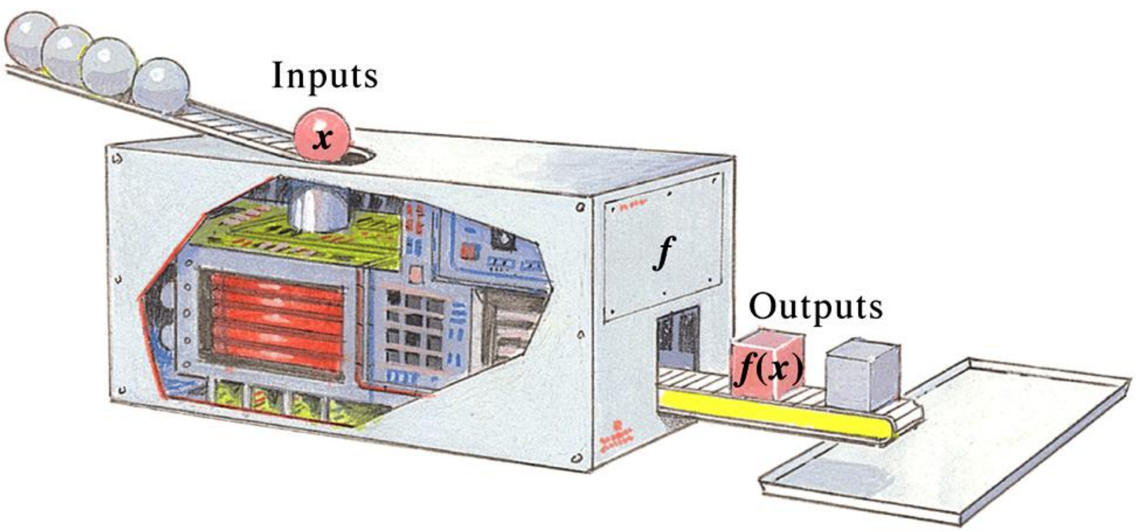
In this section we will discuss

- Functions, Composite Functions,
- Inverses, Inverse functions, Horizontal line test for to determine if Inverses are functions.
- Finding formulas for Inverses
- Finding Graphs of functions and their inverses

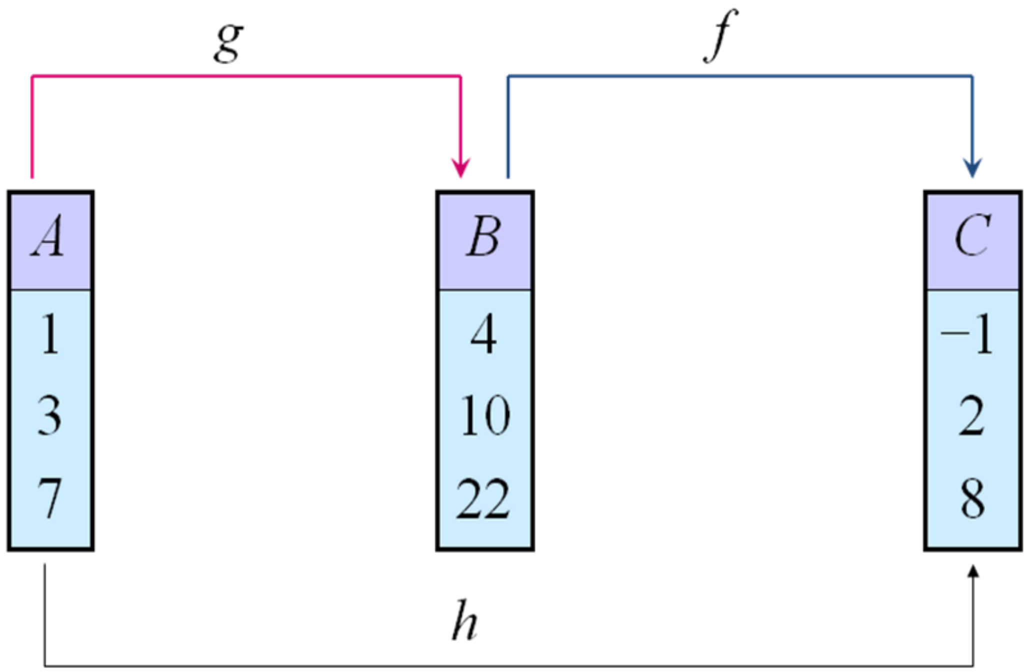
Functions & Composite Functions

A *function* is a special kind of correspondence between two sets. The first set is called the **domain**. The second set is called the **range**. For any member of the domain, there is *exactly one* member of the range to which it corresponds. This kind of correspondence is called a **function**.



