Math 400 Extra Credit on Newton's Method.

Name:_____

Score: /(10pts) (Will only be graded by instructor during finals week)

Purpose: To give you experience using Microsoft Excel and understanding Newton's Method. To utilize this software to find solutions to equations by Newton's Method.

Assignment:

Due Date: This assignment is due at the beginning of class on the day of the third exam.

1. ☐ Check box when you have Read section 3.8 and complete the HW. Attach this HW separately to this assignment.

2. (1 pts) Use the Intermediate value theorem and the first derivative test to determine two intervals containing the two zeros of the polynomial below.

3. (3pts) Use Excel to help you apply Newton's Method to find all the zeros (there are two real zeros) of the function

   $$y = x^4 - 2x^3 - 2x + 2$$

4. (1pt) What conditions must the function have in order for Newton's method to converge on a root?

5. (1pts) Can Newton's method appear to converge to a root when the value it actually converges on is not a root? If so, how can you tell if the value is a root of the function?

6. (3pts) Use Excel to help you use Newton's Method to find the intersections of the functions

   $$y_1 = Tan(x) \ \& \ y_2 = 2x$$

   over the interval $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

   (hint: Normally you would set the equations equal to each other and solve. However this is a very difficult equation to solve, so we can employ Newton's Method to solve this equation by considering the very useful/reoccurring equation

   $$g(x) = y_1 - y_2$$

   and finding its zeros. You may also be interested in making a quick sketch of the two functions to see how many intersection points there will be.)

7. (1pts) Verify that each root you find is indeed an intersection point. I.E. Be sure Newton's method has not diverged or converged upon one of those rare points of convergence which is actually not a root.

Calculate all answers until $g(x_n) < 1.0*10^{-12}$ (i.e. g(x) is is less than $1.0*10^{-12}$ or is nearly 0).

Format for your solutions:

1. Complete all the assigned parts, and attach your HW to the back of this document

2. Copy the wording for each of the assigned tasks and then show your work/answer/solutions below it.

3. Include your excel spread sheets in a format which includes AT LEAST the necessary values required to generate your next iterative guess. You may wish to add more information which will help you to make logical sense of your work and help you to be sure that you are actually answering the question; if you do be sure to include it as well. For tasks 3 & 6, b**e sure to show tables for and find all the roots**. For tasks 3 & 6 include the table used to find each root include. Then include one table for task 3 and one table for task 6 which shows the formulas used in your tables.

4. Also include all hand work (especially for problems 3 & 6), graphs (you may want to graph them to get an idea of where and how many roots there are), or written explanations that you feel will best explain the idea of what you are doing.

5. Make this assignment VERY NEAT. I will not spend ANY time trying to figure out your thought process or deciphering your work.

If you have any questions or need some help do not hesitate to come talk to me in office hours or via email. I will not grade this assignment until the very end of the semester.

Example problem

Find the positive root of $f(x)=x^2 - 2$ I.E. find the square root of 2.

	A	B	C	D	E
		f(x) =	$x^2 - 2$		f '(x) = 2x
	n	x_n	$f(x_n)$	f '(x_n)	x_{n+1}
7	0	1	-1	2	1.5
8	1	1.5	0.25	3	1.416666667
9	2	1.416666667	0.006944444	2.833333333	1.414215686
10	3	1.414215686	6.01E-06	2.828431373	1.414213562
11	4	1.414213562	4.51E-12	2.828427125	1.414213562
12	…	1.414213562	0	2.828427125	1.414213562
13	Root of f(x)				
	x =	1.414213562		sqrt(2) =	1.4142136

		f(x) =	$x^2 - 2$		f '(x) =2x
	n	x_n	$f(x_n)$	f '(x_n)	x_{n+1}
18	0	1	"=(B18)^2 - 2"	"=2*B18"	"=B18-(C18/D18)"
19	1	"=E18"	"=(B19)^2 - 2"	"=2*B19"	"=B19-(C19/D19)"
20	2	"=E19"	"=(B20)^2 - 2"	"=2*B20"	"=B20-(C20/D20)"
21	3	"=E20"	"=(B21)^2 - 2"	"=2*B21"	"=B21-(C21/D21)"
22	4	"=E21"	"=(B22)^2 - 2"	"=2*B22"	"=B20-(C22/D22)"
23	…	"=E22"	"=(B23)^2 - 2"	"=2*B23"	"=B20-(C23/D23)"
	x =	"=E23"		sqrt(2) =	"=SQRT(2)"