Goals:

- To Answer any lingering questions about why the algorithms work, and gain a deeper understanding
- To show the computational advantages of the Hindu-Arabic number system

Alternate bases

See https://betterexplained.com/articles/numbers-and-bases/ for another explanation of our numbers.

Our representations of numbers has been demonstrated not to be unique in terms of its symbols, but even the place values and digits we use to represent a number can be different too!

What if we were born with only two appendages and no fingers, like penguins? What might our number system look like?

What would a system that only uses two symbols or digits look like…Binary.

The thing to remember is that in any number system you have to start a set of symbols, called digits, that each uniquely represent zero, one, and perhaps other quantities that are larger than the previous number by one.

Then, where you place the digit will assign that digit its value!

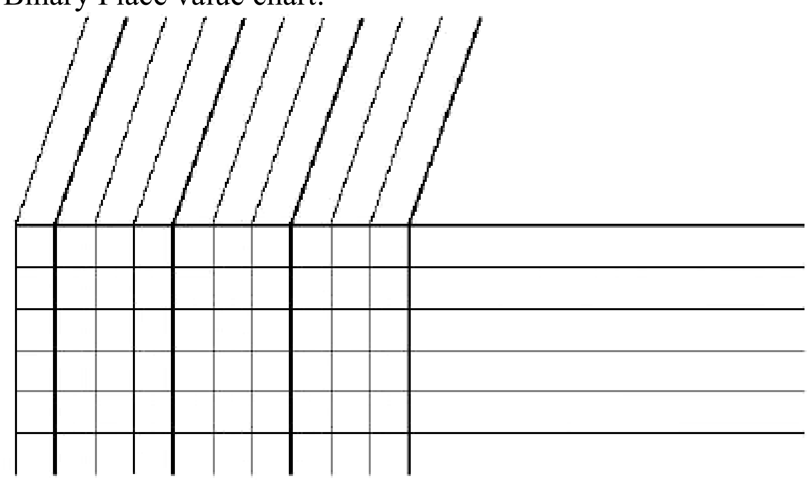
Base two (binary):

There are only TWO digits in base two, they are {0,1}

Try to remember that these are the only symbols that have any meaning now. The symbol 2 is just as meaningless in binary as a number as the symbol ☺ is to us in base ten.

So all your numbers will be constructed of 1's and 0's, and where you place the digit will give that digit its meaning in expanded notation.

Binary Place value chart:

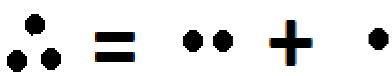


Below the illustration shows how a binary number is created by showing the place values or common groupings. It also illustrates how each number is a regrouping of place values in a expanded form.

Counting in Binary and Decimal

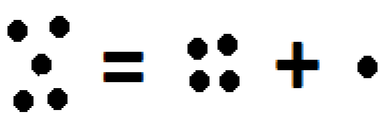
32	16	08	04	02	01	10	01
					1		1
				1	0		2
				1	1		3
			1	0	0		4
			1	0	1		5
			1	1	0		6
			1	1	1		7
		1	0	0	0		8
		1	0	0	1		9
		1	0	1	0	1	0
		1	0	1	1	1	1
		1	1	0	0	1	2
		1	1	0	1	1	3
		1	1	1	0	1	4
		1	1	1	1	1	5
	1	0	0	0	0	1	6

For example the number three is just one grouping of two and one grouping of 1.



So three is 11_2 in base two, one two and one one.

In another example five is just a regrouping into groups of four and one but without any groupings of two.



It five is 101_2 , one group of four, no groups of two, and one group of one.

Example: Go back up to the base two place value chart on the previous page, and express the number six in binary, then below express the number six in expanded form.

When writing numbers in alternate bases, the base of the number being represented is often shown in the subscript. Ex: $542_7, 7654_8, 2122_3$

All numerals/digits in those numbers should never have occurrences of numerals of the base number or larger.
Example: Circle all numbers which cannot represent actual numbers in another base.

$$2531_7, 8452_8, 1110_3, 4325_5, 9876_{10}, 8549_9, 7564_3, 546\alpha_{10}$$

Write down ten base ten numbers in the base 10 number column, then express that number in binary by placing the correct number of each groupings in the binary place value columns.
For example:

	8	4	2	1	
13 would be	1	1	0	1	one eight, one four, no twos and one one.

