*Theorem Limit Laws*

If and are real numbers and and , then

1. Sum Rule:
2. Difference Rule:
3. Product Rule:
4. Constant Multiple Rule:
5. Quotient Rule:
6. Power Rule: (If and have no common factors, , and is a real number)

Limit of a Composite Function

If are functions such that and then

*Corollary Limits of Polynomials Can Be Found By Substitution*

If then

*Theorem Limits of Rational Functions Can Be Found By Substitution If the Limit of the Denominator is Not Zero*

If and are polynomials and , then .

Hint: If , then is a factor of and so can factor from both then cancel. Ex:

Ex 1 Find the limits.

1. (\*If see , conjugate unless have diff of 2 squares, as in part c)
2.
3. (\*We see a but since there’s a diff of 2 squares, don’t conjugate)

Ex 2 Suppose that , Find:

Ex 3 Evaluate the limit for at .



*Theorem The Squeeze/Sandwich Theorem*

Suppose that for all in some open interval containing ,

except possibly at itself.

Suppose also that . Then .



\*Useful when dealing with since for all

Ex: Find

Hint: Try imagining its graph. Find its limit as x-> +inf. Is bounded (meaning does its range have a max/min value)?

 

Ex: Find

Hint: Think what could this look like x or sin x? which one is more “dominant”? For which values of x is one more dominant? Is bounded (meaning does its range have a max/min value)?

If so what is it? What does this say about ?

*Theorem*

If for all in some open interval containing , except possibly at itself, and the limits of and both exit as , then .

Note: Even if for all and and both exist, we cannot conclude that .

Ex 4 Suppose the inequalities hold for values of near 0. What, if anything, can we say about ? Give reasons for your answer.

Note: , meaning if x<2 then x but if x then it is not necessary that x<2 so squeeze theorem will apply.

Three Special limits

1. 2. 3.

Ex 5

1. (#74)
2. (#70)
3. (#30)
4. (#32)
5. (#78)