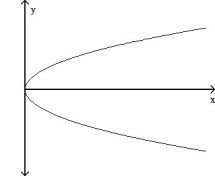
Motivation: Often you will have equations which are not functions, or are not able to be solved explicitly (y=f(x)). This does not mean that we are any less interested in finding derivatives of these curves at given points. So we have Implicit differentiation to help with this.

Ex 1 Consider the curve below. ( is *implicitly* defined) Find by solving for then next finding using “implicit differentiation”. Let . Then .

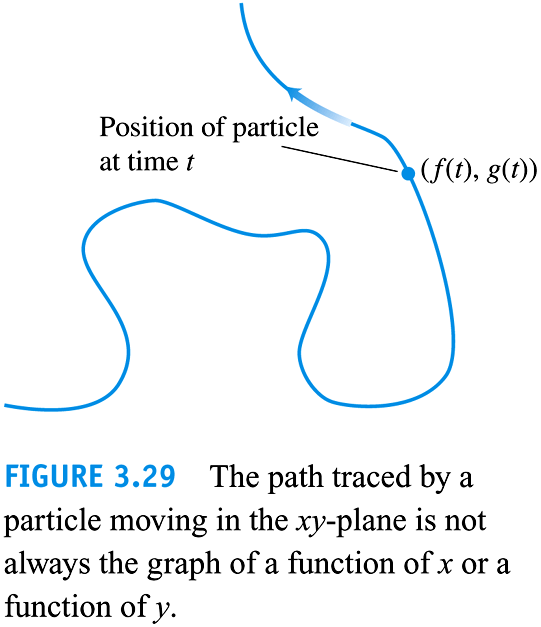
(Note that for , exists.)



so

Note: In some equations that define an *implicit* relation between and , we cannot solve for explicitly as we did above. And even if we could, it might look ugly. See below.

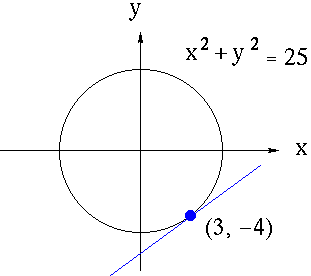
Ex 2 If , then solving for gives and

Find . (Just kidding.)

Another great motivation is if we have a curve which is not a function. For some values of x, we may have two values of y, and at each of these y values we could have a tangent line. These curves will not all have the benefit of being able to be solved explicitly for y.

However, some functions *y* are written IMPLICITLY as functions of *x* . A familiar example of this is the equation

*x*2 + *y*2 = 25 ,

which represents a circle of radius five centered at the origin. Suppose that we wish to find the slope of the line tangent to the graph of this equation at the point (3, -4) .

How could we find the derivative of *y* in this instance ? One way is to first write *y* explicitly as a function of *x* . Thus,

*x*2 + *y*2 = 25 ,

and

where the positive square root represents the top semi-circle and the negative square root represents the bottom semi-circle. Since the point (3, -4) lies on the bottom semi-circle given by

the derivative of *y* is

i.e.,

(this is for when you are looking at points in Quad. III or IV)

Thus, the slope of the line tangent to the graph at the point (3, -4) is

But this could be done easier by using implicit differentiation.

***Implicit differentiation*** is a method where we find without actually solving for . It is just an application of the chain rule.

*Implicit Differentiation*

1) Differentiate both sides of the equation wrt , treating as a differentiable function of .

2) Collect the terms with on one side of the equation.

3) Solve for .

*Let’s try the previous problem using implicit differentiation.*

*D* ( *x*2 + *y*2 ) = *D* ( 25 )

So (which works for points in any quadrant).

Now let’s find the slope of the curve at :

as we anticipated.

Ex: Use implicit differentiation to complete example 2.

Ex 4 (# 26) Use implicit differentiation to find if . (Avoid quotient rule here.)

Ex 6 Find and then if .

Ex 7 Verify that the given point is on the curve and find the lines that are **a)** tangent and **b)** normal to the curve at the given point. (The normal line is perpendicular to the tangent line at the given point.)

## Logarithmic Differentiation

Some Derivatives are easier to take the derivative of if you use properties of logs:

Ex: 3.5.70 Find if Note this would take a quotient rule and perhaps two product rules.

But we can consider this equivalent equation to make it easier to take our derivative:

Now we use implicit differentiation to take this derivative more easily!

EX:

For more good examples see the link below

<http://www.math.ucdavis.edu/~kouba/CalcOneDIRECTORY/implicitdiffdirectory/ImplicitDiff.html#PROBLEM%2014>

I am including the problems from this link below. I recommend you look at the problems and to find the solutions, you should visit the link above.

The following problems range in difficulty from average to challenging.

