Goals:

- What is a positional number system? How does it differ from simple grouping systems?

Positional values and bases for the following number systems:

- The Hindu-Arabic Number system
- Babylonian Numeration system
- Mayan Numeration system
- Greek Numeration system

Recall the Classical Chinese Numeration system:

零	一	二	三	四	五	六	七	八	九
0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
十	百	千	万						
10	**100**	**1000**	**10000**						

Example:

873 = 八百七十三
8*100 + 7*10 + 3

To express the number 873, this multiplication number system requires us to multiply a number by its grouping value for each group size.

In contrast, the simple grouping system would just have us repeatedly express the number 100 eight times, the number 10 seven times and the number 1 three times. This would require us to use eighteen symbols to write this number in the Egyptian number system.

However the Chinese system is able to reduce the number of required symbols down to five!

The next step in this progression is to use position to IMPLY a multiplied value instead of explicitly MULTIPLING each value.

If we adapt the Chinese system to a positional system, and Imply that the symbol at the end of the number is worth 1, the next symbol moving to the left is worth ten, then 100, etc. we could express 873 using three symbols and a positional number system.

八七三 = eight (hundreds) + seven(tens) + three(ones)

This is the foundation of the positional number system. There are symbols/numerals for all counting numbers up to the grouping number or BASE number, and the position of the numerals will represent the size of that group's value.

Hindu-Arabic Numeration

This is an example of a positional grouping system. Like the Egyptian number system, the size of the groups are in a base of ten, as the groups consist of powers of tens (ie, tens, tens of tens, tens of [tens of tens], etc). but unlike the Egyptian number system, the Hindu-Arabic number system uses the position of the numeral to give the size or value to it.

The value of the places starting from the position furthest to the right is valued at 1 and each position to the left is worth ten times the last. The accepted symbols or numerals are 1,2,3,4,5,6,7,8,9 and the keystone to the system is the symbol 0, which can serve as a placeholder if there are none of that grouping number present. This means we have the symbols 0-9 (ten numerals).

The place values are all powers of the base or grouping number, which is ten.

MathATube.com

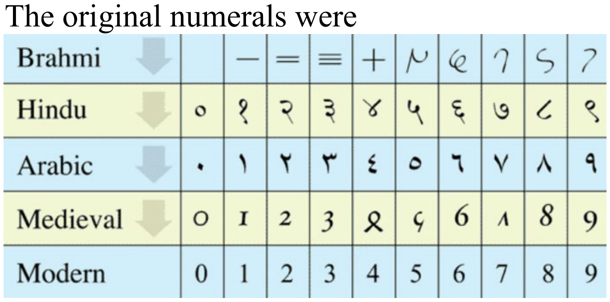
Place Value Chart

Hundred-billions	Ten-billions	Billions	Hundred-millions	Ten-millions	Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Ones

Examples:
In the spaces of the chart write the number

a) 5 million, three hundred fifty seven thousand one hundred thirty four.
b) Four billion and two
c) Fifteen thousand eight hundred

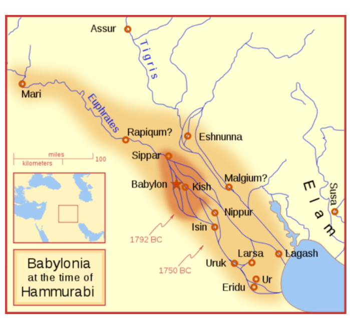
The original numerals were



Every society used slightly different symbols, but the structure was identical.

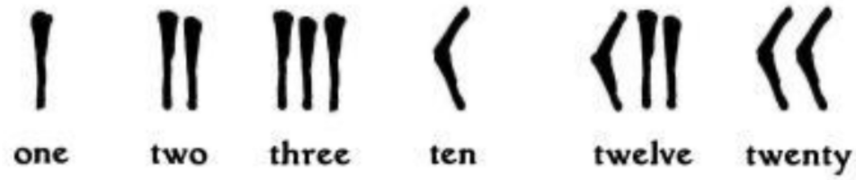
Babylonian Numeration

Ancient Babylon was located in present day Iraq 59 miles southwest of Baghdad. It is mentioned in the bible and is said to be where the tower of babel was constructed. All that is left is ruins, but it is said that its heavily vegetated walls and terraces were one of the an impressive sight at the time.