Binary Place Value Chart

	Binary Place									
Base 10 Number	512	256	128	64	32	16	8	4	2	1
	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0

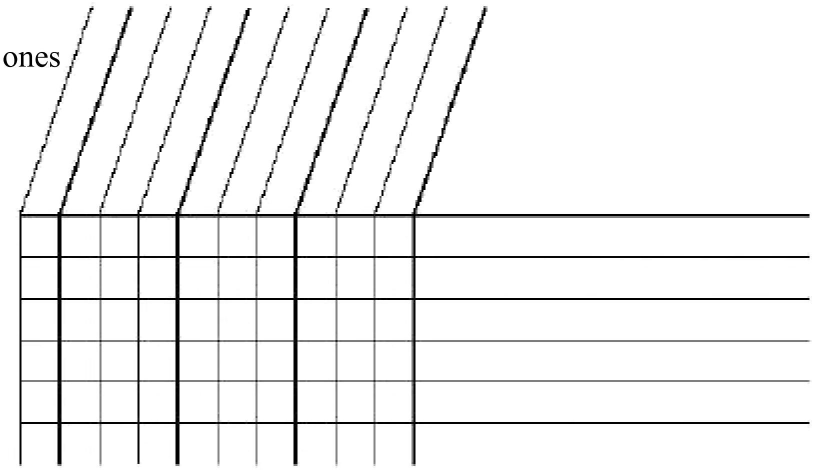
Other Bases

We can use any base number system we want. We just need the correct amount of symbols (including zero) and an appropriate place value system.

Base three

Base **three** uses **three** digits they are {0,1,2} and the place values are

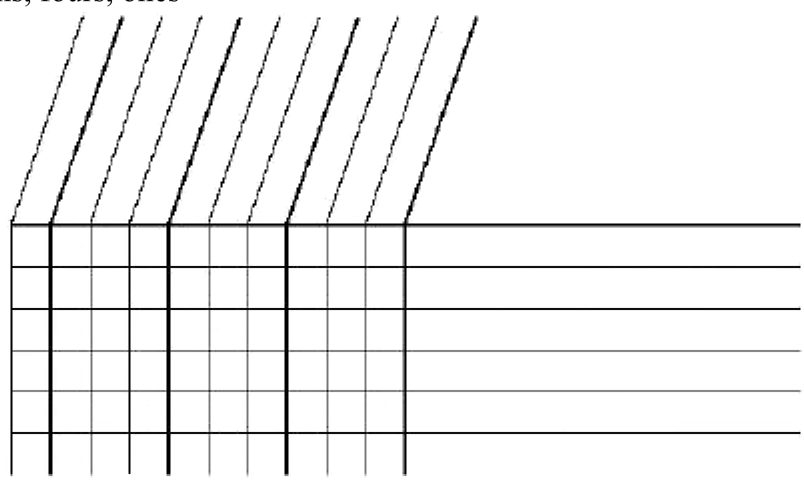
…,eighty ones, twenty sevens, nines, threes, ones



Base four

Base **four** uses **four** digits they are {0,1,2,3} and the place values are

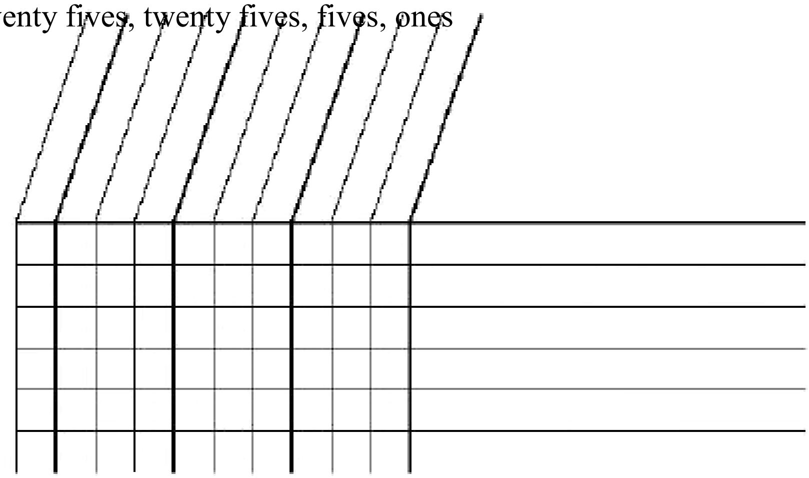
…, two hundred fifty sixes, sixty fours, sixteens, fours, ones



Base Five

Base **five** uses **five** digits they are {0,1,2,3,4} and the place values are

…, six hundred twenty fives, one hundred twenty fives, twenty fives, fives, ones



Converting from one base to another

The thing to remember is that 150 apples is the same number of apples no matter how you represent the number in any system. The only thing that changes is the way that number is represented.

