In this section we will discuss

- Different Properties of Logs
- Using the different properties together

Properties of Logs

Product Rule for Logarithms

For any positive numbers M,N and $a, a \neq 1$

$$Log_a(MN) = Log_a M + Log_a N$$

The logarithm of a product is the sum of the logarithms of the factors.

Proof:

$$Log_a(MN) = Log_a M + Log_a N \rightarrow a^{Log_a M + Log_a N} = MN$$

We need to show that this is true.

$$a^{Log_a M + Log_a N} = a^{Log_a M} \cdot a^{Log_a N} = MN$$

Ex: Express as an equivalent expression that is a sum of logarithms.

a) $Log_3(81 \cdot 27)$ b) $Log_5(25 \cdot 125)$ c)$Log_x(3yw)$

Ex: Express as an equivalent expression that is a single logarithm (it is the previous question but going backwards

a) $Log_3(x) + Log_3(y)$ b) $Log_w(4) + Log_w(5)$ c)$Log_3(7) + Log_3(7)$

Power Rule for Logarithms

For any positive numbers M,N and $a, a \neq 1$

$$Log_a(M)^P = P \cdot Log_a M$$

The logarithm of a product is the sum of the logarithms of the factors.

Proof:

$$Log_a(M)^P = P \cdot Log_a M \rightarrow a^{P \cdot Log_a M} = (M)^P$$

We need to show that this is true.

$$a^{P \cdot Log_a M} = (a^{Log_a M})^P = (M)^P$$

Ex: $Log_a(M^3)$ $= Log_a(M \cdot M \cdot M) = Log_a M + Log_a M + Log_a M = 3Log_a M$

Ex: Express as an equivalent expression that is a product

a) $Log_a r^8$ b) $Log_2 x^{1/3}$ c) $Log_b 3^{-3}$

Power Rule for Logarithms

For any positive numbers M,N and $a, a \neq 1$

$$Log_a\left(\frac{M}{N}\right) = Log_a M - Log_a N$$

The logarithm of a product is the sum of the logarithms of the factors.

Proof:

$$Log_a\left(\frac{M}{N}\right) = Log_a M - Log_a N \rightarrow a^{Log_a M - Log_a N} = \frac{M}{N}$$

We need to show that this is true.

$$a^{Log_a M - Log_a N} = a^{Log_a M + Log_a N^{-1}} = MN^{-1} = \frac{M}{N}$$

Ex: Express as an equivalent expression that is a difference of two logarithms.

a) $Log_2\left(\frac{4}{32}\right)$ b) $Log_3\left(\frac{29}{13}\right)$ c) $Log_a\left(\frac{y}{x}\right)$

Using the Different Properties Together

Ex: Express as an equivalent expression using the individual logarithms of $w, x, y, \& z$

a) $Log_a(w^2x^{-2}y)$

b) $Log_b\left(\frac{w^2x}{y^3z}\right)$

c) $Log_a\sqrt[4]{\frac{x^8y^{12}}{a^3z^5}}$

Obvious or not obvious?

The Logarithm of the Base to an exponent
For any base a,

$$Log_a a^k = \_\_\_\_\_\_\_\_\_\_$$

Note:

$$Log_a(M \cdot N) = Log_a M + Log_a N$$
$$Log_a\left(\frac{M}{N}\right) = Log_a M - Log_a N$$
$$Log_a(M)^P = P \cdot Log_a(M)$$

BUT

$$Log_a(M + N) \neq Log_a M + Log_a N$$
$$Log_a(M - N) \neq Log_a M - Log_a N$$
$$Log_a(M \cdot N) \neq Log_a(M) \cdot Log_a(N)$$
$$Log_a\left(\frac{M}{N}\right) \neq \frac{Log_a(M)}{Log_a(N)}$$