## Limits at (We discussed this section back in 2.5)

Find

This limit is asking for the behavior of as grows infinitely large ( ). This is asking for the tendency of the function. If this limit exists, it will approach what we call a horizontal asymptote. If it is infinite, we will know it will grow without bound. In this case the right and left hand side of the function as x grows without bound, the function will approach the same horizontal asymptote, . Can you think of a function where the function will approach a different horizontal asymptote on the left hand side than it will on the right hand side?

*(Informal) Defn of Limits at*

We say that has the ***limit L as x approaches infinity*** and write if, gets arbitrarily close to (as close as we want) as for all sufficiently large .

We say that has the ***limit L as x approaches minus infinity*** and write if, gets arbitrarily close to (as close as we want) as for all sufficiently small .

*(Formal) Defn of Limits at*

Let *L* be a real number.

1. The statement means that for each there exists an such that

If for all then

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If for all then

*Defn* A line is a ***horizontal asymptote*** of the graph of a function

if either or .

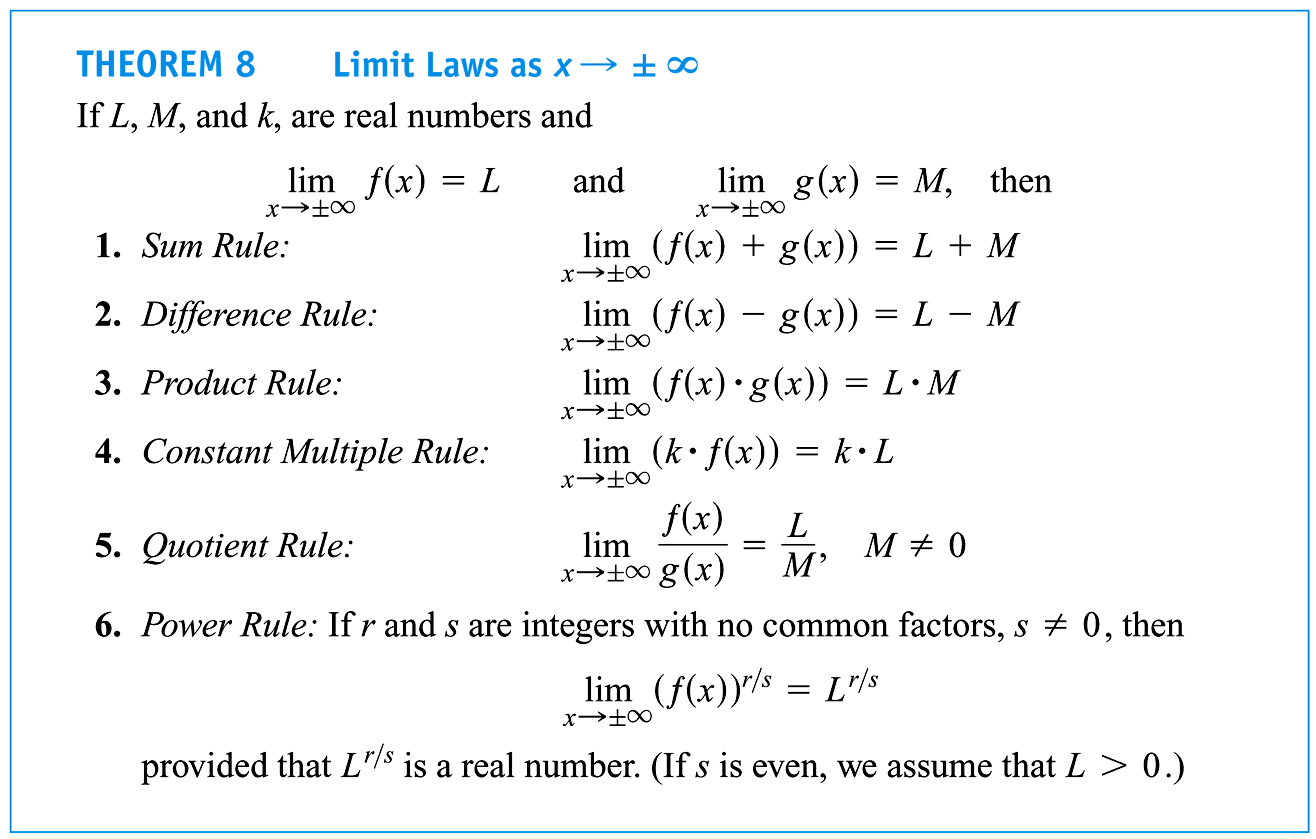
*Limits at*

1. If is a positive rational number and is any real number, then

The second limit is valid only if is defined when .

We say that has the ***limit L as x approaches minus infinity*** and write if, gets arbitrarily close to (as close as we want) as for all sufficiently small .

Ex: Find:



*Limits of Rational Functions at*

Let be a rational function.

1. If the , then and is a horizontal asymptote.
2. If , then and is a horizontal asymptote.
3. If , then and is an oblique/slanted asymptote.

Ex:

Eqn of slant asymptote.

Ex: Find

* What is the horizontal asymptote of the previous example a)?
* Can a graph ever cross it’s vertical asymptote?
* Horizontal?

*Defn* In a rational function, if the degree of the numerator is one more than the degree of the denominator, the graph has an ***oblique (slanted) asymptote***. (The oblique asymptote is found by performing long division.)

Find the slant asymptote of the function

Slant Asymptote

General function behavior (near )

Try At Home Example:

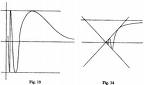
Find the slant asymptote of the function

Slant Asymptote

General function behavior

(near )

Ex Find the limit and graph of each. [Show on virtual TI-83 if time permits.]



1. as
2. as

Ex: 4.5.32 Find (need to employ Sandwich Theorem for part of this)

Ex: Find the limit:

1. 4.5.28
2. 4.5.34
3. 4.5.40
4. 4.5.41

Ex: Sketch the graph of the equation. Look for any extrema, intercepts, symmetry, asymptotes, and curvature.

Assume: