Goals:

- What are Numbers? Have people always used the same ones?
    - We will discuss historically how societies represented quantities of objects, with numbers
- Tally marks, Ancient Egyptian numbers, Ancient Roman Numbers, Classical Chinese Numbers
    - Simple grouping systems and modified simple grouping systems
    - Multiplicative grouping systems,
    - Positional number systems.
- Base/grouping size

What are Numbers?

At it very core, numbers are symbols that are used to represent the notion of quantity.

The different ways in which we can represent various quantities or numbers is through a **NUMBER SYSTEM**.
A number system will consist of symbols that represent numbers which are called **numerals or digits**.
It will also consist of a set of rules for which to construct different quantities with these numerals or digits.

Our own number system is called the Hindu-Arabic number system, it is a positional system and will be discussed further in the next section.

Early examples of number systems consisted of:
Tally sticks: |||| -four, ||||||||| - nine, etc. This is a simple grouping number system.

In simplest form a **Simple grouping system** is an assignment of symbols for, the unit number, one group of that unit number (often ten, but could be 20 as in Mayan numbers, or 2 in Binary numbers), a group of the group and so on for as many symbols are required to represent desired quantities.

Early on grouping became useful as it is difficult to recognize a number at a glance with tally marks.

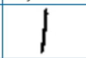
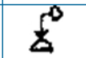
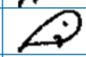
As you can see, grouping is essential to easily recognizing and potentially writing numbers

||||||||||||||| vs.  - fifteen

Ancient Egyptian Numeration

This is an example of a simple grouping system. The size of the groups are in a base of ten, as the groups consist of powers of tens (ie, tens, tens of tens, tens of [tens of tens], etc). They used symbols for single objects, | -one, and groupings of objects like ten - ∩, one hundred - 𓍢, one thousand - 𓆼, up to one million - 𓁨. You could use nine repeated digits but on the tenth, a new symbol was created.

Let's look at the symbols:

Decimal Number	Egyptian Symbol	What it is supposed to be
1 =		staff
10 =		heel bone
100 =		coil of rope
1000 =		lotus flower
10,000 =		pointing finger
100,000 =		tadpole
1,000,000 =		astonished man

Examples: convert the Egyptian numbers

EX 1: 

EX 2: 

Examples: convert the numbers to Egyptian numbers

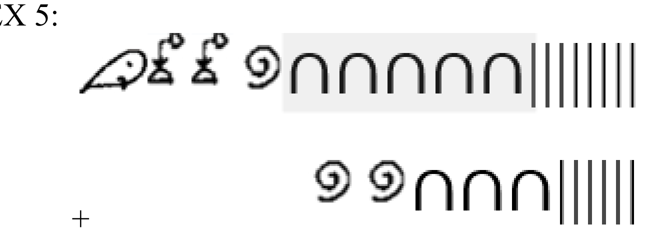
EX 3: 2,031,472

EX 4: 95,184

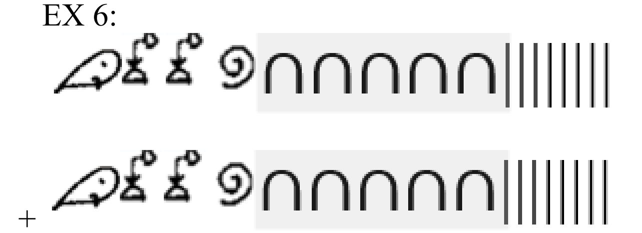
How do the operations of addition, subtraction, multiplication and division work in this number system?

Adding:

EX 5:

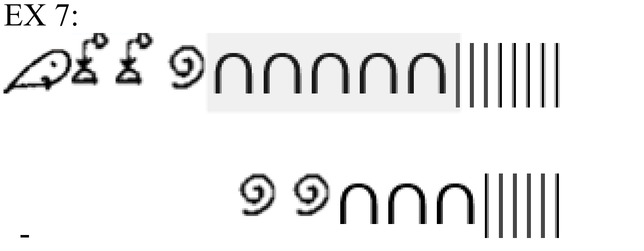


EX 6:

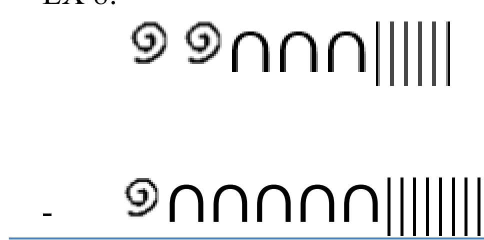


Subtracting:

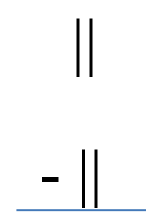
EX 7:



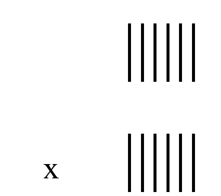
EX 8:

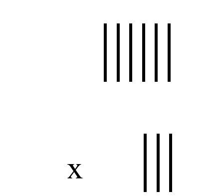


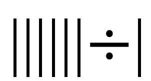
EX 9:



Multiplication and Division:







Ancient Roman Numeration

We are somewhat familiar with Roman Numerals today as they still show up on watch/clock faces, Super Bowl edition numbers, and other places mostly serving a design esthetic than for ease.

