Ex 1 Find the limit of each. (Graph the functions.)

1.

Consider example a). Although the limit as doesn’t exist, we can at least describe the behavior of near by indicating the limit is . We are not saying limit exists nor are we saying is a real number, but merely that as we approach 3 from the right along the axis, the graph of grows without bound. Examples a) & b) both have infinite limits, but in example c) we can not say it has a limit of since the limits from the right and left do not agree.

In example d) however, although the limit at 0 doesn’t exist, we can at least describe the behavior of on the right and left of 0 by indicating the limit is . We are not saying limit exists nor are we saying is a real number, but merely that as we approach 0 from both sides along the axis, the graph of grows without bound towards .

*Informal Defn*

 means approaches as approaches if grows arbitrarily large for all near .

 means approaches as approaches if grows arbitrarily small for all near .

Note: When we say the limit=infinity, we are not saying the limit exist, but merely that the function is arbitrarily large as approaches c.

Ex 2

Thomas 2.5.16

Ex 3 (# 20) Find limit of as

Ex 4 Let . Find: Note: Do not need graph, look at signs and sizes of numbers.

1.
2.
3.
4.
5. What, if anything, can be said about \_\_\_­­\_\_\_\_? And \_\_\_\_\_\_\_?\_\_\_\_\_\_?

*Defn* A line is a ***\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_*** of the graph of of a function if either or .

Ex 5 (# 36) Sketch the graph of by looking for asymptotes and intercepts. (Refer to ex 3)

Just so we can see the formal def.



## Limits at (this section is discussed until section 4.5)

Find

This limit is asking for the behavior of as grows infinitely large ( ). This is asking for the tendency of the function. If this limit exists, it will approach what we call a horizontal asymptote. If it is infinite, we will know it will grow without bound. In this case the right and left hand side of the function as x grows without bound, the function will approach the same horizontal asymptote, . Can you think of a function where the function will approach a different horizontal asymptote on the left hand side than it will on the right hand side?

*(Informal) Defn of Limits at*

We say that has the ***limit L as x approaches \_\_\_\_\_\_\_\_\_\_\_*** and write if, gets arbitrarily close to (as close as we want) as for all sufficiently large .

We say that has the ***limit L as x approaches \_\_\_\_\_\_\_\_\_\_\_*** and write if, gets arbitrarily close to (as close as we want) as for all sufficiently small .

Ex 3 Find the . (Substitute )

From page 102:



Ex 4 Find the right- and left-hand limits of each.

1. (at )
2. (at )

Ex 5 Find the limit and graph of each. [Show them virtual TI-83 if time permits.]

1. as
2. as

*Defn* A line is a ***\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_*** of the graph of a function if either

 or .

* What is the horizontal asymptote of (the first example of) example 6?
* Can a graph ever cross it’s vertical asymptote?
* Horizontal?

Ex 6 Find

1. What can we say about this limit?

What can we say about these limits?

1.

*Limits of Rational Functions at*

Let be a rational function.

1. If the , then and is a horizontal asymptote.

Ex:

1. If , then and is a horizontal asymptote.

Ex:

1. If , then and is an oblique/slanted asymptote.

Ex:

 Eqn of slant asymptote.

*Defn* In a rational function, if the degree of the numerator is one more than the degree of the denominator, the graph has an ***\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ asymptote***. (The oblique asymptote is found by performing long division.)

At Home Example:

Find the slant asymptote of the function

 Slant Asymptote

 General function behavior

Ex: Find (Sandwich Theorem)

 and by Sandwich theorem