So when you change from one system to another, you are only changing the way the number looks, you may be changing how the "apples" are being grouped (in boxes of five vs ten vs two) but that is really it.

In a positional number system, this really comes down to regrouping or "reboxing" the numbers.

In Base Ten:

In a base ten number system (we have ten digits 0-9 which means we can have between 0-9 boxes, after that we would just use a bigger box, for example if we have ten boxes of one, we could just rebox this more efficiently as one box of ten) and the size of the groupings of apples will come in groups/boxes of 1,10,10*10=100,10*10*10=1000, etc..

So 150 apples  would be boxed into

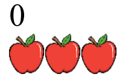
One box of 100 = 100
Five bags of 10 = 50
No single apples of 1 = 0

So we have 150_{10} = one hundred + five tens + zero ones

In Base Seven:

In a base seven number system (we have seven digits 0-6 which means we can have between 0-6 boxes, after that we would just use a bigger box, for example if we have seven boxes of one, we could just re-box (regroup) this more efficiently as one box of seven[the grouping number]) and the size of the groupings of apples will come in groups/boxes of 1,7,7*7=49,7*7*7=343, etc..

So 150_{10} apples would be boxed into

3 baskets of 49 =  = 147 apples
0 bags of 7 = 0 = no apples
3 single apples i.e. of 1 =  = 3 apples

So we have 150_{10} = 303_7 = 3*49 +0*7+3*1 = one hundred and fifty apples

Examples A:
Convert the following into base ten.
$a)\ 221_3$ $b)\ 654_7$ $c)\ 10110_2$ $d)\ 564_8$

Examples B:
Convert the following base ten numbers into the desired base.
$a)\ 123_{10} =$_________$(base\ 5)$ $b)\ 135_{10} =$ __________(base 3) $c)\ 333_{10} =$_________(seven)

Solutions A:
$221_3 = (2 \cdot 3^2) + (2 \cdot 3^1) + (1 \cdot 3^0) = 18 + 6 + 1 = 25_{10}$
$654_7 = (6 \cdot 7^2) + (5 \cdot 7^1) + (4 \cdot 7^0) = 294 + 35 + 4 = 333_{10}$
Note: calculator shortcut: $((6 \cdot 7 + 5) \cdot 7 + 4)$
$10110_2 = (1 \cdot 2^4) + (0 \cdot 2^3) + (1 \cdot 2^2) + (1 \cdot 2^1) + (0 \cdot 2^0) = 16 + 4 + 2 = 22_{10}$
Calculator shortcut: $((((1 \cdot 2 + 0) \cdot 2 + 1) \cdot 2 + 1) \cdot 2 + 0)$
$564_8 = (5 \cdot 8^2) + (6 \cdot 8^1) + (4 \cdot 8^0) = 320 + 48 + 4 = 372_{10}$

Solutions B:
$123_{10}\ (into\ base\ 5) \rightarrow \frac{123}{25} = \mathbf{4}r23 \rightarrow \frac{23}{5} = \mathbf{4}r3 \rightarrow \frac{3}{1} = \mathbf{3}r0\ so\ 123_{10} = 443_5$

$135_{10}\ (into\ base\ 3): \frac{135}{81} = \mathbf{1}r54 \rightarrow \frac{54}{27} = \mathbf{2}r0 \rightarrow$
$nothing\ more\ so\ put\ enough\ place\ holders\ so\ the\ 1\ is\ in\ the\ 81's\ place$
$135_{10} = 12000_3$

$333_{10}\ (into\ base\ 7): \frac{333}{343} = \mathbf{0}r343 (so\ the\ 343\ place\ value\ is\ not\ needed) \rightarrow \frac{333}{49} = \mathbf{6}r39 \rightarrow \frac{39}{7} = \mathbf{5}r4$
$\rightarrow \frac{4}{1} = \mathbf{4}r0\ \ so\ 333_{10} = 654_7$