Tangent to a Circle: A line is tangent to a circle if at a point if passes through and is perpendicular to the radius at .



*What does it mean to be tangent to a curve?*

*We can generalize that it means one of the following:*

* *The tangent line, L, touches the curve, C, at only one place P, for all points on the curve “near P”.*

 *Below are pictures of two lines that are tangent to the curve and one line that is not tangent.*



Give an example of a physical interpretation of the tangent line to a curve: (car driving along curve, rock thrown from a sling)

Discussion: Suppose we have a curve and we want to find the tangent to the curve at the point . We can construct secant lines joined by and points near . We choose points that approach . If the resulting slopes of the secant lines have a limit, say , we take it to be the slope of the curve at that point. The tangent line is the line that goes through and has slope .



*Defn* The ***slope of a curve*** at the point is the number

(provided the limit exists)

The ***tangent line*** to the curve at is the line through with this slope.

The expression is called the ***difference quotient of f at with increment h***.

If exists, the limit is called the ***derivative of f at*** .

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Ex 1 Find the derivative of at . What can we conclude? (Easiest derivatives to find.)

Ex 2 Let . Find the slope of the parabola at . Write an equation of the tangent to the parabola at this point.

Ex 3 (T18) Let . Find the slope of the function’s graph at . Find an equation for the line tangent to the graph there. (Outline steps)

EX 4 (T 14) Find the slope of the function at the point **and** find the equation of the tangent line to the curve at that point.

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*Notation*

Some common alternative notations for the derivative of :

The symbols and are called ***differentiation operators*** and indicate the operation of differentiation the same way indicates the operation of addition.

: “the derivative of wrt ”

 or “the derivative of wrt ”

, : “f prime”, “y prime” (respectively)

Ex 5 (T 18) Differentiate the function . Then find an equation of the tangent line at the point .

*Defn* has a ***vertical tangent*** at if or . need for hw

Ex 6

Does have a vertical tangent at ?

Is differentiable on its domain (see pg 122 of Larson 4th ed)?

Does have a vertical tangent at ?

*One-Sided Derivatives*

 is differentiable on if it is differentiable at every point in and if the limits below exist:

***Right-hand derivative at a***:

***Left-hand derivative at b***:

Ex 4 (T 38) Does the function have a derivative at ? Is it continuous at ?

If this curve were to be a road, what would it be like to drive on at x=0? Why?

What does this imply about the relationship between continuity and differentiability?

*Theorem*

If has a derivative at , then is continuous at .

Note: The converse is not true as we saw in example 4.

Proof (If time permits)

WTS: Given that exists at that is continuous at (i.e. that )

*When Does A Function Not Have a Derivative?*

 *BUT*

Continuity Differentiability

Differentiability Continuity

*Differentiable implies smooth*

Ex 5 (T 31)

The graph below is that of . At which points of the interval is not defined?

Graph the derivative of .



Ex: (L #55): On your own or if time.

Identify a function that has the following characteristics: