Goals:

- We will expand our look at conditional statements and consider Converse Statements, Inverse Statements, and Contrapositive Statements.
- You can find a truth table generator at: https://web.stanford.edu/class/cs103/tools/truth-table-tool/

Conditional Statements

Recall, Statements are sentences that can be classified as being either True or False, but not both. This is also the case for compound statements.

Also recall that a **conditional statement** is a compound statement that uses the connective, "if…then…"

Example of a conditional statement:
If you do something ridiculous, then I will roll my eyes.

This type of statement discusses the implied condition, that If p occurs, then Q will be the consequence.

As such we define the component statement **p**, the **hypothesis or antecedent.**

We then define the component statement **q**, the **conclusion or consequent.**

And symbolically we write the conditional statement if p then q as:

$$p \Rightarrow q \; or \; p \rightarrow q$$

The Converse, inverse, and contrapositive of a conditional statement

For the following definitions, consider the conditional statement $p \rightarrow q$.

The **inverse** of a conditional statement $p \rightarrow q$ is the statement $\sim p \rightarrow \sim q$

The **contrapositive** of a conditional statement $p \rightarrow q$ is the statement $\sim q \rightarrow \sim p$.

The truth tables for these are:

Inverse:

p	q	$\sim p$	$\sim q$	$\sim p \Rightarrow \sim q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

Contrapositive:

p	q	$\sim p$	$\sim q$	$p \Rightarrow Q$	$\sim q \Rightarrow \sim p$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

It is worth noting for purposes of logical argument that a conditional statement and its contra positive are logically equivalent, meaning they have the same truth values in every case.

We will utilize this when proving theorems in geometry.

Def: The **<u>converse</u>** of a conditional statement $p \Rightarrow q$ is the statement $q \Rightarrow p$.

The converse is merely another compound statement. It is important to note that regardless of the truth value of $p \Rightarrow q$, the converse may or may not be true, the converse, is simply another statemtent.

Example:

Conditional statement:
All men are human. i.e. If a person is a man, then they are a human.

Converse statement:
All humans are men. i.e. If a person is a human, then they are a man.

In this instance, the converse is a false statement.

Example:

Conditional statement:
If a polygon has three sides, then it is a triangle.

Converse statement:
If a polygon is a triangle, then it has three sides.

In this case the converse is a true statement.

This is because the original statement is a biconditional statement.

The Biconditional Statement

When a conditional statement is being made AND also its converse, we would say p if and only if q. This type of statement is called a **biconditional statement.** Symbolically we would write, $\boldsymbol{p \Leftrightarrow q}$ **or** $\boldsymbol{p \leftrightarrow q}$**.**

The truth table for this is:

p	q	$(p \leftrightarrow q)$
F	F	T
F	T	F
T	F	F
T	T	T

A biconditional statement is true with BOTH truth values are THE SAME.

Examples:

For each of the following conditional statements (or statements that can be written in the traditional if…then form) write a) the converse, b) the inverse, c) the contrapositive

1: If you lead, then I will follow.

a)

b)

c)

2: Where there is smoke, there is fire.

a)

b)

c)

Example: Identify the following as True or False

T / F $6 = 9 - 3 \ iff \ 8 + 2 = 10$

T / F George H. Bush was president if and only if George W. Bush was not president

T / F Yosemite is a park in Oregon if and only if Puerto Rico is one of the 50 states in the U.S.A.

Ans: T,F,T