Goals:

- We will learn vocabulary necessary to discuss ideas presented in this chapter on Logic
- We will discuss, compound statements, negations, and quantifiers.

Statements

Def: A **Statement** is a set of words and symbols that collectively make a claim that can be classified as True or False, but not both simultaneously.

Example: Circle T or F for one point, state why it is so for the rest of the points.

Each of the following is a statement.

1. **T / F** $x^2 \cdot x^5 = x^7$
2. **T / F** The sum of the internal angles of a triangle always is 180°
3. **T / F** $a^2 \cdot b^5 = (a \cdot b)^7$
4. **T / F** Its Time.

It is often necessary to discuss negations of statements (you may wonder what the consequences of your statement is, or what will happen if you don't do what you say.

Statement:"I always put jello in my pants"

Negation:

Statement: $2^3 = 8$

Negation:

Def: A **Variable** is a letter that represents an unknown value or number.

The statement "$x + 2 = 3$" is called an **open statement** because it may or may not be true depending on the value of the variable. In algebra class we called this an algebraic equation.

Compound Statements

In the English language we often will combine two statements forming what is called a **compound statement**. Then each statement becomes like a building block, much in the same way that sentences become building blocks to form paragraphs.

Logical Connectives, or simply Connectives are terms like **"and", "or", "if…then…", "not"** can be used to create a logical connection between two statements. These connectives modify the compound statement into specific logical conditions.
This statement is an example of a compound statement because two claims which can each be classified as

true or false were combined into one statement; which depending on how these statements were combined, the validity of the compound statement is dependent on the type of Connective used.

There are a few different types of compound statements we will discuss:

1. Conjunctions
2. Disjunctions
3. Conditional Statements or Implications otherwise seen as (If …then…) statements.

And 4. Negations (which are not on their own considered a compound statement

1. A **conjunction** is an **"AND"** statement that combines two statements together in a way that requires **both** conditions to be true for the conjunction statement to be true.

Ex: The band modest mouse has a short lead singer AND they came from Seattle.

2. If only one of the two conditions must be true this statement is called a **disjunction** . A disjunction is an **"OR"** statement, and requires **one or more** condition to be true for the statement to be true.

Ex: I have eaten sugar or I have done heroin.

$P \vee Q$(where the symbol ∨ is a short hand representation for or).

3. A **Conditional Statement** or Implication is a compound statement where one statement implies that the other statement is true.

Ex: If it is raining then I will carry an umbrella.

This type of statement has a condition or statement called the **antecedent or hypothesis** and a statement or conclusion called the **conclusion or consequent** . If the condition is met or true, we can be assured that the conclusion will also be true.

In the above example the antecedent is the statement **It is raining**, and the consequent is the statement **I will carry an umbrella**.

All conditional statements (implications) will have the symbolic form $\boldsymbol{P \Rightarrow Q}$ (where **P** is the antecedent, **Q** is the consequent, and the symbol ⇒is a short hand representation for implies). I sometimes refer to conditional statements as an **"If Then"** statement.

4. For a given statement P, the negation of P, $\sim P$, is a statement that negates the original statement, P. A negation must have the opposite truth value from the original statement.

Ex: P= I go to the park every Sunday; ~P = I do not go to the park every Sunday.

Symbolic Logic

In mathematics however, although we do use compound statements, we do not necessarily always care about what each statement is, we are sometimes more interested in uncovering the basic structure of the statement. Rather than getting lost in the potential for having multiple different ways one can use words to make the same statement, in math we often will dispense with the words and instead use symbols to let the true structure prevail by using variables as objects in a statement.

Ex: Instead of making the statement "Jack went up the hill and Jill fetched a pail of water" we can write $P \wedge Q$(where the symbol $\wedge$is a short hand representation for &).

Notice the symbol looks like an uppercase A

Doing so saves time and unclutters that statement so that we can see more easily its true form.
In order for the statement to be true we know that both Jack must go up the hill, call this statement P , AND Jill must fetch some water, call this statement Q. Then the above sentence can be symbolically written $P \wedge Q$. For this statement to be true BOTH P and Q must be true.

