Ex 1 Differentiate each function. Guess the derivative: Actual derivative:

1.

*Theorem The Chain Rule*

If is differentiable at the point and is differentiable at the point , then the composite function is differentiable at and

.

In Leibniz’s notation, if and , then

 , where is evaluated at .

Note: “Outside-Inside” Rule is helpful way to remember Chain Rule.

If , then .

 Derivative of outside Derivative of Inside

The “outside” function is and the “inside” function is .

To take the derivative of the composite function, , we:

“ take the derivative of the outside **times** the derivative of the inside.”

Note: When we differentiate the “outside” function , we leave alone and plug it into .

You can think of this as a unit conversion as seen in Leibniz’s notation:

* Using the Chain Rule, we see the following:

If is a function of , then .

*The General Power Rule*

, where is a function of .

Note: Regarding the Chain Rule, practice makes perfect. Important to be solid with algebra.

Ex 2 (# 4) Given and , where find 1) find 2) Find

Ex 3 Write the function in the form and . Then find as a function of .

1. (# 14)
2. (# 22)

Ex 4 Find the derivative of each function. Find for part c).

1. (#49)
2. (# 60)
3. (# 38)
4. (# 66)

Ex 5 (# 70) Find the value of at the given value of .

## The Derivative of the Natural Logarithmic Function

*Theorem Derivative of the Natural logarithmic function*

 or

 In particular,

Proof:

Ex 5 Find the derivative of wrt or , as appropriate.

1.
2.
3.
4.
5.

1.
2. (Recall: and )

## Bases Other than e

Definition of Exponential function to Base

If is a positive real number () and is any real number, then the **exponential function to the base**  is denoted by and is defined by

If is the constant function .

Definition of Logarithmic Function to Base

If is a positive real number () and is any real number, then the **logarithmic function to the base**  is denoted by and is defined by

*Theorem Derivatives for Bases other than e*

 In particular,

 In particular,

Proof: First proof

Ex 6 Find the derivative wrt to the independent variable.

1. (# 78) (Use Change of Base – change to base )
2. (# 86) (Use Change of Base – change to base )
3.

## A Brief Discussion on Parametric Equations (Book does not cover this section until later)

Given a curve in 2-space as in Figure 3.29

Describing this curve can be difficult, especially if the curve cannot be described by a

Function. To remedy this, we can use Parametric Equations to describe the curve.

Def: Parametric Curve

If are given as functions

Over an interval of t-values (i.e. on domain of t) then any point in 2-D can be

defined by the equivalent point and in this format the curve is a

**parametric curve**. The equations are **Parametric Equations** for the curve.

A physical example of parametric equations is and etch-a-sketch.

Think of how an etch-a-sketch works. One knob is the parametric equation for the x direction, the other knob is the parametric equation for the y direction. To move the sketcher you must independently rotate (t) the knobs.

Then the position of the sketch is a function of the x-knob and the y-knob.

Ex: Mathematically describe a circle centered at the origin with radius=1 using only one variable.

What we need to do is to use a single variable to describe every point on the circle. We will use the hour hand of a clock moving counter clock wise as the idea. The variable will be the angle of the hour hand.

From Trig recall that any point on a unit circle can be described by the angle the radius makes with the positive x axis.

This is the general idea. So.

The curve described by

Can be described by the parametric equations, and for

To check we plug in our parametric equations into the equation:

Ex: Mathematically describe an ellipse centered at the origin with vertical radius of 2 and horizontal radius of 3.

I.E. Describe the curve

By using parametric equations

Let and for

Then

Ex: The position of a particle moving in the xy-plane is given by the equations and parameter interval

 for

Identify the path traced by the particle and describe the motion.

 So the parametric equations are describing the right half of a parabola w/ vertex at (0,0)

Derivatives of Parametric Curves:

A parameterized curve, is differentiable at if are differentiable at

Derivatives of Parametric Curves:

A parameterized curve, is differentiable at if are differentiable at , and is given by:

Provided that all three derivatives exist and .

Parametric Formula for :

If all the equations

define y as twice-differentiable function of x, then at any point where