Goals:

- We will attempt to apply our skills in logic to analyze an argument by reducing it to symbolic form and testing the argument's validity using truth tables.
- You can find a truth table generator at: https://web.stanford.edu/class/cs103/tools/truth-table-tool/

Modus Ponens: the law of detachment

Forming and adhering to a logical argument is not necessary in life, because there are so many ways to "win" an argument. There are many different types of logical fallacies one can commit which will often either convince or encourage others to agree with your point of view. We will talk about fallacies more later. For the moment let's just consider "fighting fair". What I mean by this is, let us look at how to construct an argument that is logically valid and can hold up to the rigor of an unbiased metric, like a truth table.

In logic, the **law of detachment** says that if the following two statements are true:

1) If p, then q
   AND
2) p

Then we must infer a third statement

3) q

It says what we all intuitively believe to be true, that **if you have a conditional statement AND the hypothesis is true, then the conclusion must be true.**

Example:

If $\underbrace{you\ take\ one\ step\ closer\ to\ me}_{p}$, then $\underbrace{I\ will\ either\ punch\ you\ or\ kiss\ you}_{q}$.

$\underbrace{You\ took\ one\ step\ closer\ to\ me}_{p}$.

Therefore,

$\underbrace{I\ will\ either\ punch\ you\ or\ kiss\ you}_{q}$.

Symbolically the argument looks like this:

$$\big((p \rightarrow q) \wedge p\big) \rightarrow q$$

No matter how we feel about this argument (like your mom always says "you can't go around kissing strangers!" or like the law may dictate "you can't go around hitting people"), what we would like to know is if logically speaking, we have created a logically sound argument. The law of detachment says YES!

Instead of just taking my word or the texts word for it (look up the fallacy of false authority)

lets examine the validity of this argument with a truth table:

p	q	$p \to q$	$(p \to q) \wedge p$	$[(p \to q) \wedge p] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

The Law of Detachment:
1) If p, then q

AND

2) p

Then we must infer

3) q

Even though it may be illegal to kiss or punch a person for taking a step nearer you, it is logically valid to do so when the law of detachment is present.

Morality and legality are subjective constructs created by a society, but logic is objective, unable to change with the times, it is permanent, like gravity on earth, like the equations of a circle, or the existence of matter. Logic is a universal truth!

We would say that this argument is a **tautology** (taa·**taa**·luh·jee), which means a statement that is true by necessity or by virtue of its logical form.

Reasoning by Transitivity: the Law of Hypothetical Syllogism

(**Syllogism**:= logical transitive argument)

Example: *Determine where the argument is valid or invalid:*

If I drop my toast, then it will fall on the floor.

If my toast falls on the floor, then it will land butter side down.

Therefore, if I drop my toast, then it will land butter side down.

Solution:
Let p = I drop my toast, q = it will fall on the floor r = it will land butter side down

Symbolic argument:

$$\begin{array}{c} p \to q \\ q \to r \\ \hline p \to r \end{array}$$

p	q	r	$p \to q$	$q \to r$	$p \to r$	$(p \to q) \wedge (q \to r)$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$
T	T	T					
T	T	F					
T	F	T					
T	F	F					
F	T	T					
F	T	F					
F	F	T					
F	F	F					

This is a **Valid/Invalid** form of logical argument. (Circle the correct one)

This type of logic is called **reasoning by transitivity** and is a valid logical argument.

The Moral of the story: Murphy's Law: Whatever bad thing can happen, will happen.

To check your answer, copy this $((p \to q) \wedge (q \to r)) \to (p \to r)$ into this link. You can also use this to check your other component statements too!

Reasoning by Modus Tollens: the Law of Contraposition, or Indirect Reasoning

This is a rather natural style of argument utilized when a more direct approach may be more difficult or when a slightly more complex argument is desired.

Example:
Determine whether the argument is valid or invalid

If he is a man, then he is a human.

He is a dog, not a human.

He is not a man.

Symbolically:
Let p = he is a man, q = he is a human, He is a dog = ~q

$$p \to q$$

$$\sim q$$

$$\sim p$$

p	q	$\sim q$	$p \to q$	$\sim p$	$(p \to q) \wedge (\sim q)$	$[(p \to q) \wedge (\sim q)] \to \sim p$
T	T	F		F		
T	F	T		F		
F	T	F		T		
F	F	T		T		

This is a **Valid/Invalid** form of logical argument. (Circle the correct one)

This style of logic is called, indirect reasoning, the law of contraposition, or Modus Tollens.

To check your answer, copy this $((p \to q) \wedge (\sim q)) \to \sim p$ into this link.

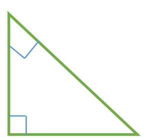
Reasoning by Contradiction: Reductio Absurdum (reduction of an argument to absurdity)

Often in logic, as it is in internet trolling, it is useful or necessary to simply show that something is necessarily false.

