Goals:

- We will narrow our view of statements and compound statements to symbolic logical representations.
- We will consider the value of these symbolic statements and compound statements as True or False, and look at Truth Tables as a way to determine the value of a compound statement.
- We will discuss logical equivalences to symbolic statements.
- You can find a truth table generator at: https://web.stanford.edu/class/cs103/tools/truth-table-tool/

Recall, Statements are sentences that can be classified as being either True or False, but not both. This is also the case for compound statements.
For example the compound statement "$1 = 2 \text{ } or \text{ } 3 < 4$" is a true statement, since one of the two component statements was true.

In logic, it is important to understand the possible outcomes of a conditional statement, and depending on the validity of each component statement, what the value of the entire conditional statement is. To help us understand the logic without getting confused by the context of the statement, we use truth tables.

Truth Tables

Let us first consider the conjunction, and

Given two component statements, p & q, the conjunction, "and" stipulates that for the compound statement $p \wedge q$ to be true, both p & q must each be true.

Notice that for the component statement p there are only two outcomes, T or F.
The same goes for q.
However, when considering a compound statement involving both p & q, we must look at all possible outcomes.

This is the **truth table** which illustrates the value of the conjunction of p & q for all possibilities of the values of p & q.

p	q	(p ∧ q)
F	F	**F**
F	T	**F**
T	F	**F**
T	T	**T**

Note, the conjunction is ONLY true when both p & q are true.

Example:
Let p represent the statement yesterday was St. Patrick's Day, and
Let q represent the statement Today is New Year's Day.
Then the compound statement $p \wedge q$ is only true if the day after St. Patrick's Day is New Year's Day.

Let us now consider the validity of the disjunction, or

A disjunction, or "or" statement unlike the "and" statement, can be true with either component statement being true as well as with both component statements being true.

There can be some ambiguity in the use of the word "or" as either exclusive or inclusive. The ambiguity lies in the case of both component statements being true. What of the statement, "you can go to a movie or you can stay home"? In English this use of "or" would be the exclusive "or" as it is assumed that one could not do both, however, logically the use of the disjunction would imply that they indeed could do both go to a movie and then stay home. This is the inclusive case as it includes the option to do both.

The **exclusive "or"** would not be true if both component statements are true.
The **inclusive** "or" will be true if both component statements are true.

A disjunction, is assumed to be the inclusive "or" unless it is specified to be the exclusive "or".

The truth values of the disjunction of p & q are:

p	q	(p ∨ q)
F	F	**F**
F	T	**T**
T	F	**T**
T	T	**T**

The disjunction $p \vee q$ means $p\ or\ q\ or\ both$, it is the inclusive "or"

Negations

The **negation** of a statement, p, symbolized as $\sim p$, must have the opposite truth value to p.

The truth table for the negation is:

p	¬p
F	**T**
T	**F**

Example:
Let p represent a true statement, let q & r represent false statements.
Find the truth value of the following compound statements.

1. $(q \vee \sim r) \wedge p$
2. $(\sim p \wedge q) \vee \sim r$
3. $(\sim r \wedge \sim q) \vee (\sim r \wedge q)$
4. $\sim[r \vee (\sim q \wedge \sim p)]$

Ans: 1. T 2. T 3. T 4. T

Example:
Determine whether each statement is true or false.
1. For some real number, $x, x > 5$ and $x < 7$
2. For all real numbers, $x, x > 5$ and $x < 7$
3. For all real numbers, $x, x^2 > 0$
4. For every real number $x, x > 1$ or $x < 2$

Ans: 1. T (6) 2. F 3. F (0) 4. T

Constructing Truth Tables

If there are n, number of component statements, then the number of rows in a truth table should always be 2^n

This will ensure that each possible outcome has been considered.

Example:
Construct the truth table for each compound statement:

$$(\sim p \wedge q) \vee \sim q$$

There are two component statements with both having a negative.
We will need 2^2 or 4 rows in order to consider every case.
We will first start with the component statements, then we will consider their negations, then we will consider the smaller compound statements as we build up to what we wish to understand.

p	q	$\sim p$	$\sim q$	$(\sim p \wedge q)$	$(\sim p \wedge q) \vee \sim q$
T	T	F	F	F	F
T	F	F	T	F	T
F	T	T	F	T	T
F	F	T	T	F	T

De Morgan's Laws for Logical Statements

De Morgan's Laws discusses the logical equivalence of the negation of a conjunction and a disjunction.

It defines the negation of a conjunction as

$$\sim(p \wedge q) \equiv \sim p \vee \sim q$$

An example of this is, if you are not able to $p = go\ to\ the\ movies$ "and" $q = eat\ pizza$, then you may not go to the movies "or" you may not eat pizza.

For the disjunction, the negation of a disjunction looks like this:

$$\sim(p \vee q) \equiv \sim p \wedge \sim q$$

If you may not (go to the movies or eat pizza), then you may not go to the movies and you may not eat pizza.

We can confirm De Morgan's Laws by looking at the truth table values and recognizing that these two compound statements have the same logical outcomes, which makes them logically equivalent.

DEMORGAN'S LAWS

1. $\sim(P \wedge Q) = (\sim P) \vee (\sim Q)$
2. $\sim(P \vee Q) = (\sim P) \wedge (\sim Q)$

The first of DeMorgan's laws is verified by the following table. You are asked to verify the second in an exercise.

P	Q	$\sim P$	$\sim Q$	$P \wedge Q$	$\sim(P \wedge Q)$	$(\sim P) \vee (\sim Q)$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

Verify the second of De Morgan's Laws

p	q	$\sim p$	$\sim q$	$(p \vee q)$	$\sim(p \vee q)$	$(\sim p) \wedge (\sim q)$
T	T	F	F			
T	F	F	T			
F	T	T	F			
F	F	T	T			