The Babylonian numeration system from about 4000 BC, at which point it could be said that it was the Sumerian system, is a positional number system where the base or grouping numbers are powers of base 60.

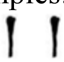
Though this would imply that there were 60 unique symbols, originally in fact the numerals for 1-59 were actually represented with two symbols a vertical stroke and a horizontal wedge.



These numerals would be an example of a simple grouping system, within a positional system. By the year 2000 BC it is thought that the Babylonians had more symbols which were adaptations of these.

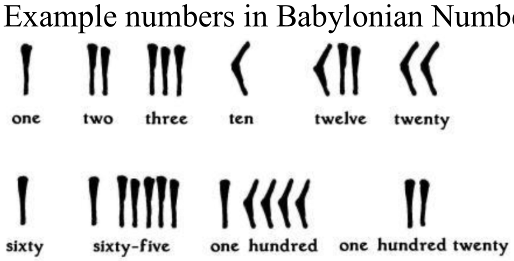
The grouping or base of this number system was 60, so the place values were 1,60,60*60=3600,60*60*60=216000,etc.

______ ______ ______ ______
216000 3600 60 1

Examples:
61 =  2= 

One very interesting missing digit was the place holder, or zero. This left for interpretive issues or ambiguities when writing numbers like 1= , 60=  , 3600=  , and the like (did you notice the space after the vertical strike is larger on 60 and even larger on 3600 than on the 1? No? Well that's a problem).

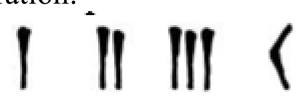
Example numbers in Babylonian Numbers



Examples:
Express the following Hindu-Arabic numbers in Babylonian numeration:

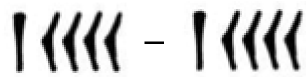
57: 540: 3599: 84,562: 500,000:

Express the following Babylonian numbers in Hindu-Arabic numeration:

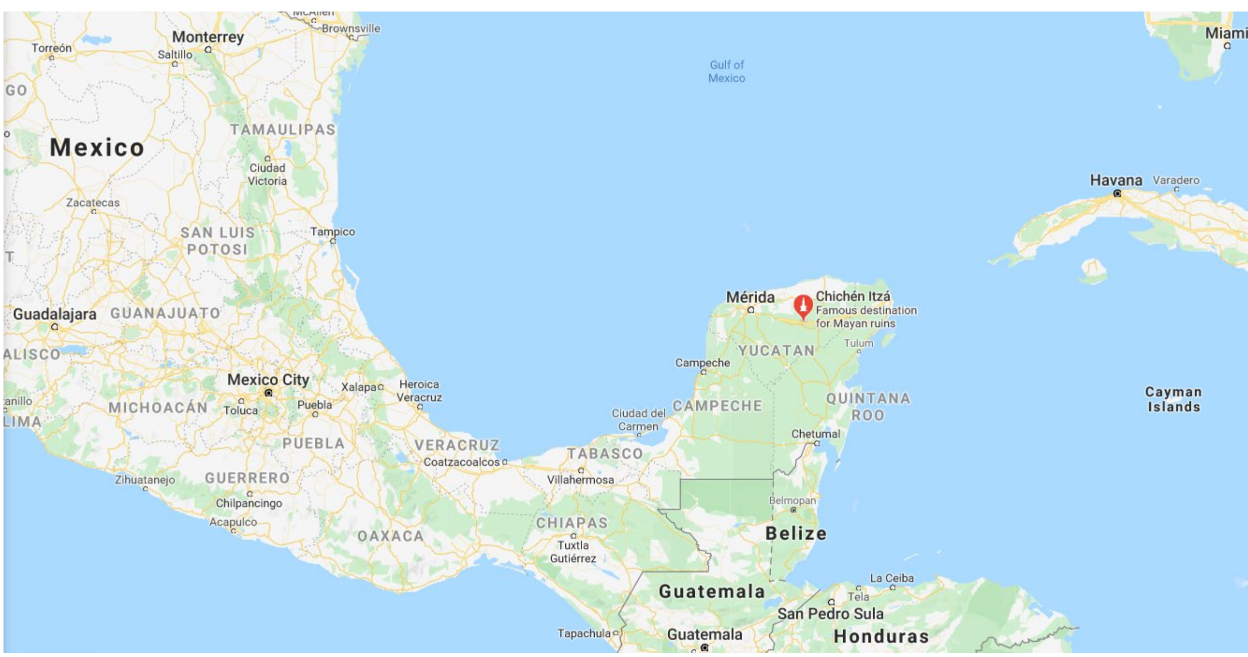
For further reading you can look at the article about Plimpton 332 which is a 3700 year old clay tablet. Watch this video below explaining the significance of Plimpton 332.
https://www.popsci.com/this-mysterious-ancient-tablet-could-teach-us-thing-or-two-about-math/

