In 2.2 we began to look at functions. In 2.3-2.5 we looked in depth at very specific functions; linear functions. Now in this section we will go back to functions in general, both linear and non-linear. We will consider some very general properties of functions and examine how we add, subtract, multiply, and divide functions. We will also look at what these operations will do to the domain of these sums, differences, products, quotients, of functions.

## The Algebra of Functions

What we mean by this is simply, adding, subtracting, multiplying, and dividing functions.

Ex: Let $f\left(x\right)=x+1 \& g\left(x\right)=x^{2}$

Lets create a new function $(f+g)\left(x\right)=f\left(x\right)+g(x)$.

Question: is $(f+g)\left(4\right)=f\left(4\right)+g\left(4\right)?$ obviously yes

It turns out that adding, subtracting, multiplying, and dividing functions is really intuitive.

## The Algebra of Functions:

If $f and g$ are functions and x is in the domain of both functions, then:

1. $f\left(x\right)+g\left(x\right)=\left(f+g\right)\left(x\right)$
2. $f\left(x\right)-g\left(x\right)=\left(f-g\right)\left(x\right)$
3. $f\left(x\right)∙g\left(x\right)=\left(f∙g\right)\left(x\right)$
4. $f\left(x\right)/ g\left(x\right)=\left(f/g\right)\left(x\right)$ only if $g(x)\ne 0$

Ex: Let $f\left(x\right)=x^{2}-2 \& g\left(x\right)=5-x$. Find:

1. $\left(f-g\right)\left(3\right)$
2. $\left(f+f\right)\left(0\right)$
3. $\left(\frac{f}{g}\right)\left(5\right)$
4. $(f+g)(a)$

Question: What if x is not in the domain of both functions? Can you find and example so we can look at what happens? (if do not recognize any functions like $\frac{1}{x} or x^{\frac{1}{2}}$ then ok to wait on this question)

Now lets consider what happens to the domain and range of these functions.

Ex: Let $f(x)$ be the function that has this graph:

 -1 0 1 3

Let $g\left(x\right)$ be the function that has the graph:

1. What is the domain of $f\left(x\right)? g\left(x\right)? \left(f+g\right)\left(x\right)?$

 $\left\{x:-1\leq x\leq 1\right\};\left\{x:0\leq x\leq 3\right\};\left\{x:0\leq x\leq 1\right\}$ SET NOTATION or $[-1,1]∩[0,3]=[0,1]$

1. How does the domain of $f\left(x\right) \& g\left(x\right) $relate to the domain of$ \left(f+g\right)\left(x\right)?$ (think in terms of the union/intersections of sets)

Ex: Let $f\left(x\right)=x^{2}+4;g\left(x\right)=x-1;h\left(x\right)=x+2;k\left(x\right)=2x+3$

Find the domain of each and also find the domain of $\frac{f}{g}\left(x\right);\frac{f}{h}\left(x\right);\frac{f}{k}\left(x\right);(f-g)(x)$

$$\frac{f}{g}\left(x\right):\left\{x:x\in R \& x\ne 1\right\};\frac{f}{h}\left(x\right):\left\{x:x\in R \& x\ne -2\right\};\frac{f}{k}\left(x\right):\left\{x:x\in R \& x\ne -\frac{3}{2}\right\};$$

$$\left(f-g\right)\left(x\right): \left\{x:x\in R \right\}$$