Ex Given the function and below, answer the following.

* Find
* Find
* Find
* Although does not have a limit at 0, we can still give some info about it’s behavior near 0 by calculating a “left-hand” and “right-hand” limit.
* is not defined to the left of so it does not have a limit there (see definition of limit on next page). However, it seems it should have a limit of if we restrict ourselves to where it’s defined.
* We extend the definition of a “two-sided” limit to a “one-sided” limit.

Informal Definition of a Limit Suppose is in the domain of . We say that the limit of as is , written if gets arbitrarily close to forall close to .

In order to have a limit as , both a left-hand and a right-hand limit must exist.

Informal Defns

means as from the left side

means as from the right side

Ex: (2.4.18) where

Ex: (2.4.22)

Ex: (2.4.26)

Ex: (2.4.28)

*Theorem* The Existence of a Limit

A function has a limit as approaches iff it has left-hand and right-hand limits there and these one-side limits are equal.

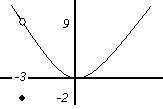
and

Ex:

Ex 1 Find the right- and left-hand limits. Next, find .



1. Jump Discontinuity at (at )



1. Removable Discontinuity at (at )
2. when x = 0 & (at c = 2)
3. When x = 0 & (at c = 0)

## Continuity at a Point and on an Open Interval

Why are , , and discontinuous. How you remedy these discontinuities?



***Continuity Test***

A function is continuous at an interior point of its domain iff it meets the following 3 conditions:

1. exists ( lies in the domain of )
2. exists ( has a limit as )
3. (the limit equals the function value)

Ex 1 sketch the following functions that are continuous and discontinuous.

a) b) c) d)

Ex 2 Why are the discontinuous functions in example 1 discontinuous? Give reasons using Continuity Test.

*Types of Discontinuities (in order from least worst to worst)*

1. Removable : exists but (give example)
2. Jump: (give example)

1. (Essential) Infinite: Either LHL or RHL equals (give example)
2. (Essential) Oscillating: oscillates too much to have a limit (give example)

(Essential discontinuities occur when either RHL or LHL DNE.)

*Definition* (Formal) of a Limit on a Closed Interval

In the interval , if then is an ***interior*** point. and are ***endpoints***.

A function is ***continuous at an interior point c*** of its domain if .

A function is ***continuous at left endpoint a*** or is ***continuous at a right endpoint b*** of its domain if or , respectively.

If a function is not continuous at , we say is discontinuous at and is a ***point of discontinuity.***

Ex 3 Given graph below, answer the following:

1. Does exist?

1. Does exist?
2. Does exist?

1. Is continuous at ?

1. At what values of is continuous?

Note: A function is not continuous at a point unless it is an interior point. A function is called continuous if it is continuous

## Properties of Continuity

*Theorem* Properties of Continuous Functions

If the functions and are continuous , then the following combinations are continuous at .

1. , for any number
2. provided
3. , provided it is defined on an open interval containing , where and are integers

*Corollary* Polynomial and rational functions are continuous. (Rational functions are continuous where they are defined.)

*Theorem* If is continuous at and is continuous at , then the composite is continuous at .

Ex Where is continuous? Why? (Explain with theorems.)