Exercises:
What is the largest number you can express using only 1 place value? Using 2 place values?

How do you express the result of the subtraction problem 

Make up two one problem for each of the 4 operations (+ − ×÷) using Babylonian number system. Are there any operations that seem adaptable to our experiences of the algorithm(s) for the method to carry out the operation?

Mayan Numeration



The Mayan people lived a vibrant life with a rich knowledge of planetary movement, solar movement, and mathematics in support of these endeavors. Mayan territories spanned from what is modern southern Mexico into Guatemala and Belize as well as parts of Honduras and El Salvador. The time period spans from as early as 8000 BC to as late as the arrival of the Spanish in 1511AD and about on to about 1697

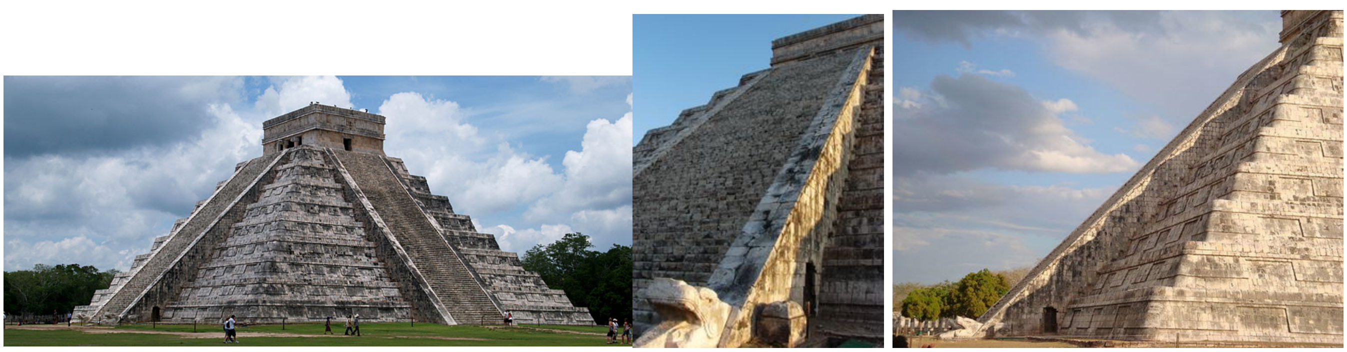
AD. This is a very long time to span, and the number system was not consistent throughout this span of time. The system we will discuss from the text is the number system used for their calendar system, which was selected to be advantageous for describing one year. They had other number systems that were a standard base 20 positional number system as well.

The Mayan calendar number system is a positional number system where the numbers are written from top to bottom just as the Chinese number system. Though, due to geographical distances these two number systems and cultures are believed to have grown independently and without influence between them.

Their predictions of the solar year were amazingly accurate as they estimated one solar year to be approximately 365 solar days. Interestingly, they had two different calendars, a solar calendar known as the Haab', and a 260 day calendar year known as the Tzolkin.
By the Tzolkin, which is still used by some Mayan communities in the Guatemalan Highlands, months consisted of 20 days, and one year consisted of 13 months.

However, the solar calendar consisted of 18 months each lasting 20 days (360 days) plus a 5 day period known as the Wayeb' or "Nameless days", creating a 365 day solar calendar. This calendar starts on the Winter Solstice.

