Overview:

* What is a rational function?
* The Evolution of a the numbers lead to fractions (rational numbers) and in the same way the evolution of polynomials leads us to polynomial fractions (rational expressions).
* The great thing about this chapter is that we already know the secrets to how to do it all.
* The bad thing is the secret is knowing how to add subtract multiply and divide with fractions.

Def: An expression that consists of a polynomial divided by a nonzero polynomial is called a **rational expression.** In other words, rational expressions are simply fractional polynomials.

Examples:

## Multiplying Rational Expressions:

First lets recall how to multiply Rational Numbers:

$\frac{3}{2}∙\frac{5}{7}=$ So In general given some integers $a,b,c,d where c,d\ne 0 $ $\frac{a}{b}∙\frac{c}{d}=$

Well Same goes for multiplying rational expressions:

$$\frac{x}{2}∙\frac{x^{2}}{y}=$$

 And also in general, given some polynomials $a,b,c,d where c,d\ne 0 $ $\frac{a}{b}∙\frac{c}{d}=$

Ex: multiply these rational expressions:

$\frac{2x-3}{3y+2}∙\frac{4x^{2}}{y-1}=$

What is the best/easiest way to simplify this multiplication: $\frac{12}{5}∙\frac{15}{2}$

Ex: Multiply these rational expressions:

$$\frac{(x-3)}{(2x+1)}∙\frac{(x-7)}{(x-3)}$$

## Simplifying Rational Expressions

First lets examine how we simplify fractional numbers.

Simplify: a) $\frac{108}{45}$ b) $\frac{36}{27}$ c) $\frac{5}{100}$

Now simplify:

1. $\frac{x^{2}}{3x^{5}}$ b) $\frac{\left(x+7\right)^{2}}{\left(x+7\right)^{3}}$

 c) $\frac{\left(x+1\right)\left(x+2\right)}{(x+2)}$ d)$\frac{x^{2}-1}{x^{2}+2x+1}$

 e) $\frac{8t^{2}-4t}{4t^{2}}$ e) $\frac{9x^{2}+6xy-3y^{2}}{12x^{2}-12y^{2}}$

 f) $\frac{3-x}{x-3}$ g) $\frac{x}{x+1}$

Rational functions are functions that are described by rational expressions.

Examples:

Ex:

Find the domain of the rational function $f\left(x\right)=\frac{\left(x-1\right)\left(x-2\right)}{\left(x-3\right)\left(x+4\right)\left(x-5\right)}$

Ex: Consider the function:$ f\left(x\right)=\frac{x^{2}-3x+2}{\left(x-2\right)}$

1. Find $f\left(0\right),f\left(1\right),f(2)$
2. Find the domain of the function
3. Simplify the rational function
4. Find the domain of the simplified rational function.
5. Why are they different, how can we fix it?

Are the functions $f\left(x\right)=\frac{x^{2}-x-30}{x-6}$ & $g\left(x\right)=x-5$ with restricted domain $\{x:x\ne 6\}$ equivalent?

Why would we want to restrict a simplified functions domain?

Ex: Consider the function given by $M\left(t\right)=\frac{3t^{2}-2t}{4t-2}$, which describes the time, in hours it takes for two machines, working together to complete a job that one machine could do alone in $t$ hours and the other machine could do in $3t-2$ hours.

1. Find the time it will take the two machines working together to complete a job in the time, t, one machine could do in 2 hours.
2. Find the time it will take the two machines working together to complete a job in the time, t, one machine could do in 5 hours.
3. Find the domain of $M(t)$

(translation): $t$ is the time it takes for one machine to do a certain job. $M(t)$ is the time it takes for both machines to do that job.

1. Find M(2) b) Find M(5) c) Find where M(t) does not exist.

Solution:

## Dividing Rational Expressions

First lets remember how to do it with rational numbers (fractions)

$$\frac{1}{2}÷\frac{3}{5}$$

So dividing rational numbers is really just the same as \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Now lets try it with rational expressions.

Ex:

Divide and simplify your answer.

$$\frac{4a^{2}-1}{a^{2}-4}÷\frac{2a-1}{a-2}$$

Ex:

Simplify the rational function and be sure to list all restrictions to the domain.

6.1.74 $g\left(n\right)=\frac{n+5}{n-5}÷\frac{n^{2}-1}{2n+2}$

Things to remember:

* Rational functions are just fractions with polynomials
* You multiply them straight across
* You divide by multiplying by the reciprocal
* You can cancel out common factors in the numerator and denominator (magic one’s)
* If you have a function and you cancel terms, you MUST restrict the domain of your function
* You don’t have to do this for expressions (things without the = sign)