In this sections we will discuss

* Multiplying Radical Expressions
* Simplifying by Factoring
* Multiplying and Simplifying

## Multiplying Radical Expressions

## The Product rule for Radicals:

For any real numbers $\sqrt[n]{a}$ and $\sqrt[n]{b}$,

$$\sqrt[n]{a}∙\sqrt[n]{b}=\sqrt[n]{a∙b}$$

I.E. The product of the nth roots is the nth root of the product

Proof:

$$\sqrt[n]{a}∙\sqrt[n]{b}=a^{1/n}∙b^{1/n}=\left(a∙b\right)^{1/n}=\sqrt[n]{a∙b}$$

Note: when no index is written on a radical symbol it is understood to be a square root.

Ex: $\sqrt[3]{2}\sqrt[3]{5}$ $\sqrt{x-2}\sqrt{x+2}$ $\sqrt[4]{\frac{y}{5}}\sqrt[4]{\frac{7}{x}}$ $\sqrt[3]{2}\sqrt[4]{5}$

$$\sqrt[3]{2}\sqrt[4]{5}$$

$$=\sqrt[4]{\frac{7y}{5x}}$$

$$=\sqrt{x^{2}-4}$$

$$=\sqrt[3]{2∙5}=\sqrt[3]{10}$$

Recall $\left(x+y\right)^{2}\ne x^{2}+y^{2}$ and so also $\left(x+y\right)^{1/2}\ne x^{1/2}+y^{1/2}$ which means $\sqrt{x^{2}+4}\ne \sqrt{x^{2}}+\sqrt{4}$

## Simplifying by Factoring

Square roots are really good (it is their only purpose) at one thing, Finding what number squared gives you the radicand. So it takes radicands and simplifies things that are squared.

Cube roots are really good (it is their only purpose) at one thing, Finding what number cubed gives you the radicand. So it takes radicands and simplifies things that are cubed.

Fourth roots are really good (it is their only purpose) at one thing, Finding what number to the fourth power gives you the radicand. So it takes radicands and simplifies numbers raised to the fourth power.

Nth roots are really good (it is their only purpose) at one thing, Finding what number raised to the nth power gives you the radicand. So it takes radicands and simplifies things that are raised to the nth power.

## Using the product rule to simplify:

$$\sqrt[n]{a∙b}=\sqrt[n]{a}∙\sqrt[n]{b}$$

For any real numbers $\sqrt[n]{a}$ and $\sqrt[n]{b}$,

I.E. The product of the nth roots is the nth root of the product

$\sqrt{12}$ $\sqrt{200}$ $\sqrt{18x^{3}}$ $\sqrt{20x^{8}y^{6}z^{3}}$ $\sqrt[3]{-72}$ $\sqrt[4]{162x^{6}}$

$$\sqrt[4]{81x^{4}}∙\sqrt[4]{2x^{2}}=3x\sqrt[4]{2x^{2}}$$

Ex: $\sqrt[7]{2^{8}x^{9}y^{14}z^{22}}$

Remember: To simplify an nth root, identify factors in the radicand with exponents that are multiples of that root, n.

## Multiplying and Simplifying

What if we have a product where each factor does can not be simplified. Does that mean the product can not be simplified?

Consider the example $\sqrt{6}\sqrt{15}$

Neither factors can be simplified, however, when multiplied, we discover that each factor had a square root of 3. When multiplied this made a product which has a square root of $ 3^{2}$ which can simplify.

So $\sqrt{6}\sqrt{15}=\sqrt{6∙15}=\sqrt{2∙3∙3∙5}=3\sqrt{10}$

Recall that we saw this with rational expressions too.

$$\frac{x-2}{x+3}∙\frac{4}{x-2}$$

Individually neither factor can be simplified, but after we multiply them we find a common factor which can be simplified.

So
$$\frac{x-2}{x+3}∙\frac{4}{x-2}=\frac{4}{x+3}$$

Ex: $\sqrt[3]{16ab^{2}}∙\sqrt[3]{a^{2}b^{14}}$

Ex: $\sqrt[4]{8a^{3}b^{5}}∙\sqrt[4]{4a^{2}b^{3}}$