

ἀγεωμέτρητος μηδείς εἰσίτω

“Let no one untrained in geometry enter here” ~Plato



Inscribed above the entrance of Plato's Academy in Athens Greece in approximately 387 BC
<https://www.iep.utm.edu/academy/>

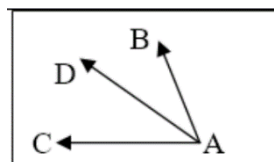
It creates pause to imagine that the foundation of the Greek education would stand upon geometry. Considering the Academy did not focus only on mathematics, why did Plato believe so heavily on geometry? Was it the formulations and knowledge of triangles and lines, or was it the formal approach and strict adherence to logic that Plato revered?

Vocabulary:

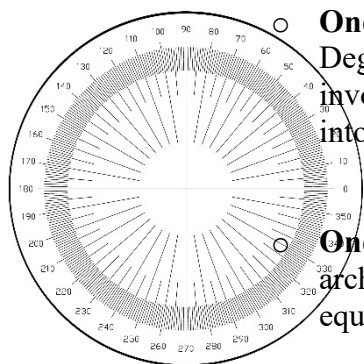
- **Point:** “that which has not part” Euclid’s Elements, Book I, Definition 1. A point has a location in space but no size or mass. Points are represented by a dot and by a capital letter. \cdot A
- **Line:** In infinite set of points with the requirement s.t. for any two points on the line there is a third point on the line between the first two points. Written as \overleftrightarrow{AB} or with a lower case letter l .
 - Two or more points on the same line are said to be **collinear**.
 - **Parallel lines** are lines in the same plane that never intersect and are always the same distance apart.
 - When two lines have a point in common they are said to **intersect**.
 - Two non-parallel lines will intersect, at a single point.
 - When two lines meet to form congruent adjacent angles the lines are said to be **perpendicular**.
- **Def:** A line which starts at a point, A and goes off in a particular direction, towards B to infinity is called a **Ray**. It is denoted \overrightarrow{AB} .
- **Line segments:** written \overline{AB} . The *length* of this line segment can be written as \overline{AB} or $|AB|$. It is the length of line containing and between points A & B.
- **Def:** A **plane** is a 2-D shape that has infinite length and width but no thickness.
- Planes can be parallel, these are called parallel planes, and they are defined as any two planes that do not intersect.
- The intersection of two non-parallel planes will always be a line.
- Two lines that lie in separate planes and never intersect are called **skew lines**.

- The **midpoint** of a line segment is the point that separates the line segment into two line segments each having equal measure.
- **Def:** An **angle** is the union of two rays that share a common endpoint.
- **Vertex of an angle:** Given two non parallel lines the point of their intersection is called the **vertex** of the angle made by the two lines. It is the pointy part of the shape and is labeled $\angle ABC$ where point **B** is always the vertex.

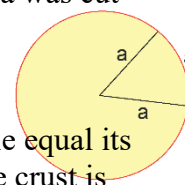
If rays \overrightarrow{AC} & \overrightarrow{AB} are “united”, the resulting Angle is $\angle BAC$.
 $m\angle BAC$ means the “measure of Angle BAC”.



- The measure of an angle is the amount of rotation one line is rotated from another. There are two traditional units of measurement for angles, **degrees** and **radians**

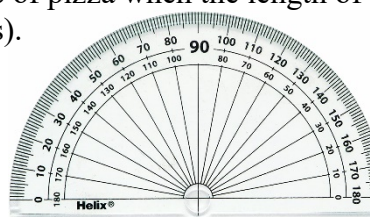


- **One degree, 1°**, is $1/360^{\text{th}}$ of a full rotation of one side/ray of an angle back onto itself. Degrees originated from the Babylonians, whose calendar system and numeration system involved quantities of 360. One degree amounts to one slice of pizza if the pizza was cut into 360 equal sized slices.

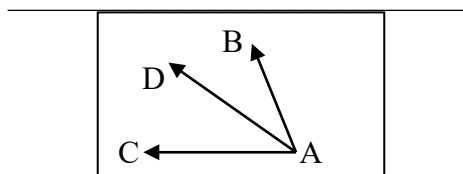


- **One radian, 1 rad**, is the amount of angle required to make the radius of a circle equal its arc length. In food terms, it's the size of a slice of pizza when the length of the crust is equal to the length of the side of the slice (radius).

- **Protractor:** Instrument used to measure angles.

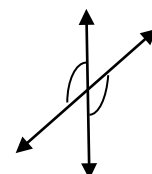


Ex: 1.2.7 Name all the angles in the figure below.



