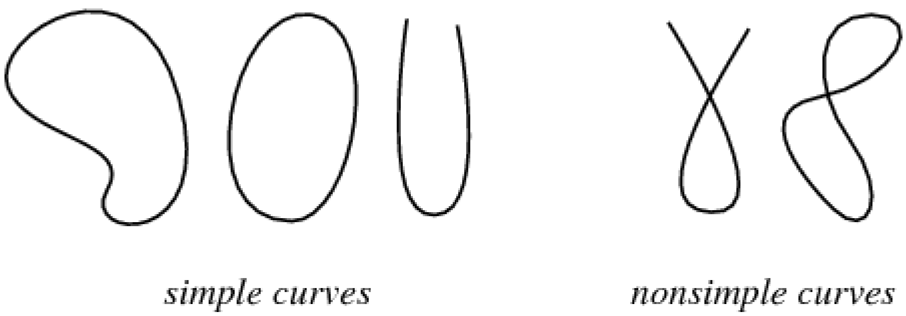
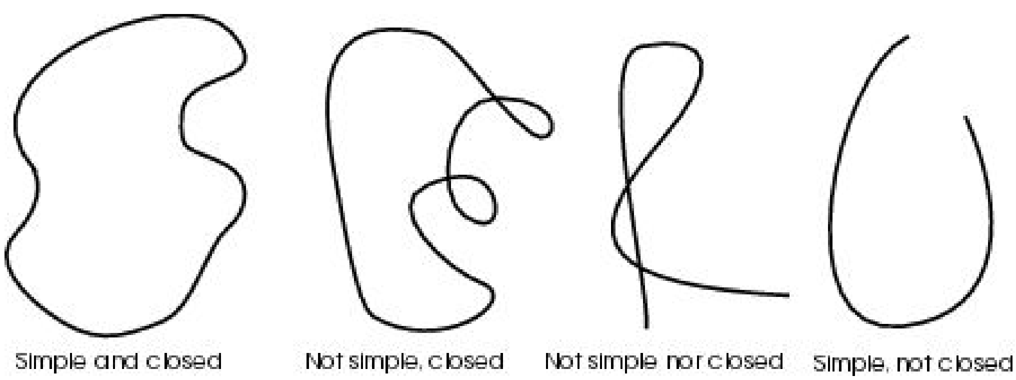
Goals:

- To learn about curves, polygons and Circles by learning introductory definitions and theorems.
- To attempt to "Prove" the logical correctness of some theorems.

Definition: A **simple curve** is a non-straight line that can be drawn continuously without gaps, and the line never intersects itself. This curve lives entirely in a single plane.

Definition: A **closed curve** is a curve in a plane that has the same point as the starting and ending point. It must be drawn continuously.

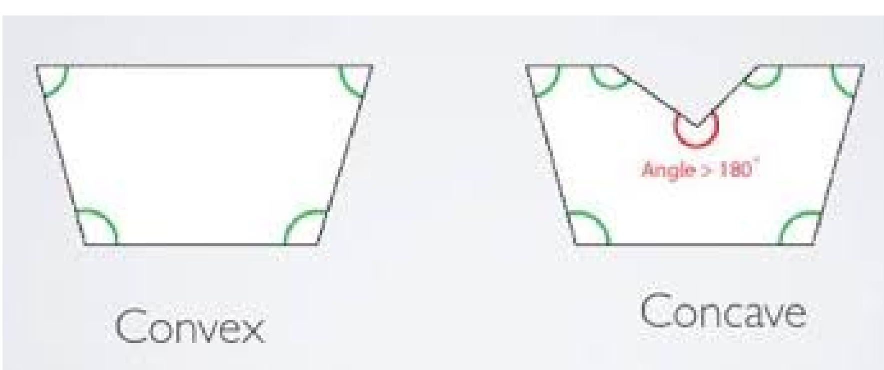




Definition: A simple closed curve is said to be **convex** if, for any two points A & B inside the figure, the line segment AB is always contained as a subset of points inside the simple closed curve.