Their predictions and calculations of astronomy were very accurate and were infused into architecture, art, and culture. In Chinchin Itza on the Yucatan peninsula in southern Mexico, the pyramid of El Castillo is a monument to the start of their solar year on the winter solstice.



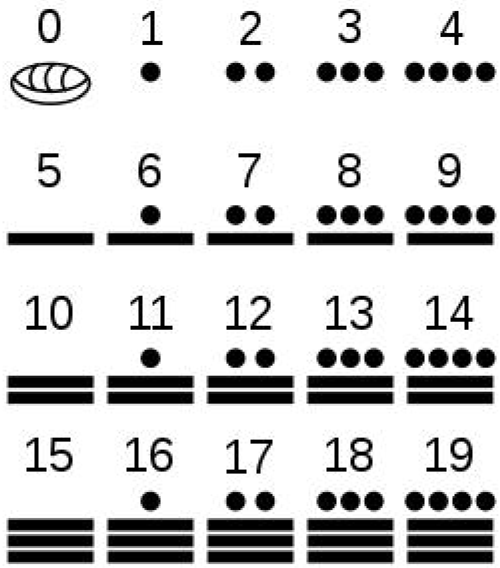
The Descent of Kukulkan: a simithsonian time lapse video of the solar alignment during an equinox on the pyramid of El Castillo in Chinchen Itza, located on the Yucatan peninsula.

https://www.youtube.com/watch?v=Zvv9EnBuem4

Based upon this calendar is the Mayan number system we will consider. It is positional, and the place values follow the calendar, so there is a ones place, 20's place, 18x20=360's place, 18x20x20's 7200's place, 18x20x20x20=144,000, and so on as a modified base 20 number system. Numbers were written in a vertical format from top to bottom as was common in the Chinese number system. Each numeral below the other represented a new place value.

Their digits consisted of a simple grouping system as a set of 20, and they made use of a zero digit before this was common practice in the West.

The Mayan digits were as follows:



An Example of the Place Values were:

2,880,000:

144,000:

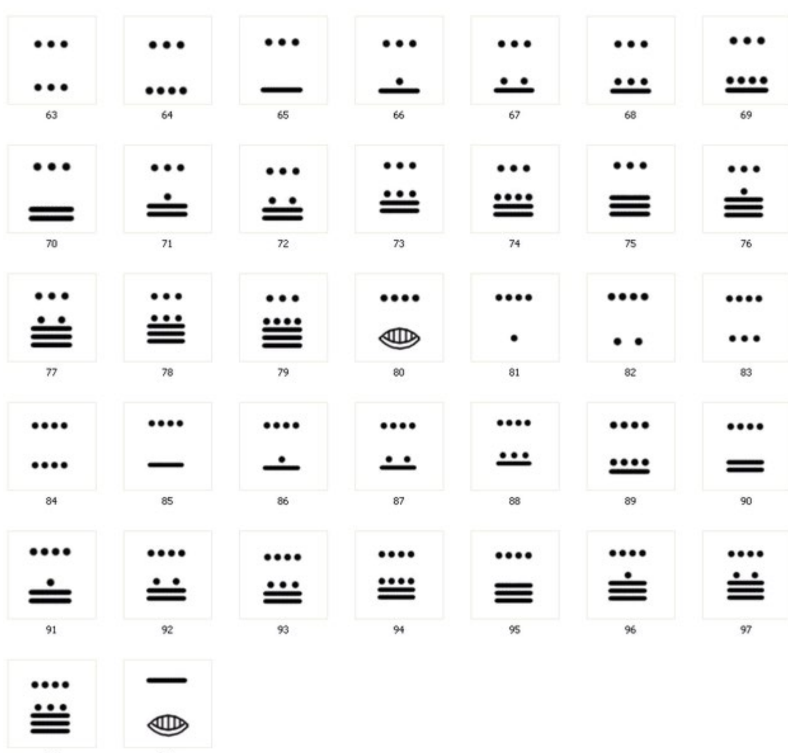
7,200:

