We’ll learn 2 theorems that relates a function and its derivative (so that we can make deductions about if we know information about and vice-versa).

* Consider: is a continuous function on and . What do you notice?

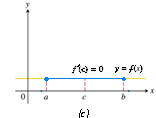
*Theorem Rolle’s Theorem*

Suppose that

1. is continuous on
2. is differentiable on

Then there exists a number in such that .

Geometrically, it makes sense:



Proof:

a)

b)

c) on . Will have any horizontal tangents, if so where?

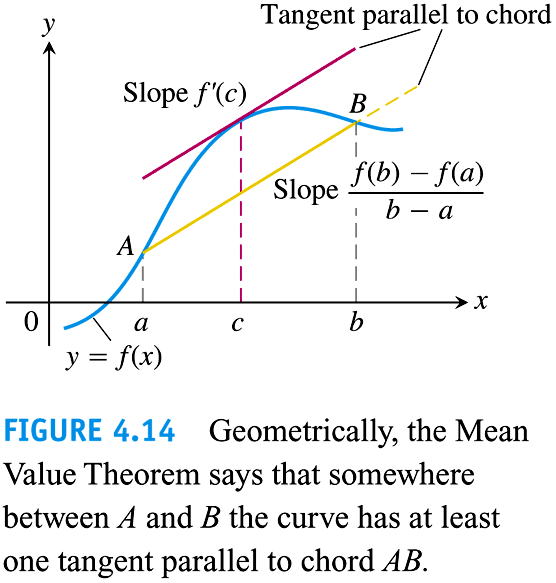
On your own:

Ex (4.2.21) Repeat the above directions for on .

Consider the interval . is cont. on this interval and diff. on the open interval .

By Rolle’s Theorem there exist a number namely such that .

Next, give 3 examples (graphs ok) where is never 0 because one of the conditions in Rolle’s Theorem is not met.



*Theorem Mean Value Theorem (MVT)*

Suppose that

1. is continuous on
2. is differentiable on

Then there exists a in such that .

The MVT says that if conditions 1 and 2 are met, then the slope of the secant line and the slope of the tangent line at some point in the interval are the same.

* Driving example. If represents a position, then MVT says at some point in that time interval, the (instantaneous) rate of change is the same as the average rate of change.

Ex: If you drive between 2 checkpoints on the freeway with an average speed of 80mph, then you can get a ticket for driving this speed because at some point you must have been driving 80 mph.

Ex A trucker handed a ticket at a toll booth showing that in 2 hours she had covered 160 miles on a toll road with speed limit 65 mph. The trucker was cited for speeding. Why?

Ex 2 Determine whether the Mean Value Theorem (MVT) can be applied to on . If so, Find all values of in that satisfy the conclusion of the MVT.

1. (4.2.50) on
2. (4.2.51)

On your own:

Ans: YES, Students must show why. when

Ex 3 Which of the functions satisfy the hypotheses of the MVT on the given interval and which do not? Give reasons for your answer. (Graphing can help here.)

1. on

1. on

1. on
2. on

Ex 4 (5-8) Show that the function has exactly one zero in the interval . HINT:(Use IVP to show it has at least one 0. Next, show the derivative is always + to verify that it cannot have any more than 1 real root.)

*Corollary 1*

If at every point in an open interval , then for all , where is a constant.

This helps us identify where a function is constant (if at all).

*Corollary 2*

If at every point in an open interval , then there exists a constant such that for all . That is, is a constant on .

This tells us that two functions whose derivatives are the same differ only by a constant.

Ex 5 Suppose that and that for all . Must for all ? Give reasons for your answer.

Ex 6 Find all possible functions with the given derivative. (This is a process we will later call Integration)

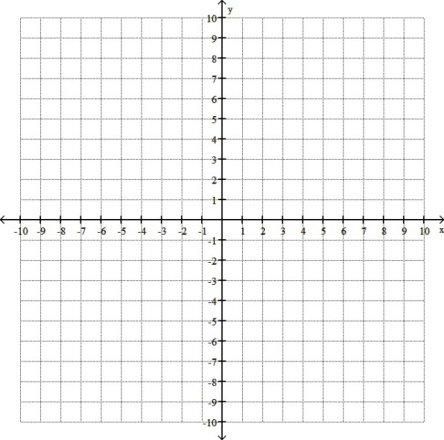
Ex 7 Find a function that has the derivative and whose graph passes through the point (0,1)

If time in class go over the next two examples, if not students should do on their own.

Ex: 4.3.64 & 66 The graph of is given. Sketch the graph of over the top of the graph of .



Ex: 4.3.70 Use the graph of (shown) to sketch the graph of .



Ex: (4.2.73) Determine the values of such that the function satisfies the hypotheses of the MVT on the interval [0,3]

Diff on (0,3):

\*\*\*WRONG\*\*\*

@ x=0:

Correct:

@x=1:

Solving for c from \*\*

Cont. on [0,3]:

@x=0:

&

@x=1:

So \*\*

On you own: Do problem 4.2.74

Determine the values of such that the function satisfies the hypotheses of the MVT on the interval . Where