Goals:

- We will narrow our view of statements to conditional statements.
- We will consider the possible value of conditional statements and what is required for an conditional statement to be True or False.
- We will look at the negation of a conditional statement and its logical equivalences.
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- You can find a truth table generator at: https://web.stanford.edu/class/cs103/tools/truth-table-tool/

Conditional Statements

Recall, Statements are sentences that can be classified as being either True or False, but not both. This is also the case for compound statements.

Also recall that a **conditional statement** is a compound statement that uses the connective, "if…then…"

Example of a conditional statement:
If you say "Candyman" 5 times in the mirror, then Candyman will appear and give you a lollypop.

This type of statement discusses the implied condition, that If p occurs, then Q will be the consequence.

As such we define the component statement **p**, the **hypothesis or antecedent.**

We then define the component statement **q**, the **conclusion or consequent.**

And symbolically we write the conditional statement if p then q as:

$$p \Rightarrow q$$

In logic, it is important to understand the possible outcomes of a conditional statement, and depending on the validity of each component statement, what the value of the entire conditional statement is. To help us understand the logic without getting confused by the context of the statement, we use truth tables.

Truth Values of a Conditional Statement

Given two component statements, p & q, the conjunction, "and" stipulates that for the compound statement $p \wedge q$ to be true, both p & q must each be true.

Notice that for the component statement p there are only two outcomes, T or F.
The same goes for q.
However, when considering a conditional statement involving both p & q, we must look at all possible arrangements of p & q and determine if the entire conditional statement is T or F.

Since the conditional statement is itself a statement, its truth value is either T or F.

Lets look at the truth values of a conditional statement through an example:
Let p be the component statement: You will sign up for my class
And let q be the component statement: You will get an A.

Then if an instructor makes the conditional statement

$$p \Rightarrow q$$

"If you will sign up for my class, then You will get an A."

When assessing the validity of this statement, we want to determine if the instructor is

T, telling the truth OR F, they have told a lie.

Let us look at the truth table for all the possible scenarios in this conditional statement:

Scenario:	p	q	$p \Rightarrow q$
1	T	T	T
2	T	F	F
3	F	T	T
4	F	F	T

In Scenario 1.
P is True, so you will have signed up for their class, also q is true so you did get an A.
The instructor's statement $p \Rightarrow q$ was TRUE because they did not lie.

In Scenario 2.
P is True, so you will have signed up for their class, but q is false so you did NOT get an A.
The instructor's statement $p \Rightarrow q$ was then FALSE because they DID lie.

In Scenario 3.
P is False, so you will have NOT signed up for their class, while q is true so you did get an A (presumably in another instructor's course).
The instructor's statement $p \Rightarrow q$ was TRUE because THEY DID NOT LIE!

In Scenario 4.
P is False, so you will have NOT signed up for their class, while q is False so you did NOT get an A (presumably in another instructor's course).
The instructor's statement $p \Rightarrow q$ was TRUE because THEY DID NOT LIE!

This then is the truth table for ANY conditional statement $p \Rightarrow q$

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Example:
Given that $p, q, \& r$ are all F, determine the truth value of the following conditional statements.

1. $q \to r$
2. $(p \to \sim q) \to \sim r$
3. $(p \to \sim q) \to (\sim r \to q)$

Negating a conditional statement.

This is easiest to visualize by looking at the truth table:

p	q	$p \Rightarrow q$	$\sim(p \Rightarrow q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

But when we try to think of how this may work out in an example, it gets harder.

If it is not true that "if you sign up for my class then you will get an A"

What would be an equivalent way of saying this?

Which sounds equivalent: Express this symbolically
If you do not sign up for my class then you will not get an A? Symbolically: __________________
If you do sign up for my class then you will not get an A? Symbolically: __________________
You will sign up for my class AND you will NOT get an A? Symbolically:
____
You will sign up for my class OR you will NOT get an A? Symbolically: __________________

Can you think of any others?

