At certain times we may wish to know, what can I multiply $(x+1)$ by to get $(x^{4}+4x^{3}+6x^{2}+4x+1)$.

This sounds very similar to the question what can I multiply 3 by to get 72? The answer of course lies in long division.

In this section we will learn how to perform long division on polynomials.

We will focus on two situations.

1. Dividing a polynomial by a monomial
2. Dividing a polynomial by another polynomial

## Dividing a polynomial by a monomial

To divide a polynomial by a monomial we regard the division as a sum of quotients of monomials.

In the division problem $\frac{x^{5}+2x^{4}-3x^{3}}{x^{2}}$ we are asked to recognize the fact that $x^{2}$ is called a common denominator. This means it is a denominator that is common to all terms. So we will regard the division as a sum of quotients of monomials.

$$Since\frac{A}{C}+\frac{B}{C}=\frac{A+B}{C}, we know that \frac{A+B}{C}=\frac{A}{C}+\frac{B}{C}$$

Ex: 6.6.10 $\frac{-25x^{3}+20x^{2}-3x+7}{-5x}$

## Division by a Monomial

To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

Ex:6.6.11 $\frac{16y^{4}z^{2}-8y^{6}z^{4}+12y^{8}z^{3}}{-4y^{4}z}$

## Dividing a polynomial by a polynomial

To do this, we will perform long division. This is the exact same procedure that we use for long division with numbers, however it will feel a little more awkward since it involves variables.

To understand this process lets look closely at how long division works for numbers:

Consider the long division problem $\frac{108}{9}$

Lets use this as a model for what we need to do to perform long division with polynomials.

Ex: Divide and check your answer$\frac{x^{2}+2x+1}{x+1}$.

Ex: Divide and check your answer. $\frac{3x^{2}-4x-15}{x-3}$

Now lets look at two division problems where they do not divide evenly:

$\frac{15}{4}$ $\frac{110}{9}$

Now lets try it with polynomials:

Ex: 6.6.30 $\frac{3x^{3}-5x^{2}-3x-2}{x-2}$

Ex: 6.6.28 $\frac{x^{3}-2x^{2}+4x-5}{x+3}$

Ex:6.6.32 $\frac{x^{3}-x+6}{x+2}$

Ex:6.6.58 $\frac{x^{4}-x^{3}y+x^{2}y^{2}+2x^{2}y-2xy^{2}+2y^{3}}{x^{2}-xy+y^{2}}$

Ex: 6.6.44 Find a simplified expression for $F\left(x\right)=\frac{f\left(x\right)}{g\left(x\right)}$ be sure to list all restrictions on the domain.

$$f\left(x\right)=x^{4}-3x^{2}-54, g\left(x\right)=x^{2}-9$$