Types of Angles:

- **Right:** An angle whose measure is **90°**
- **Acute:** “a cute little tiny angle” So it is **less** than 90°, i.e. any angle which is **less than** 90°.
- **Obtuse:** “an angle that is hard to understand” because it is more than right but less than straight, i.e. any angle whose measure is between **90°** and **180°**.
- **Strait Angle:** a single angle whose measure is exactly **180°**. I.E. An angle whose sides form a **straight line**.
- **Reflex Angle:** An angle whose measure is between **180°** and **360°**.
- **Vertical Angle:** Two non-adjacent angles formed from the intersection of two lines.



- **Complementary Angles:** Two acute angles that are just right together, they sum to 90° .
- **Supplementary Angles:** Two angles that together make a straight angle, they sum to 180° .
- In general shapes can be labeled by their adjacent vertices. Ex: Rectangle $xyzw$ (can not be expressed as $xzyw$ because \overline{yw} is not a side of the rectangle, but it can be expressed as $yzwx$). Triangles can be labeled with ABC (where A,B,C are vertices of the three angles).
- **Congruent:** has corresponding properties. Lengths of Sides, angles, etc.
This is similar to the idea of equality, but two different lines are not equal unless they are the very same lines, so we say they are congruent, meaning, same but different.
 - \cong means congruent:
- **Bisect:** to separate something into **two parts of equal measure**. This can apply to lines and angles.

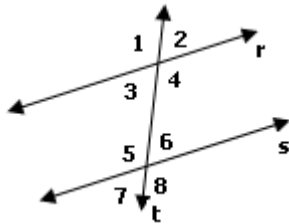
Suppose lines r & s are \parallel and are cut by the transversal t .

Postulate: VERY IMPORTANT POSTULATE (Transversal Postulate for Corresponding Angles)

If two parallel lines are cut by a transversal, then the corresponding angles are congruent.

There are 5 very important and useful theorems pertaining to parallel lines with a transversal.

If two parallel lines are cut by a transversal, Then



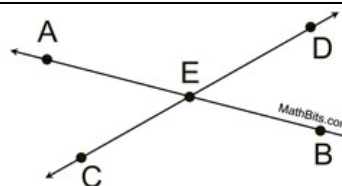
- 1) *the corresponding angles are congruent.*
- 2) *the alternate interior angles are congruent*
- 3) *the alternate exterior angles are congruent*
- 4) *the interior angles on the same side of the transversal are supplementary*
- 5) *the exterior angles on the same side of the transversal are supplementary.*

Examples:

Decide whether each statement is T or F:

- a) T / F If A & B are distinct points on a line, then ray \overrightarrow{AB} and ray \overrightarrow{BA} represent the same set of points.
- b) T / F If two lines intersect, they lie in the same plane.
- c) T / F If two lines do not intersect, they must be parallel.
- d) T / F There is no angle that can be its own complement.
- e) T / F line segment \overline{PQ} is equal to line segment \overline{QP} .

Use the image to answer the following questions:

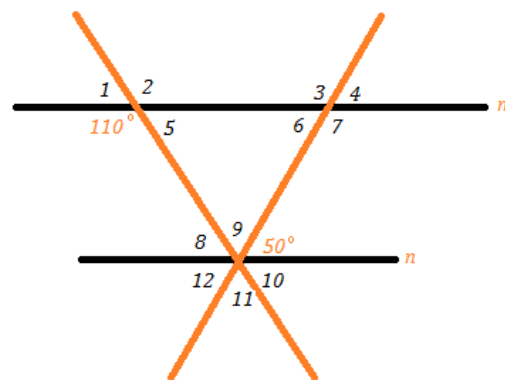


- a) If $m\angle AED = 137^\circ$ then find $m\angle DEB$ & $m\angle AEC$ & $m\angle BEC$
- b) If $m\angle AEC = (5x - 129)^\circ$ & $m\angle DEB = (2x - 21)^\circ$
- c) If $m\angle CEA = (x + 1)^\circ$ & $m\angle AED = (4x - 56)^\circ$
- d) If angles A and B are complimentary, and angle B has a measure that is twice that of angle A, what is the measure of each angle.

Answers:
 a) $m\angle DEB = 43^\circ, m\angle AEC = 43^\circ, m\angle BEC = 137^\circ$
 b) $m\angle AEC = 51^\circ = m\angle DEB$
 c) $m\angle CEA = 48^\circ$ & $m\angle AED = 132^\circ$
 d) $m\angle A = 30^\circ, m\angle B = 60^\circ$

Use the sketch to find the measure of each numbered angle. Assume that $m \parallel n$.

- | | |
|----|-----|
| 1. | 7. |
| 2. | 8. |
| 3. | 9. |
| 4. | 10. |
| 5. | 11. |
| 6. | 12. |



Answers: All measurements are in degrees.
 1:70 2:110 3:130 4:50 5:70 6:50 7:130 8:70 9:60 10:70 11:60 12:50

The supplement of an angle measures 25° more than twice its complement. Find the measure of the angle.

Let $x =$ _____