The function pictured has been named f. Here x represents an arbitrary input, and $f(x)$ – read "f of x," "f at x," or "the value of f at x" represents the corresponding output.

In the real world, functions frequently occur in which some quantity depends on a variable that, in turn, depends on another variable. Functions like this are called **composite functions.**

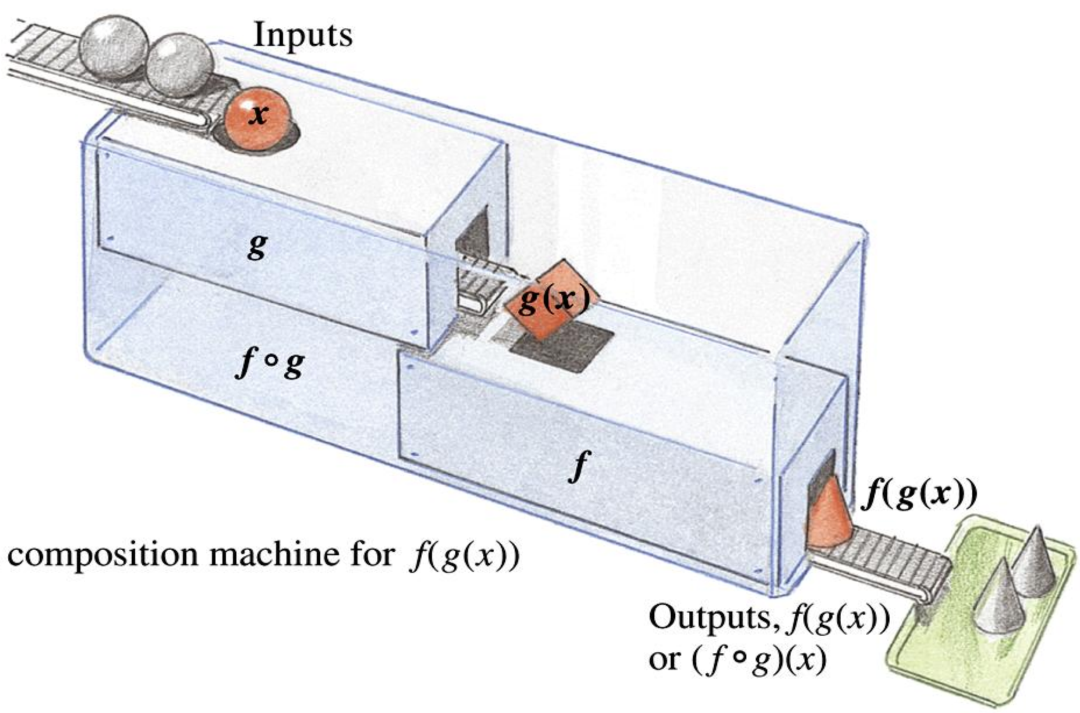


$h(x) = ?$

Composition of Functions

The *composite function* $f \circ g$, the *composition* of f and g, is defined as

$$(f \circ g)(x) = f\left(g(x)\right).$$



A composition machine for $f(g(x))$

Ex: Given $f(x) = \sqrt{x}$ and $g(x) = x + 3$ find

a) $f(g(1))$ b) $(g \circ f)(1)$ c) $(g \circ g)(1)$ d) $f(g(x))$ e) $(g \circ f)(x)$

Inverses, Inverse functions, Horizontal line test for to determine if Inverses are functions.

