

Adding and subtracting in Alternate Bases

Each Base has its own set of digits for which all numbers in that base are made of. The digits are:

Base Ten = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

Base two = {0, 1}

Base four = {0, 1, 2, 3}

Base six = {0, 1, 2, 3, 4, 5}

Base eight = {0, 1, 2, 3, 4, 5, 6, 7}

Base Three = {0, 1, 2}

Base five = {0, 1, 2, 3, 4}

Base seven = {0, 1, 2, 3, 4, 5, 6}

Base nine = {0, 1, 2, 3, 4, 5, 6, 7, 8}

Place value:

Each base has a different set of place values. They all start with a ones place, then each new place value to the left of the previous one is a multiple of that base.

Example: Base ten: one, $10^1 = \text{ten}$, $10^2 = \text{hundred}$, $10^3 = \text{thousand}$, $10^4 = \text{ten thousand}$, etc.

Base two: one, $2^1 = \text{two}$, $2^2 = \text{four}$, $2^3 = \text{eight}$, $2^4 = \text{sixteen}$, $2^5 = \text{thirtytwo}$,...

Writing numbers in base ten, you may notice the repetitive pattern of our numbers.

Fill in the table below, and pay attention to any patterns you notice in writing our numbers

| | | | |
|---|----|----|--|
| 0 | 10 | 20 | |
| 1 | 11 | | |
| 2 | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| 9 | | | |

Can you describe the way the repeating pattern works?

Does this pattern exist in other bases?

Fill out this table of numbers in base five.

Note the only symbols we have for digits are $\{0,1,2,3,4\}$

| | | | | |
|---|----|----|--|--|
| 0 | 10 | 20 | | |
| 1 | 11 | | | |
| 2 | | | | |
| | | | | |
| 4 | | | | |

Do you notice any patterns in writing these consecutive numbers? Is it a familiar pattern similar to base ten?

Is there a similar pattern in base 3?

Fill out the table below.

| | | | | |
|---|----|----|--|--|
| 0 | 10 | 20 | | |
| 1 | 11 | | | |
| 2 | | | | |

What about base two?

Fill out the table below

| | | | | | | | | |
|---|----|-----|--|--|--|--|--|--|
| 0 | 10 | 100 | | | | | | |
| 1 | 11 | | | | | | | |

This may seem different because of how quickly we jumped to a third and fourth place values, but is there a repeating pattern?

Now go back and try adding in these bases.

a) $8_{ten} + 3_{ten} =$

b) $4_{five} + 3_{five} =$

c) $4_{five} + 2_{five} =$

d) $4_{five} + 4_{five} =$

e) $2_{three} + 2_{three} =$

f) $2_{three} + 1_{three} =$

There is an underlying relationship here between the sum, the way you write that sum, and the base you are in. Can you think of any ways that they all fit together?