Examples:
Let p represent the statement "I have made rad stuff at the FLC Maker Space"
let q represent the statement "I love the sushi at the Roost"
Convert each compound statement into symbols:

1. I have made rad stuff at the FLC Maker Space and I love the sushi at the Roost

2. I have not made rad stuff at the FLC Maker Space or I love the sushi at the Roost

3. I have not made rad stuff at the FLC Maker Space nor do I love the sushi at the Roost

   $\sim p \wedge \sim q \equiv \sim(p \vee q)$
4. I hate the sushi at the Roost and I have made rad stuff at the FLC Maker Space

5. If I make rad stuff at the FLC Maker Space then I love the sushi at the Roost.

Examples:
Circle if the given sentence is a Statement "S" or a Compound Statement "CS". If it is a Compound Statement, then classify it as a Conjuncton "and", Disjunction "or", Conditional Statement "if/then" or a Negation, "not".

6.	**S CS**	Every Dog has it day.	**AND OR IF/THEN NOT**
7.	**S CS**	There is no rain anywhere in Ca today.	**AND OR IF/THEN NOT**
8.	**S CS**	Behave yourself and sit down.	**AND OR IF/THEN NOT**
9.	**S CS**	You will put the lotion in the basket or you'll get the hose again.	**AND OR IF/THEN NOT**
10.	**S CS**	Negative numbers are smaller than positive numbers and 4 is less than 5.	**AND OR IF/THEN NOT**
11.	**S CS**	We were just headed over the boarder for some French fries and gravy sir.	**AND OR IF/THEN NOT**
12.	**S CS**	You are not funnier than me.	**AND OR IF/THEN NOT**
13.	**S CS**	If you look at me again there's gunna be a fight.	**AND OR IF/THEN NOT**

Quantifiers

Quantifiers are used to indicate how many cases of a particular situation exists.

Universal Quantifiers are used to specify everything from the object is being considered or nothing. Example universal quantifiers are words like **all, each, every, and no, or none.**

Existential Quantifiers refer to the one or more but not all or none of the object is being considered. Words and phrases like **some, there exists, and for at least one**, imply that of all the objects, there are somewhere between all and none being discussed, but not all or not none.

Example: For every child, there is at least one text book.

Relative to the object of children, we are considering all of them, but for text book, we are considering one or more.

Forming Negations of Quantified Statements

It may be misleading at times what the negation of a quantified statement is.

The negation of a negation of a statement is simply the statement itself. In other words, two negations make cancel each other out like a double negative in speech, “don’t not go”, “ain’t got none”, “not nothing”

Examples:
1. Quantified Statement: I always smell my shoes after I work out.
   Negated Statement:

2. Quantified Statement: I never talk to strangers who wear tutus in public.
   Negated Statement:

Student Exercises:
Write a negation for each statement.
1. The flowers are due to be watered
2. All students present will get an A.
3. Some people have all the luck.
4. This is not a negation to a statement.

Common Number Sets

$\mathbb{N}$ = Natural Numbers = {1, 2, 3, ...}

$\mathbb{W}$ or $\mathbb{N}_0$ = Whole Numbers = {0, 1, 2, 3, ...}

$\mathbb{Z}$ = Integers = {..., -1, -2, -3, 0, 1, 2, 3, ...}

$\mathbb{Q}$ = Rational Numbers = {p/q ; p and q are integers}

$\mathbb{I}$ = Irrational Numbers = {non-rational number}

$\mathbb{R}$ = Real Numbers = {All of the above number sets}

Imaginary Numbers = {Numbers containing $i = \sqrt{-1}$ }

$\mathbb{C}$ = Complex Numbers = {*a* + *bi* ; a and b are real, $i = \sqrt{-1}$ }

Deciding Whether Quantified Statements are True or False

T or F There exits a whole number that is a rational number.
T or F Every integer is a whole number.
T or F Some Real Numbers are irrational numbers.