 120

|  |  |
| --- | --- |
|  | 120 |
| 2 | 60 |
| 2 | 30 |
| 2 | 15 |
| 3 | 5 |
| 5 | 1 |

 So the prime factorization of 120 is: $12=2∙2∙2∙3∙5$

# Exponents

Exponents are a short hand notation for repeated multiplication.

Example:

$$2^{3}$$

2- is called the base number, it is the number that is getting repeatedly multiplied

3-is called the exponent, it is the number that tells how many factors of the base are being multiplied.

$$2^{3}=2∙2∙2$$

Example:

$$5^{2}=5∙5$$

The Base is 5

The Exponent is 2

Example: find the prime factorization of 600, express your answer in exponential form.

|  |  |
| --- | --- |
|  | 600 |
| 2 | 300 |
| 2 | 150 |
| 2 | 75 |
| 3 | 25 |
| 5 | 5 |
| 5 | 1 |

So $600=2^{3}∙3∙5^{2}$

Why are Prime number so important?

How may we find prime numbers? ([sieve of Eratosthenes](Sieve_of_Eratosthenes_animation.gif), divisibility rules)

#### [Sieve of Eratosthenes](http://www.visnos.com/demos/sieve-of-eratosthenes)



#### Divisibility Rules:

This is very useful!!!

# Unique Factorization Theorem

Also known as the Fundamental Theorem of Arithmetic

I like to think of it as the **fingerprint theorem**

This is a very wonderful theorem that states that every whole number can be *uniquely factored* as a product of primes.

So basically every whole number has a **unique fingerprint**, that finger print is its prime factorization.

$6=2∙3$

This factorization is unique, if you add or remove any factors, the number stops being 6! So 6 cannot be expressed in prime factorization in any other way except 2 & 3 (or 3 &2 but that is really the same thing).