Goals: To understand the meaning of a fraction with regards to multiplication.

* What does it mean to divide with fractions?
* Illustrating division with fractions
* Important Fact: Division with fractions is the same as multiplication with the divisors reciprocal

# What does it mean to divide with fractions?

Recall, that division is repeated subtraction, that in the division problem

 $\frac{24}{6}=4$

24 is call the dividend

6 is called the divisor

4 is called the quotient

And the problem is asking the question, “How many 6’s are in 24?”

So then what does it mean when we see the division problem

$$10÷\frac{1}{2}$$

Division has not changed, it still means the same thing: “**How** many $\frac{1}{2}^{'}s$ are there in **10** **whole things**?”

Lets draw a picture of what this question is asking.



Show all the $\frac{1}{2}'s$ and then just count how many there are. How many are there?

Ex: Fill in the blanks to complete the sentence describing the fundamental question that the division problem is asking.

1. For $3÷\frac{1}{4}$, then **use the picture** to determine the quotient of $3÷\frac{1}{4}$.

\_\_\_\_\_\_\_ many \_\_\_\_\_\_\_\_ are there in \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_­­­­­\_\_\_\_\_\_\_\_\_?

 Answer: \_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. For $2÷\frac{2}{3}$, then **use the picture** to determine the quotient of $2÷\frac{2}{3}$.

\_\_\_\_\_\_\_ many \_\_\_\_\_\_\_\_ are there in \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_­­­­­\_\_\_\_\_\_\_\_\_?

 Answer: \_\_\_\_\_\_\_\_\_\_\_\_\_\_

Definition: Let $a,b $be any non-zero numbers. We say the **reciprocal** of the fraction $\frac{a}{b}$ is the fraction$\frac{b}{a}$.

Fact: The product of every non-zero fraction and its reciprocal is always 1.

Example: Find the reciprocal of the following fractions and what is the product of the fraction and its reciprocal?

1. $\frac{3}{5} $ Reciprocal: \_\_\_\_\_\_\_, Product: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
2. $\frac{7}{3} $ Reciprocal: \_\_\_\_\_\_\_, Product: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
3. $\frac{2}{3} $ Reciprocal: \_\_\_\_\_\_\_, Product: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

We have noticed that in the problem $10÷\frac{1}{2} $dividing by $\frac{1}{2} $is the same as multiplying by 2

2 is called the reciprocal of $\frac{1}{2}$

 $10÷\frac{1}{2}=10∙$\_\_\_\_\_\_, also $4÷\frac{1}{6}=4∙$\_\_\_\_\_\_, & $2÷\frac{2}{5}=10∙$\_\_\_\_\_\_,

What does it look like then to divide a fraction by a fraction?

Example:

1. For $\frac{7}{3}÷\frac{1}{3}$, then **use the picture** to determine the quotient of $\frac{7}{3}÷\frac{1}{3}$.

\_\_\_\_\_\_\_ many \_\_\_\_\_\_\_\_ are there in \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_­­­­­\_\_\_\_\_\_\_\_\_?

 Answer: \_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. For $\frac{8}{9}÷\frac{4}{9}$, then **use the picture** to determine the quotient of $\frac{8}{9}÷\frac{4}{9}$.

\_\_\_\_\_\_\_ many \_\_\_\_\_\_\_\_ are there in \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_­­­­­\_\_\_\_\_\_\_\_\_?

 Answer: \_\_\_\_\_\_\_\_\_\_\_\_\_\_

What we have noticed in our division problems is not a coincidence, it is just yet another way in which we can interpret division with fractions.

Example:

# Important Fact:

Let $a,b,c,d$ be non-zero numbers.

When dividing with fractions, $\frac{a}{b}÷\frac{c}{d}$, it is the same as multiplication by the divisors reciprocal.

$$\frac{a}{b}÷\frac{c}{d}=\frac{a}{b}∙\frac{d}{c}$$

Example: Divide the two numbers. Reduce your answer.

1. $\frac{-3}{2}÷\frac{2}{5}$ b) $\frac{15}{8}÷\frac{40}{16} $ c) $\frac{100}{27}÷\frac{500}{9}$

Notice, again, before you multiply, it is best to divide out all the “magic one’s” FIRST.

1. $-\frac{22}{13}÷\frac{11}{-39}$ e) $\frac{-12}{35}÷\frac{28}{70}$ f) $34÷\frac{17}{-8}$

Challenge problem. Let $a$ be a specific but unspecified non zero number (we call this a constant)

$-\frac{4a^{2}}{5}÷\frac{8a^{3}}{5^{2}}= $