*PROBLEM 1 :* Assume that *y* is a function of *x* . Find *y*' = *dy*/*dx* for *x*3 + *y*3 = 4 .

*PROBLEM 2 :* Assume that *y* is a function of *x* . Find *y*' = *dy*/*dx* for (*x*-*y*)2 = *x* + *y* - 1 .

*PROBLEM 3 :* Assume that *y* is a function of *x* . Find *y*' = *dy*/*dx* for .

*PROBLEM 4 :* Assume that *y* is a function of *x* . Find *y*' = *dy*/*dx* for *y* = *x*2 *y*3 + *x*3 *y*2 .

*PROBLEM 5 :* Assume that *y* is a function of *x* . Find *y*' = *dy*/*dx* for *exy* = *e*4*x* - *e*5*y* .

*PROBLEM 6 :* Assume that *y* is a function of *x* . Find *y*' = *dy*/*dx* for.

*PROBLEM 7 :* Assume that *y* is a function of *x* . Find *y*' = *dy*/*dx* for .

*PROBLEM 8 :* Assume that *y* is a function of *x* . Find *y*' = *dy*/*dx* for

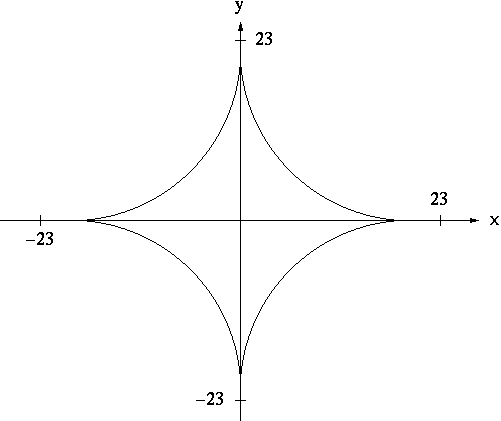
*PROBLEM 9 :* Assume that *y* is a function of *x* . Find *y*' = *dy*/*dx* for .

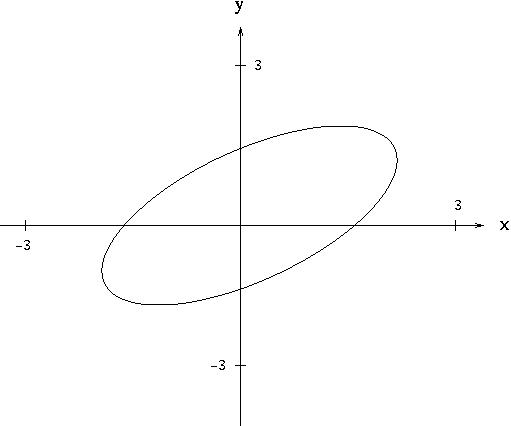
*PROBLEM 10 :* Find an equation of the line tangent to the graph of (*x*2+*y*2)3 = 8*x*2*y*2 at the point (-1, 1) .

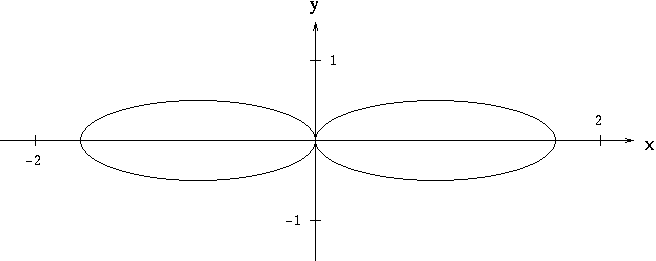
*PROBLEM 11 :* Find an equation of the line tangent to the graph of *x*2 + (*y*-*x*)3 = 9 at *x*=1 .

*PROBLEM 12 :* Find the slope of the graph of *x*2*y* + *y*4 = 4 + 2*x* at the point (-1, 1) .

*PROBLEM 13 :* Consider the equation *x*2 + *xy* + *y*2 = 1 . Find equations for *y*' and *y*'' in terms of *x* and *y* only.

*PROBLEM 14: (similar to problem 3.5.44)*  Find all points (*x*, *y*) on the graph of *x*2/3 + *y*2/3 = 8 (See diagram.) where lines tangent to the graph at (*x*, *y*) have slope -1 .

*PROBLEM 15 :* The graph of *x*2 - *xy* + *y*2 = 3 is a "tilted" ellipse (See diagram.). Among all points (*x*, *y*) on this graph, find the largest and smallest values of *y* . Among all points (*x*, *y*) on this graph, find the largest and smallest values of *x* .

*PROBLEM 16 :* Find all points (*x*, *y*) on the graph of (*x*2+*y*2)2 = 2*x*2-2*y*2 (See diagram.) where *y*' = 0.