Goals: To understand the meaning and application of Proportions.

- Purpose and value of using proportions
  - Definition and application
- Solving Proportions
  - Extremes vs. Means Theorem
  - Using proportions to solve application problems.

Purpose and Value of Using Proportions:

Definition:
A **proportion** is defined as a statement that equates two ratios or rates. So, a proportion is a statement that says that two ratios or rates are equivalent fractions.

$$\frac{a}{b} = \frac{c}{d}$$

Also, *if two ratios are equal*, we say they are **in proportion**, and are therefore **equivalent fractions** too.

For example, we know that the fraction $\frac{4}{8}$ can be reduced to $\frac{1}{2}$. This means that $\mathbf{\frac{4}{8} = \frac{1}{2}}$. This true statement is actually considered a **proportion** because it is equating two ratios or rates.

We might consider an application involving pay. Say that we get paid the same rate every day. One day we worked for 2 hours and earned $20 and then on another day we worked 5 hours and earned $50. This information can be written as a proportion in two different ways.

$$\frac{\$20}{2\ hrs.} = \frac{\$50}{5\ hrs.} \quad \text{or} \quad \frac{2\ hrs.}{\$20.} = \frac{5\ hrs.}{\$50.}$$

Notice the importance or where we write the units! When we wrote the first rate with the dollar amount on top and hours on the bottom, we needed to write the equivalent fraction with the dollar amount on top and hours on the bottom. In the second proportion, we included the hours on top and the dollars on bottom. Both proportions are accurate representations of the information in the problem!

Can you think of two examples that could represent proportions?
1.

2.

Exploring Proportions:

Develop: Proportions can be used to solve many interesting problems! To understand how, we need to explore the facts about proportions! Let's start with a couple of basic proportions!

How can we determine that the following statements are true?

$\frac{4}{8} = \frac{1}{2}$ $\frac{5}{6} = \frac{30}{42}$ $\frac{6}{9} = \frac{20}{30}$

Note: In each of the above proportions, we were able to easily show by multiplication that the statements are true. What if we encounter a proportion where this is not true?

Ex. $\frac{4}{8} = \frac{1}{2}$ This proportion represents a fact. We were able to show that above! Let's consider another property that is true about this proportion. Take a look at what we get if we cross-multiply the numbers in this fraction.

$\frac{4}{8} = \frac{1}{2}$

4 x 2 = 1 x 8

8 = 8

Interesting! When we cross multiply the values in the proportion, we get equal values! Let's try it in a second example!

Ex. $\frac{\$20}{2\ hrs.} = \frac{\$50}{5\ hrs.}$

20 x 5 = 2 x 50

100 = 100 It worked again! Is this always true?

Let's try and start with our formal definition of a proportion,

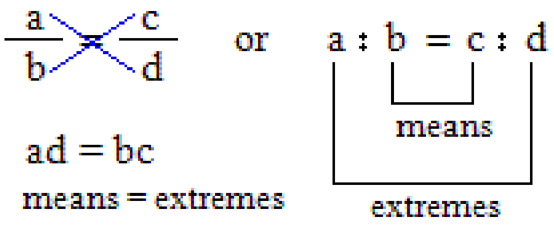
$\frac{a}{b} = \frac{c}{d}$ Now, let's multiply both sides of the equation by $\frac{bd}{1}$.

$\frac{bd}{1} \cdot \frac{a}{b} = \frac{c}{d} \cdot \frac{bd}{1}$ As long as we multiply both sides of the equation by the same value $\left(\frac{bd}{1}\right)$, our equation is still true! Now, after simplifying, we have the following.

$a \cdot d = b \cdot c$ So, it wasn't a coincidence that we got equivalent values when cross multiplying within our proportion. This fact is called the "Extremes vs. Means Theorem".

Extremes vs. Means Theorem:

More formally, for any proportion $\frac{a}{b} = \frac{c}{d}$, the product of the means (a and d) is equivalent to the product of the extremes (b and c). In other words,



Checking the Validity of a Proportion:
We can use the Extremes vs. Means Theorem to see if a proportion is valid! If a proportion is true, then the product of the means must be equal to the product of the extremes. Check the following proportions to see if they are valid.

1. $\frac{3}{4} = \frac{15}{20}$ 2. $\frac{3}{7} = \frac{4}{8}$ 2. $\frac{9}{7} = \frac{4}{3}$ 3. $\frac{12}{13} = \frac{36}{39}$

Solving Proportions:

Some proportions can be solved using basic math. Others we need to solve using the Means vs. Extremes Theorem.

Examples:

1. **Fill in the blank to make each proportion true.**
   a. $\frac{75}{100} = \frac{\quad}{400}$ b. $\frac{4}{20} = \frac{36}{\quad}$ c. $\frac{15}{18} = \frac{\quad}{72}$

How do we set up proportions?

Examples:

Use a proportion to set up each of the following problems. Then use the Means-Extremes Theorem to solve each proportion.

Note: *When setting up proportions, we need to be very careful that our units line up.*

1. Robbie went to the store to buy hamburgers for the family barbeque. The hamburgers were on sale for $3.99 for 4 burgers. Robbie needed 22 burgers for the barbeque. How much money did he spend?

2. Amanda read 18 pages in 23 minutes. At this rate, how many pages will she read in 45 minutes?

3. After working 22 hours in your new job, your paycheck is $223. Next week, you are scheduled to work 31 hours. How much should your paycheck be?