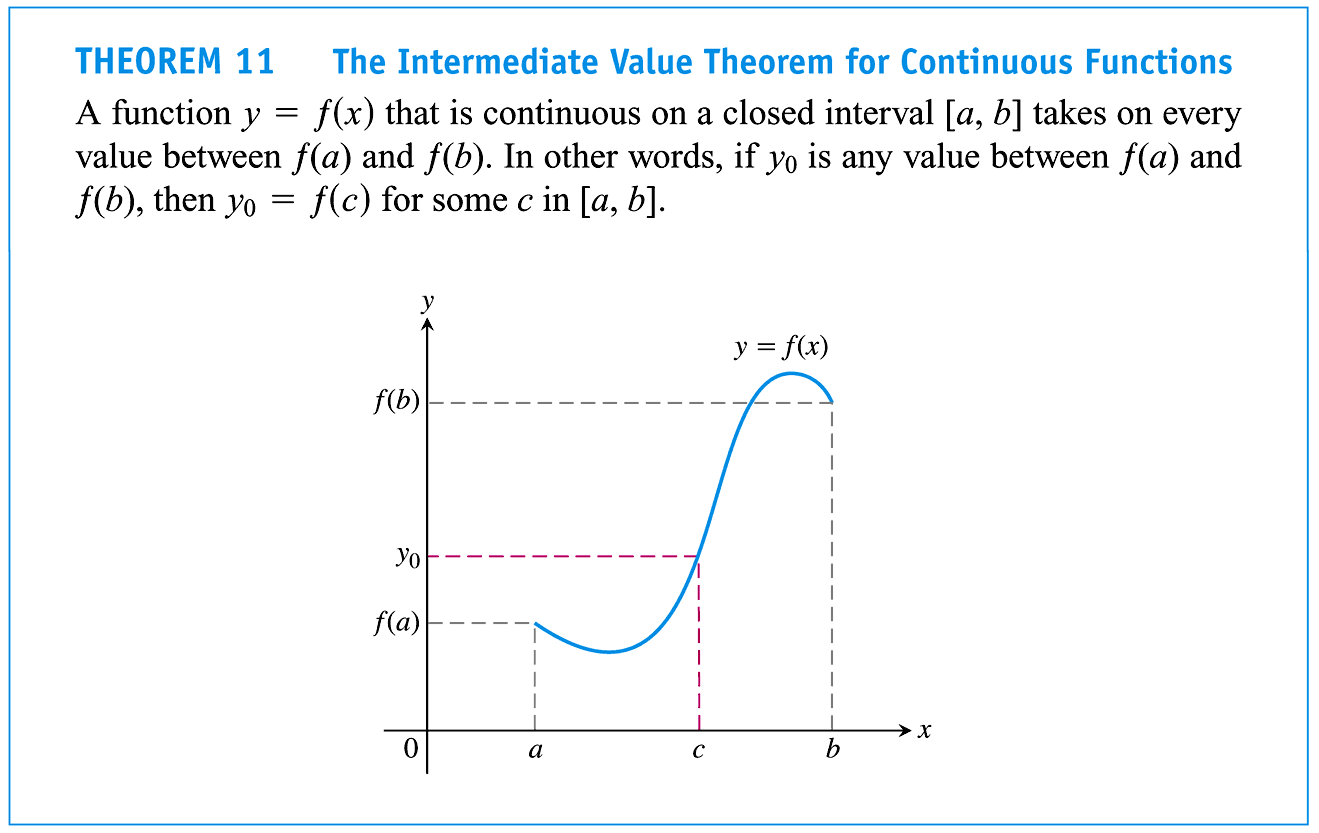
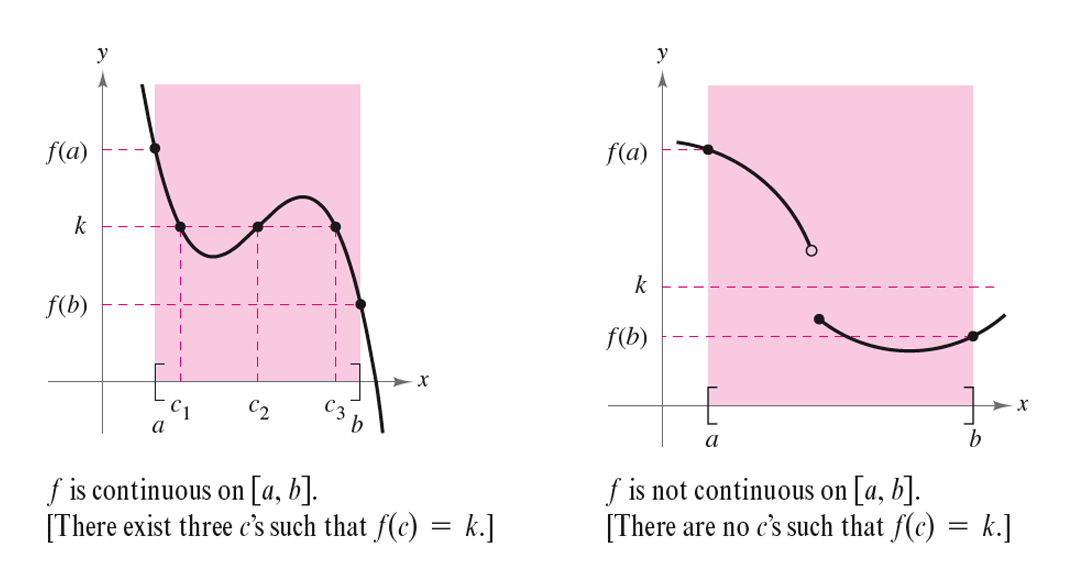
Ex (ex 7 pg 96): Describe the intervals on which each function is continuous:

## The Intermediate Value Theorem

*Theorem* ***Intermediate Value Property for Continuous Functions (IVP)***

A function that is continuous on a closed interval takes on every value between and . That is, if is between and , then we can find in such that .

Functions that have this property are said to have the IVP.



Note: As a consequence of the IVP: If is continuous on and takes on both a positive and a negative value on , then it must have a root/zero.