Will any of these have the same truth values? Put them in a Truth table and see if they are logically equivalent (have the same truth values) to $\sim(p \Rightarrow q)$

Exercise:
Fill in each truth value and circle which symbolic compound statement is equivalent to $\sim(p \Rightarrow q)$

p	q	$p \Rightarrow q$	$\sim(p \Rightarrow q)$
T	T		
T	F		
F	T		
F	F		

$\sim p \to \sim q$	$p \to \sim q$	$p \wedge \sim q$	$p \vee \sim q$		

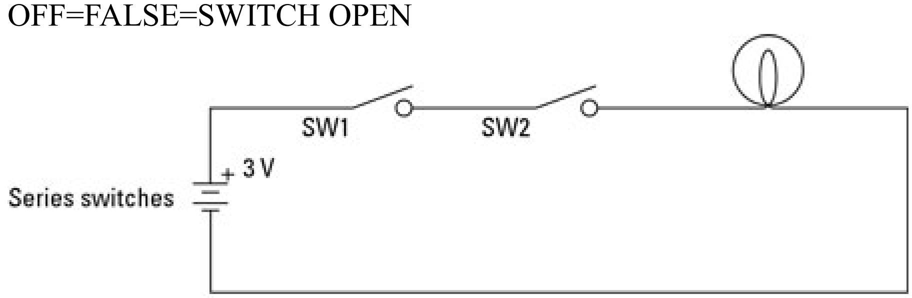
Therefore the negation of $p \Rightarrow q$ is $\sim(p \Rightarrow q) \equiv$ ________________________

Logic Circuits

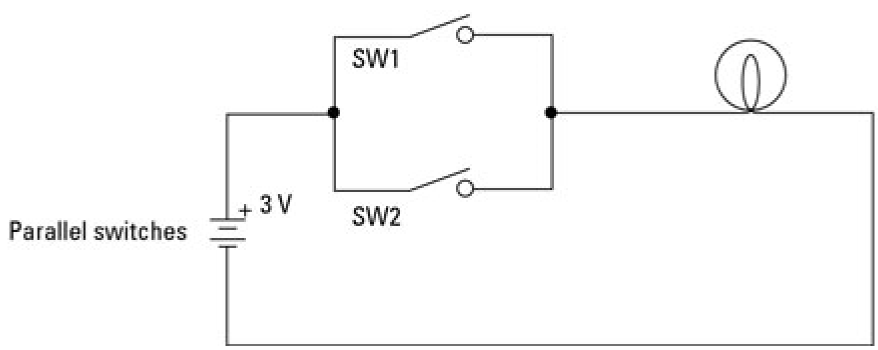
From the text *Mathematical Ideas, 12th ed.* by Miller, Heeren, Hornsby:

> One of the first nonmathematical applications of symbolic logic was seen in the master's thesis of Claude Shannon in 1937. Shannon showed how logic could be used to design electrical circuits. His work was immediately used by computer designers. Then in the developmental stage, computers could be simplified and built for less money using the ideas of Shannon.

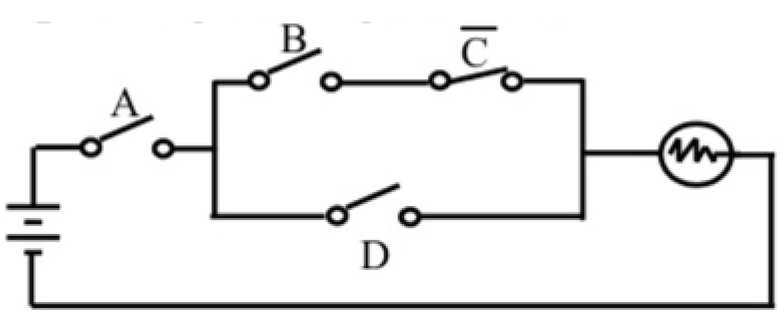
As we can see below, electricity and voltage also has a binary representation as does truth values.
ON =TRUE=SWITCH CLOSED
OFF=FALSE=SWITCH OPEN