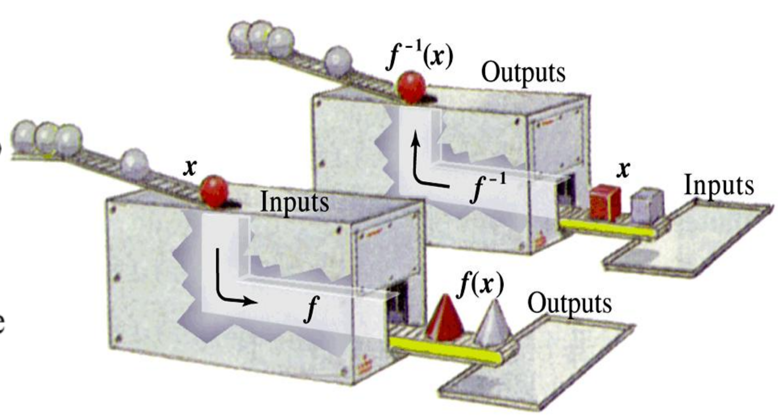
Consider the Relation NFL Teams to Hometowns and sports teams to Hometowns.

Note both are functions, both have inverses, but only one has an inverse that is a function.

What keeps that function from having an inverse that is a function?

What does an inverse function do?

Let's consider inverses of functions in terms of function machines. Suppose that a one-to-one function f, has been programmed into a machine. If the machine has a reverse switch, when the switch is thrown, the machine performs the inverse function, f^{-1}. Inputs then enter at the opposite end, and the entire processed is reversed.



The graph of an inverse function is the reflection across the line $y = x$

Graphing functions and their Inverses:

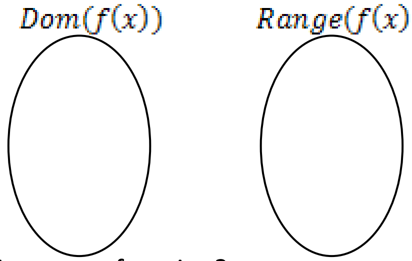
Graph the function $g(x) = -2x$ $\quad h(x) = x^3$ $\quad k(x) = x^4$ $\quad f(x) = x^2 + 2x - 4$ and then sketch its inverse.

Of these inverses which ones are functions? What do all the ones whose inverse is a function have in common and what does the one(s) whose inverse is not a function have that makes it not a function?

One-To-One Functions

A function f is *one-to-one* if different inputs have different outputs. That is, if for a and b in the domain of f with $a \neq b$, we have $f(a) \neq f(b)$, then the function f is one-to-one. If a function is one-to-one, then its inverse correspondence is also a function.

Ex: Graph $f(x) = x^2$ Is this function 1-1? What does its inverse look like? Is it a function? Why not, and how does that relate to the function being or not being 1-1? $\quad Dom(f(x)) \quad Range(f(x))$



Why did the Vertical line test work to test if an equation was a function?

Is there some way to tell if the graph of a function will have an inverse that is a function?

The Horizontal Line Test

If it is impossible to draw a horizontal line that intersects a function's graph more than once, then the function is one-to-one. For every one-to-one function, an inverse function exists.

Finding formulas for Inverses

Again, what does an inverse function do?

To Find a Formula for f^{-1}

First make sure that f is one-to-one. Then:

1. Replace $f(x)$ with y.

2. Interchange x and y. (This gives the inverse function.)

3. Solve for y.

4. Replace y with $f^{-1}(x)$. (This is the inverse function notation.)

Ex: Determine if each function is 1-1 and if it is, find $f^{-1}(x)$, its inverse function.

a) $f(x) = x + 5$ b) $g(x) = 3x + 2$ c) $h(x) = \frac{x-2}{3}$ d) $k(x) = x^2 + 2x - 4$

When the inverse of f is also a function, it is denoted f^{-1} (read "f-inverse").

Caution! The -1 in f^{-1} is *not* an exponent!

Suppose a function is described by a formula. If its inverse is a function, we proceed as follows to find a formula for f^{-1}.

Composition and Inverses

If a function f is one-to-one, then f^{-1} is the unique function for which

$$\left(f^{-1} \circ f\right)(x) = f^{-1}\left(f(x)\right) = x \quad \text{and}$$
$$\left(f \circ f^{-1}\right)(x) = f\left(f^{-1}(x)\right) = x.$$

Ex: Verify through composition that examples b) & c) are inverses (use composition).