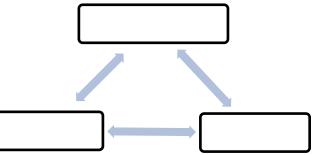Goals: To better understand the meaning of percents, and their relationship with fractions, proportions and decimals.
- The Meaning of Percent.
- Percents relationship with fractions and decimals
  - Changing percents to fractions
  - Changing percents to decimals
  - Changing decimas to percents
  - Changing fractions to percents

The Meaning of Percent

Interestingly but perhaps not surprisingly the word percent was taken from the Latin adverbial phrase "Per Centum" meaning "by the hundred" or "for each 100". The term Per is a preposition meaning for each or for every, or by means of, where as centum is Latin for the number 100.
So literally translated percent is "for every 100"

Today we use the term to represent the amount that is present for every 100 of that amount. So it is a ratio or fraction,

$$\frac{your\ part/amount}{for\ every\ 100} = \frac{part}{100} = \text{"part per 100"} = percent$$

The meaning of Percent: the part out of 100

Example: 42% means $\frac{42}{100}$, or forty two hundredths. Literally 42% is a ratio of 42 out of 100 things.
42 problems correct out of 100 possible problems, 42 dollars off for every 100 dollars, etc.

Example: if you score 15 correct out of 20 questions on a test, then you scored 14/20 on that test.

The real thing we will want to know is how good is that grade? What does that grade mean? Is it an A,B,C,D, or F? Because that will help you understand the true meaning of your exam grade.

So rather than write your grade out of 20, we can write it equivalently as _____/100, and this score now is standardized into a format that you already understand, whereas 14/20 is not a meaningful ratio.

Definition: Percent is defined mathematically as a ratio or fraction with a denominator of 100.

Ex1   Translate each percent literally to a fraction in simplest terms. (just remember what percent means!)

a) 27%   b) 7%   c) 173%   d) 211%   e) 75%

Now consider the example 90%

From here, we can do many things,

- if we reduce the fraction to simplest form, then what we have is equivalent fractions or a proportion.

  o $90\% \; by \; definition = \frac{90}{100}$

  By simply expressing the percent in its literal form, we have "converted the percent to a fraction"

  However, it is common to reduce fractions to simplest form

  $$90\% \; by \; definition = \frac{90}{100} = \frac{9}{10}$$

  So we could say that 90/100 is proportional to 9/10 or in other math words we could say that they are just equivalent fractions, but again, in this section we say we are "converting the percent to a fraction". However, now, you and I both know it's the same thing.
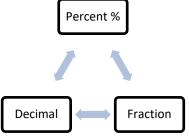
- if we divide, then what we have done is to "convert the percent to a decimal".

  $90\% \; by \; definition = \frac{90}{100} = \frac{9}{10} = $

$$
\begin{array}{r}
0.\,9 \\
10\overline{)9.0} \\
-\underline{0} \\
9\,0 \\
-\underline{9\,0} \\
0
\end{array}
= 0.9
$$

- of course if we started with a decimal (either terminating or repeating decimal), we could always express that as a fraction, and then if we created an equivalent or proportionate fraction with a denominator of 100, then we would be converting from decimals to percents!

And this embodies the true meaning and relationships between percents, decimals, and fractions.

Changing percents to fractions

To change a percent to a fraction, we need only remember the definition or meaning of a percent.

a) 35%   b)   125%   c) $11\frac{1}{9}\%$ d) 87.5 %

Changing percents to decimals

To change a percent to a decimal, we only need to again remember how a percent is defined, and then remember how to divide a number by 100.

Change the following percents to decimals.

a) 27%   b) 7%   c) 215%

d) 0.9%   e) $\frac{1}{4}\%$ f) $12\frac{1}{2}\%$

Changing decimals to percents

To change a decimal to a percent one need only recall what a percent is, a part out of 100, and what equivalent fractions are, two fractions that represent the same number.

 Change the following decimals to percents.

a) 0.09   b) 0.65   c) 1.7   d) 0.0375

e) 7.025   f) $0.\overline{6}$

Changing Fractions to percents

Again, if you recall the meaning or definition of a percent, then this task is the same as finding an equivalent fraction but with a denominator of 100. And then the fraction that you are given and the one you find are simply equivalent fractions or proportions.

There are really two ways to think of this.

Method 1: Find a magic one to make your fraction equivalent to a part per 100 and express it as a percent.

Example $\frac{3}{5} = \frac{x}{100} \rightarrow \frac{3}{5} \cdot \frac{20}{20} = \frac{60}{100}, so \frac{3}{5} = 60\%$

Method 2: Just divide your numbers and convert the following decimal to a percent.

Example $\frac{1}{3} = 0.3333 ... = 0.3333 ... \cdot \frac{100}{100} = \frac{33.\overline{3}}{100} = 33.\overline{3}\%$

Change the following fractions and mixed numbers to percents.

a) $\dfrac{16}{100}$

b) $\dfrac{3}{4}$

c) $\dfrac{1}{20}$

d) $\dfrac{7}{10}$

e) $\dfrac{6}{25}$

f) $2\dfrac{3}{5}$

g) $7\dfrac{11}{20}$

h) $\dfrac{5}{9}$

i) 5/6

j) $1\dfrac{2}{6}$