A **logical contradiction** is the conjunction of a statement p, with its logical negative, $\sim p$. In symbolic logic, it is the statement $(p \wedge \sim p)$. This statement will ALWAYS be valued as false.

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

Example: Determine whether the argument is NECESSARILY false.



All right triangle cannot have two right angles.

But it is a right triangle and it does have two right angles

Symbolically:
Let p = it is a right triangle,
q = it cannot have two right angles
it does have two right angles = ~q

$$p \to q$$

$$p \wedge \sim q$$

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p	q	$\sim q$	$p \to q$	$p \wedge \sim q$	$[(p \to q) \wedge (p \wedge \sim q)]$
T	T				
T	F				
F	T				
F	F				

So this argument **IS/ISNT** necessarily false. (circle one)

To check your answer, copy this $((p \to q) \wedge (p \wedge \sim q))$ into this link.

Logical Fallacies: Follow this link to a video of some examples

Unfortunately, when we step outside of the theoretical, as in life, things become more complicated. Arguments are often not logical at all.

A few effective but easily seen logical fallacies are:

Appeal to Force: 2+2 = 5, but if you don't agree with me, maybe we can talk it out over a lunch of knuckle sandwiches.

Appeal to Emotion: You should not give me a speeding ticket for driving 90 mph, because I was running late to my charity fundraiser for sick kids with single parents who have fostered abused animals that work as service animals to the elderly.

Appeal to the Majority: You should really try shooting up heroin, all the cool kids are doing it.

Circular Argument: Circular arguments are fun! They are fun because they are circular.

https://youtu.be/kAqIJZeeXEc: Link to *Idiocracy* clip about electrolytes.

Some stronger arguments may be used as a way to manipulate an argument in ones favor, in a way that may be less obvious.

This style of argument is more dangerous in that a person or observer may not even realize that the argument is logically unsound, but they may concede to it in ignorance or in spite.

The Strawman Argument: When you attempt to defeat an opponent's argument by attacking a position the opponent doesn't really hold.
Example:

The Ad Hominem Fallacy (against the man) Click this link to view video example: When you attempt to defeat or subvert an argument by questioning the opponents character, religious or personal beliefs, or their personal associations, rather than evaluating the soundness and validity of the argument itself.

Example: In an online article from VeloNews: *The Journal of Competitive Cycling*.
After an article was published commenting on the retirement of Lance Armstrong; one commenter wrote that Armstrong was a great athlete and that was evidenced by his long and consistent record of accomplishments.
Another commenter posted in response:
"He's not a great athlete; he's a fraud, a cheat, and a liar."
This may be true, but it does not address the persons claim.

Perhaps the hardest logical fallacies to detect are some forms that are not so overt as to dodge an argument, but rather come from a mistreatment of logical arguments.

The Fallacy of the Converse

Recall: the converse of a conditional statement $p \to q$ is the statement $q \to p$

Symbolically:
Let p = I eat dairy,
q = I have a stomach ache

$$\begin{array}{c} p \to q \\ q \\ \hline p \end{array}$$

Example:
Determine if the argument is valid or invalid.

If I eat any dairy, it gives me a stomach ache.
Last night my stomach was hurting.

I must have eaten dairy.

p	q	$p \to q$	$(p \to q) \wedge q$	$[(p \to q) \wedge q] \to p$
T	T			
T	F			
F	T			
F	F			

This is a **Valid/Invalid** form of logical argument. (Circle the correct one)

What is tricky about this argument is that it is mostly valid, but there are some instances when this argument is not true. Just because the argument works ¾ of the time, does not make it acceptable logic. At best you could say that the outcome may be "probable" provided all instances of p & q are equally likely, but definitely not guaranteed.

To check your answer, copy this $((p \to q) \wedge q) \to p)$ into this link.

The Fallacy of the Inverse

Recall, the inverse of a conditional statement $p \to q$ is the statement $\sim p \to \sim q$

Example:
Determine if the argument is valid or invalid

If I get straight A's, then I can get into university.
I did not get straight A's.

I will not get into a university.

Symbolically:
Let p = I get all A's,
q = I will get into university

$$p \to q$$

$$\sim p$$

$$\sim q$$

p	q	$\sim p$	$\sim q$	$p \to q$	$(p \to q) \wedge \sim p$	$[(p \to q) \wedge \sim p] \to \sim q$
T	T					
T	F					
F	T					
F	F					

This is a **Valid/Invalid** form of logical argument. (Circle the correct one)

Because this is not a tautology, it is again not a valid logical form, as its truth value depends on the values of the component statements. It is not true for all cases.
Just like the last argument, this is tricky because it is mostly valid, but there are also some instances when this argument is not true.

To check your answer, copy this $((p \to q) \wedge \sim p) \to \sim q$ into this link.