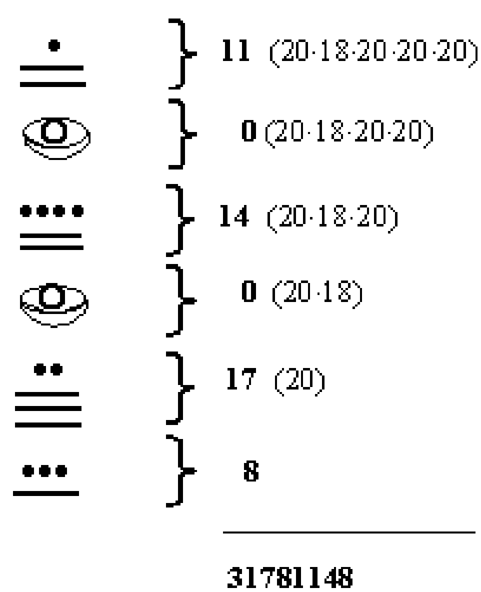
360:

20:

1:

As an example:



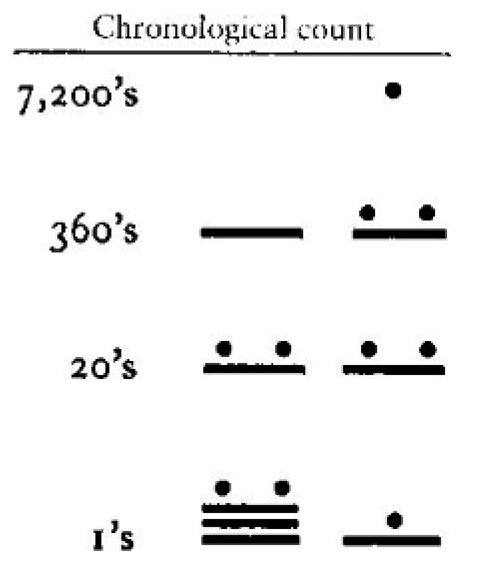


Example:
Express the following Hindu-Arabic numbers in Mayan numeration: [Use a calculator the returns remainders]

231:
Note: 231=20*10+13

454:
Note: 454=360*1+154
=360*1+20*7+14

Express the following Mayan numbers in Hindu-Arabic numeration:



Ans: 1,957 9,866

Examples:
Express the following Hindu-Arabic numbers in Mayan numeration: [Use a calculator the returns remainders].

57: 540: 3456: 84,562: 500,000:

Express the following Mayan numbers in Hindu-Arabic numeration:

Mayan Number	Meaning	Value
•		
• • • •		
▬ ▬		
▬ ▬ ▬		
	Total =	

Mayan Number	Meaning	Value
▬		
▬		
▬ ▬		
• •		
	Total =	

Greek Numeration

The classic Greek number system was a base 10 number system but differs from the other systems we have looked at. The advantage of this system is the economy for which you can write numbers less than 1000.

The numerals are as follows:

1	α	alpha	10	ι	iota	100	ρ	rho
2	β	beta	20	κ	kappa	200	σ	sigma
3	γ	gamma	30	λ	lambda	300	τ	tau
4	δ	delta	40	μ	mu	400	υ	upsilon
5	ϵ	epsilon	50	ν	nu	500	ϕ	phi
6	ϛ	vau*	60	ξ	xi	600	χ	chi
7	ζ	zeta	70	o	omicron	700	ψ	psi
8	η	eta	80	π	pi	800	ω	omega
9	θ	theta	90	ϙ	koppa*	900	ϡ	sampi

*vau, koppa, and sampi are obsolete characters

The rules for this numeration system is as follows:

- Symbols next to each other are additive ex: $\sigma\mu\alpha = 200 + 40 + 1 = 241$
- Multiples of 1000 (up to 9000) are indicated with a small stroke, " , " next to a units symbol (in this way it is similar to a multiplicative system, ex: 8000 = $,\eta$
- Multiples of 10,000 are indicated by the letter M (from the word *myriad*, meaning ten thousand) with the multiple shown above the M. ex: 30,000 = $\overset{\lambda}{M}$