In previous sections we have been dealing with systems involving two equations of two variables.

Now instead of 2 variables and 2 equations, we solve systems with 3 variables and 3 equations. Since the equation is linear in 3 variables, the graphs are not lines but planes. Also, solutions are *ordered triples*. Here are possible solutions (refer to page 183):

* a unique solution – occurs when 3 planes intersect at exactly one point (figure a)
* infinitely many solutions
* can occur when the 3 planes intersect at all points on a line (figure b) (we won’t have problems like this – will in Linear Algebra)
* can occur when all 3 planes intersect at all points on the plane (all 3 planes are really the same) This is the case where we say the equations are dependent.

Def: A system of equations that has at least one solution is said to be **consistent**.

* no solution
* can occur when all 3 planes are “parallel” to each other (like a stack) (figure c)
* planes intersect but not all three at one or more points (figure d)

Def: A system of equations that has at least one solution is said to be **consistent**.

## Solving Systems of Three Linear Equations

(Using the elimination method)

1. Write all equations in standard form $Ax+By+Cz=D$
2. Clear any decimals or fractions
3. Choose a variable that is easily eliminated. Use any two equations to eliminate that variable.
4. Pick two more equations (at least one can not be the same equation used in step 3). From these two equations eliminate the chosen variable.
5. You should now have a reduced system consisting of two equations of the same two unknowns. Solve this system by your favorite method.
6. Plug in your solutions from step 5 into one of your original equations to find the value of the third unknown.
7. Express your answer as an ordered triple.

Remember,

In order for an ordered triple to be a solution to the system of eqns it must satisfy ALL three eqns!

Ex 1 Determine whether $\left(3,2,1\right)$ is a solution to the system.

$x+y+2z=7$ Yes

$2x+y+z=9$ Yes

$3x+4y-2z=13$ NO 15$\ne $13

Ex 2 Solve each system.

Eliminate x:

1. 1) $x-2y-z=4$ $-2\left(1\right) -2x+4y+2z=-8$ $-1\left(1\right) -x+2y+z=-4$

 2) $2x+3y+z=3$ $ \left(2\right) 2x+3y+z=3$ $\left(3\right) x-y-2z=-1 $

3) $x-y-2z=-1$ A)$ 7y+3z=-5$ (B) $ y-z=-5$

1. $a=c-b$

$$3a-2b+6c=1$$

$$c=4-3b-7a$$

Eliminate c:

1. $a+b-c=0$ 1) $ a+ b -c=0$ 6(1) $6a+6b-6c=0$
2. $3a-2b+6c=1$ 3)$ 7a+3b+c=4$ 2) $3a-2b+6c=1$
3. $7a+3b+c=4$ A)$8a+4b =4$ B) $9a+4b =1$

Eliminate z:

1. 1) $ 6x-2y+2z=2$ 1) $ 6x-2y+2z=2$ 2) $4x+8y-2z=5$

2) $4x+8y-2z=5$ 2) $4x+8y-2z=5$ 2(3) $-4x-8y+2z=-4$

3) $-2x-4y+z=-2$ B) $10x+6y =7$ $0=1$

 No Sol. These two planes are parallel

 d) 1) $-3x+ 4y -z=-4$ 1) $-3x+ 4y -z=-4$ 4(2) $4x+ 8y+ 4z=16$

2) $x+ 2y+ z=4$ 2) $x+ 2y+ z=4$ 3) $-12x+16y-4z=-16$

3) $-12x+16y-4z=-16$ $-2x+6y=0$ $-8x+24y=0$

 $y=\frac{1}{3}x$ $y=\frac{1}{3}x$

 Inf. Sol.