What logical compound statement would represent this circuit? ANS: ______________________

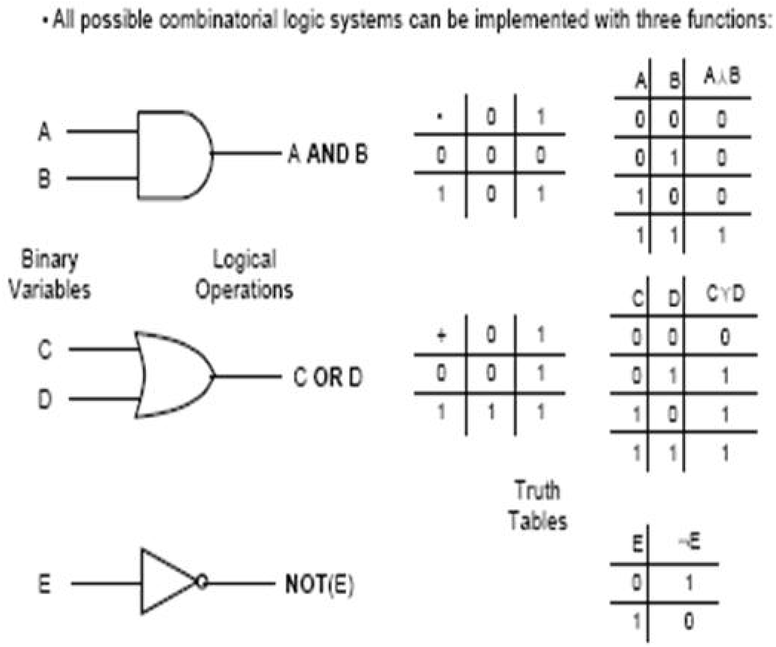


What logical compound statement would represent this circuit? ANS: ______________________



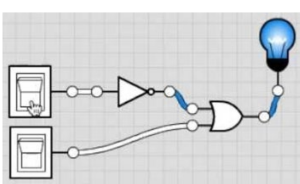
What logical compound statement would represent this circuit? ANS: ______________________

In electronics there are logic gates that are constructed of these simple switches. These gates have their own symbolic representation as seen below, but they are constructed from the circuits above.