Roman Numeration is a modified *simple grouping system*. It uses simple grouping, the groups are mostly of base ten but adds some extra rules and symbols.

For example, III - three but IV- four (not IIII as would be expected of a true simple grouping system)

First let's look at the grouping sizes:

Roman Numeral	Number
I	1
V	5
X	10
L	50
C	100
D	500
M	1000

These are the main numerals/symbols used in this system, the exception being that adding a Bar over the numeral multiplies the value of the number by 1000. A double bar over the numeral multiplies the number by $1{,}000^2$ *or* $1{,}000{,}000$. Like tally marks and this system has no symbol for zero.

The rules are as follows:

- Numbers are read from Left to Right
- Symbols should not be repeated past three (though it was common practice to break this rule) Ex: four should not be IIII as it breaks the rule of three.
- If a smaller number precedes a larger one, then the smaller number is meant to subtract from the larger. Ex IV –four
  - In this rule I precedes only V or X
  - X is preceded by only L or C
  - C is preceded only by D or M

  Examples:

  45 is written XLV and not as VL

  95 is written XCV and not VC

  490 is written CDXC and not XD

  999 is written CMXCIX and not IM
- If a bar is place over a numeral(s) then multiply the value by 1,000.
- A double bar indicates that the value is multiplied by 1,000,000.

  Examples:

  15,090 - $\overline{XV}XC$

  4,532,444 - $\overline{\overline{IV}}\,\overline{DXXXII}CDXLIV$

Examples:
Express the following numbers in Roman Numerals

84 - __________ 99 - _________ 454 - ___________ 3,445 - ___________ 4,969 - __________

Addition:

II + III = _________ XC + L = __________ XLVII + LIII = _________ LXXXIX + I = __________

Subtraction:

III – II = _______ L – X = _______ C – X = __________ $\bar{V} - \text{I}$ = ___________________

Multiplication and Division:

Egyptian Multiplication Algorithm:

EX 10: A room is XXI by XXX cubits, find the area of the room.

Soln:	1	**30**	8	**240**
	2	**60**	16	**480**
	4	**120**		

Notice:

The bold faced numbers are multiples of 30 or rather they are double each previous bold faced number.

If one wished to find three time thirty one could express three as the sum of the multiple 1 + 2.

Ex: 3 x 30 = (1+2) x 30 this will be 1***30 + 2*30 = 30 + 60** = 90.

Ex: 7 x 30 = (1+2+4) x 30 = **30+60+120** = 210.

So 21in these terms = (1+4+16), thus 21 x 30 = **30+120+480** = 630 = DCXXX square cubits

EX 11: Find the area of a room which measures XII by LIII cubits.

ANS: DCXXXVI

Observations? Strengths and Weaknesses?

Classical Chinese Numeration

This system of numeration is the predominant Chinese version, though other versions existed. This system in contrast to the others is called a multiplicative grouping system. From our perspective it may feel like a more evolved system which is getting closer to our own number system, even though it is still far different.

The numerals used in this system can be expressed similarly to this, though one should note that there are other versions of these numerals.

零	一	二	三	四	五	六	七	八	九
0	1	2	3	4	5	6	7	8	9
十	百	千	万						
10	100	1000	10000						

Example:

873 = 八百七十三
8*100 + 7*10 + 3

Another version uses a more familiar symbol for zero

Other versions use a symbol similar to the Greek letter λ for eight.

The rules for numeration in this system are as follows:

- The numbers are written top to bottom in vertical lines, though for ease of these notes I will express them Left to Right (your book shows examples written vertically.
- Numbers start with the largest values and then move down (or start L and move R in these notes)
- The numbers come in pairs of symbols, the first symbol is a single numeral valued from 0-9, the second numeral is a multiplier for that numeral.
    - The numbers are additive by the grouping of powers of ten. Ex: **873** = 八百七十三 **8*100 + 7*10 + 3**
    - If the multiplier is 1, then no multiplier will be present and will be represented by not a pair but a single numeral (as seen with the 3 in the previous example and the following example. Ex: 七 十 六 - 7 tens and 6 ones or 76.
    - In the tens pair, if the multiplier is 1, the symbol for 1 can be ommitted.

- Each power of ten will be represented in the number, if a particular power of the base (ten) is totally missing, the symbol for zero will appear as a place holder.
    - If more than one power of the base is missing then one single symbol for zero will hold the place of all the missing values. EX: 五 千 零 四 = 5*1000+0*100+0*10+4 = 5004
    - If there are no ones, tens, and/or any other values is missing from the end of the number, then a the numeral for zero need not be denoted at all. Ex: 三 万 = 3*10,000 + 0 = 30,000

Examples:
Write the given Hindu-Arabic numbers in the Traditional Chinese form (use the vertical format).

9: 42: 875: 7654: 9050: 38,600: 78,103:

Can you think of another way of expressing the numbers that is simpler or may not require as many digits to represent them?

For reference: These are some example historical number systems.