## Inverse Functions

Given a function , and some point on it , the inverse function, written , is what you get when you switch the x and y values, so is a point on

tells you how to get from x to y. tells you how to get back, so it tells you how to get from y to x.

If you think of a function as a unique set of driving directions, then if tells you how to get from my house to yours, then will be the directions that tell you how to get back. It tells you how to get from your house to mine via, the exact route taken from my house to yours only going the other direction.

*Def* Suppose that is a 1-1 function on a domain with range .

Let .

The ***inverse function***  is defined by if . The domain of is and the range is .

*Def* A function is ***\_\_\_\_\_\_\_\_\_\_\_\_\_\_*** on a domain if whenever in . In other words, if we think of functions as sets of ordered pairs, each -coordinate corresponds to *exactly one* -coordinate.

Ex 1 Explain why the following functions are not 1-1.

*Horizontal Line Test for One-to-One Functions*

A function is one-to-one if and only if its graph intersects each horizontal line at most once.

Ex 2 Prove that each function is 1-1.

Why doesn’t the same technique work for ?

Note: A function need not be 1-1 in order to have an inverse. If is not 1-1, its inverse is a relation but not a function. A function from its domain onto its range needs to be 1-1 in order for its inverse relation to be a function. For our purposes, we will only speak of inverses if the original function is 1-1.

**The Existence of an Inverse Function**:

A function from its domain onto its range has an inverse function if and only if it is 1-1.

Warning: does NOT equal ; that is, the -1 is NOT an exponent.

and “undo” each other. Simple example: If we start with a number, say 13, and we add 2 then subtract 2, we arrive at where we started, namely 13. So a function that adds 2 and a function that subtracts 2 are inverses; that is and are inverses.

One way to tell if two functions are inverses is if they do in fact undo each other. The way to test this is to compose them to see if you do one then do the other, you get back to where you started.

i.e. put x into then put this result into to see if it will undo it.

Ex: and be inverses. Prove that they are inverses by composing them.

***Alternate Definition for inverse functions:***

*A function is the inverse function of the function if*

1. for all in the domain of

And

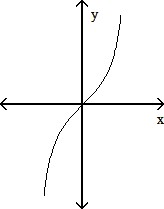
1. for all in the domain of

Since switches the x and y variables, the graph of will look like that of , but with all the values of x & y in the solution points being switched. What this will end up looking like is a reflection across the line .

Therefore, the graphs of and are mirror reflections of each other across the line . Thus, we can create the graph of from the graph of .

T/F If lies on the graph of then lies on the graph of .

Ex 3 Use symmetry with respect to the line to add the graph of to the sketch. (It is not necessary to find the formula for .) Identify the domain and range of .



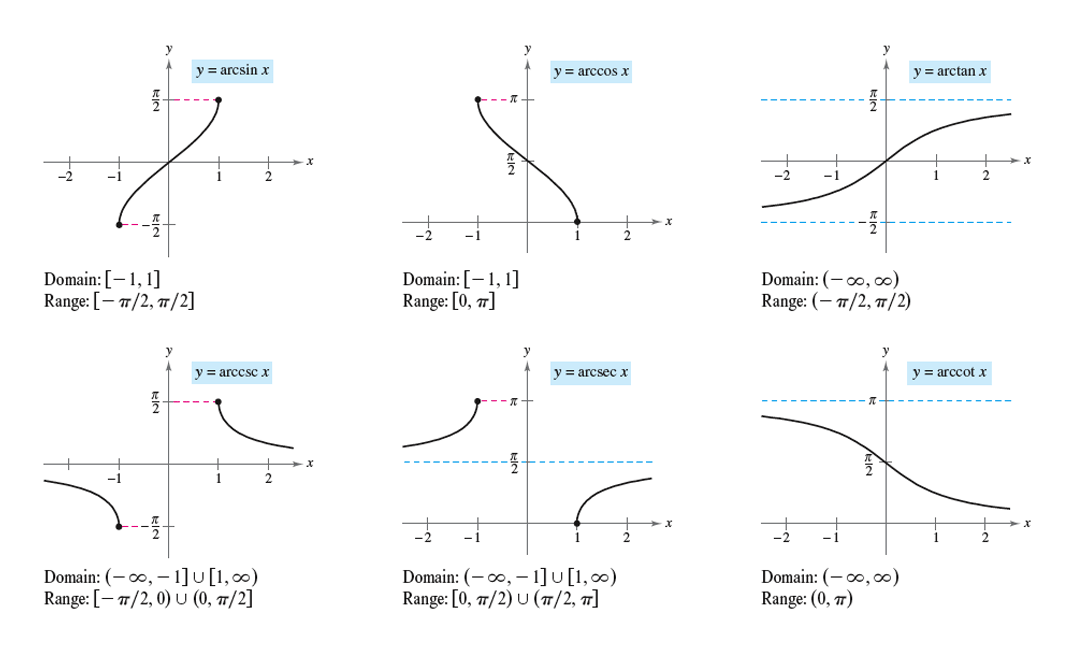
Ex 4 Find the formula for if .

## Inverse Trigonometric Functions

If you consider trig functions to be the transcendental functions that map angles of a right triangle to the ratio of particular legs of that triangle, then

Inverse Trig functions are the transcendental functions that map the ratios of legs of a right triangle to their angles.

List the 6 trig functions and their inverses. Discuss their lack of being 1-1 and how we can restrict the domain of these functions to the inverses will be functions. List the Dom & Range of each inverse function. Pg 41.



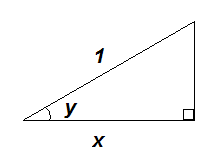
Trig functions have angle inputs and ratios of legs for out puts. The inverse trig functions have ratio of legs for inputs and angle for out puts.

Ex: Evaluate:

a) b) c) d) e)

Ex: (1.5.118) Solve the equation for x:

Ex: Given find



*Defn* The ***logarithm function with base a***, , is the inverse of the base exponential function .

Recall What are the bases of the common and natural log functions?

*Natural log*

*Common log*

This implies and . Also, and in particular, . See “Inverse Properties for and on page 52.)

*Properties of Logarithms*

For any numbers and , the natural logarithm satisfies the following rules:

1. *Product Rule*
2. *Quotient Rule*
3. *Reciprocal Rule (special case, when b=1)*
4. *Power Rule*

*Change of Base Formula*

Ex 5 (# 32) Find simpler expressions for the quantities.

E x 6 Solve for in terms of .

Review pages 55-58. (trig stuff)