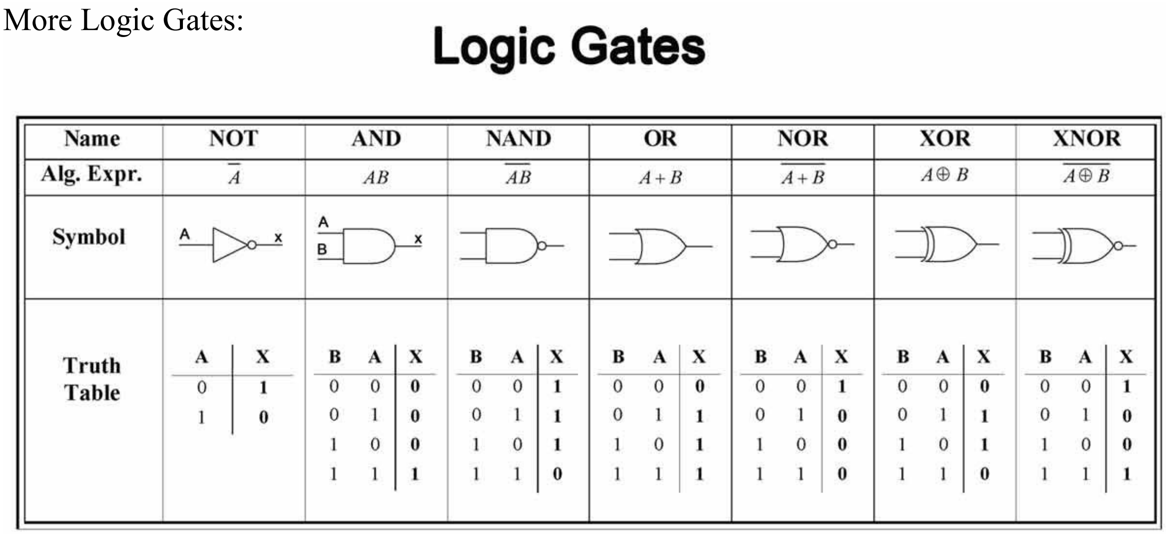


Considering the truth tables, draw a simple circuit that would be logically equivalent to $p \rightarrow q$

p	q	(¬p ∨ q)	(p → q)
F	F	T	T
F	T	T	T
T	F	F	F
T	T	T	T



More Logic Gates:

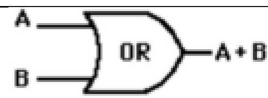
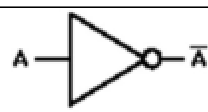


Logic Gates

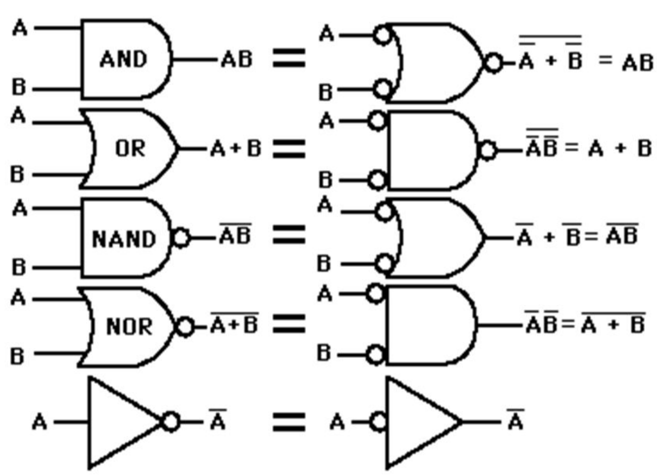
Name	NOT	AND	NAND	OR	NOR	XOR	XNOR
Alg. Expr.	$\bar{A}$	AB	$\overline{AB}$	$A+B$	$\overline{A+B}$	$A\oplus B$	$\overline{A\oplus B}$
Symbol							
Truth Table	A | X 0 | 1 1 | 0	B A | X 0 0 | 0 0 1 | 0 1 0 | 0 1 1 | 1	B A | X 0 0 | 1 0 1 | 1 1 0 | 1 1 1 | 0	B A | X 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 1	B A | X 0 0 | 1 0 1 | 0 1 0 | 0 1 1 | 0	B A | X 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 0	B A | X 0 0 | 1 0 1 | 0 1 0 | 0 1 1 | 1

In mathematics there is a branch of algebra called Boolean Algebra, which is an algebra where the variables have a binary value or 0,1 or T,F. The English mathematician George Boole created this algebra system as a way to describe logic, particularly the description of logic as laid out by Aristotle. So the ideas of logic are important, and there applications and convenient notations created by people to help accomplish goals, or explore ideas in different terms. But the ideas do not change, it is only the representation and the application that changes.

For example look at the table below:

Symbolic Logic	Boolean algebra	Circuits and logic gates	Truth Table
$A \wedge B$	AB		p q (p∧q) F F F F T F T F F T T T
$A \vee B$	$A + B$		p q (p∨q) F F F F T T T F T T T T
$\sim A$	$\bar{A}$		p ¬p F T T F

Below is a great example of De Morgan's Laws for Logic and application of it in circuits.



$AB = A \wedge B =$ ____________________

$A + B = A \vee B =$ ____________________

$\overline{AB} = \sim(A \wedge B) =$ ____________________

$\overline{A + B} = \sim(A \vee B) =$ ____________________

$\bar{A} =$ ________________