Another definition of **convex polygons** is that every interior angle has a measure that is less than 180°.

Definition: A simple closed curve that is not convex, is said to be **concave**. It will then have at least one interior angle that is more than 180°, and doing so will create the existence of two points, M & N such that the line segment $\overline{MN}$ is not completely contained inside the concave curve.



Definition: A polygon, is a closed simple curve made up of only straight line segments. These line segments are called sides, the points were the different segments meet are called vertices.

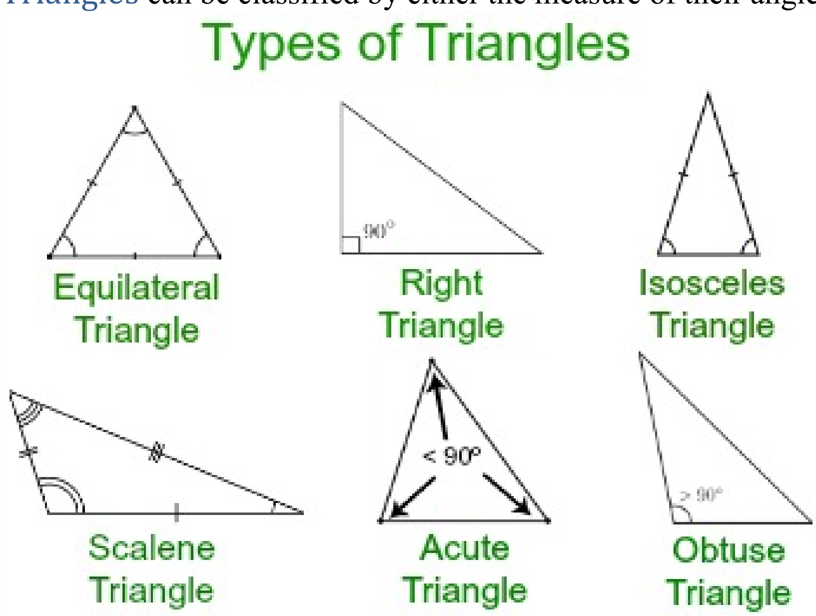
Polygons are classified by the number of sides or vertices there are in the polygon.

The addition of the term regular to the polygon means that the polygon will have all side lengths and interior angle measurements being equal.

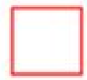
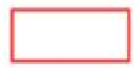
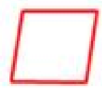
The names of the planer polygons are given below, the interior angle measure is for the regular version of the polygon, where all the angles have equal measure:

Number of Sides	Polygon Name	Interior Angle Measure
3	Triangle	60°
4	Quadrilateral	90°
5	Pentagon	108°
6	Hexagon	120°
7	Heptagon	128.571°
8	Octagon	135°
9	Nonagon	140°
10	Decagon	144°
11	Hendecagon	147.273°
$\vdots$	$\vdots$	$\vdots$
n	n - gon	$\left(\frac{(n-2)\cdot 180}{n}\right)^\circ$

Triangles can be classified by either the measure of their angles or by the lengths of their sides.



Quadrilaterals can be varied in their classifications as well. Most often this classification depends on the nature of the non adjacent sides to be parallel or not.

Sides	Angles	Name Of Quadrilateral
1. Opposite sides are equal and parallel.	Opposite angles are equal.	Parallelogram 
2. All four sides equal and two pairs of opposite sides parallel.	Each angle equals to 90°.	Square 
3. Two pairs of opposite sides are equal and parallel.	Each angle equals to 90°.	Rectangle 
4. All four sides equal and two pair of opposite sides parallel.	Two pairs of opposite angles equal.	Rhombus 
5. One pair of opposite sides parallel.	All four angles unequal.	Trapezium 

Our very first Theorem from geometry.
This comes from Euclids Elements, book 1, proposition 32

It is as follows:

Proposition 32

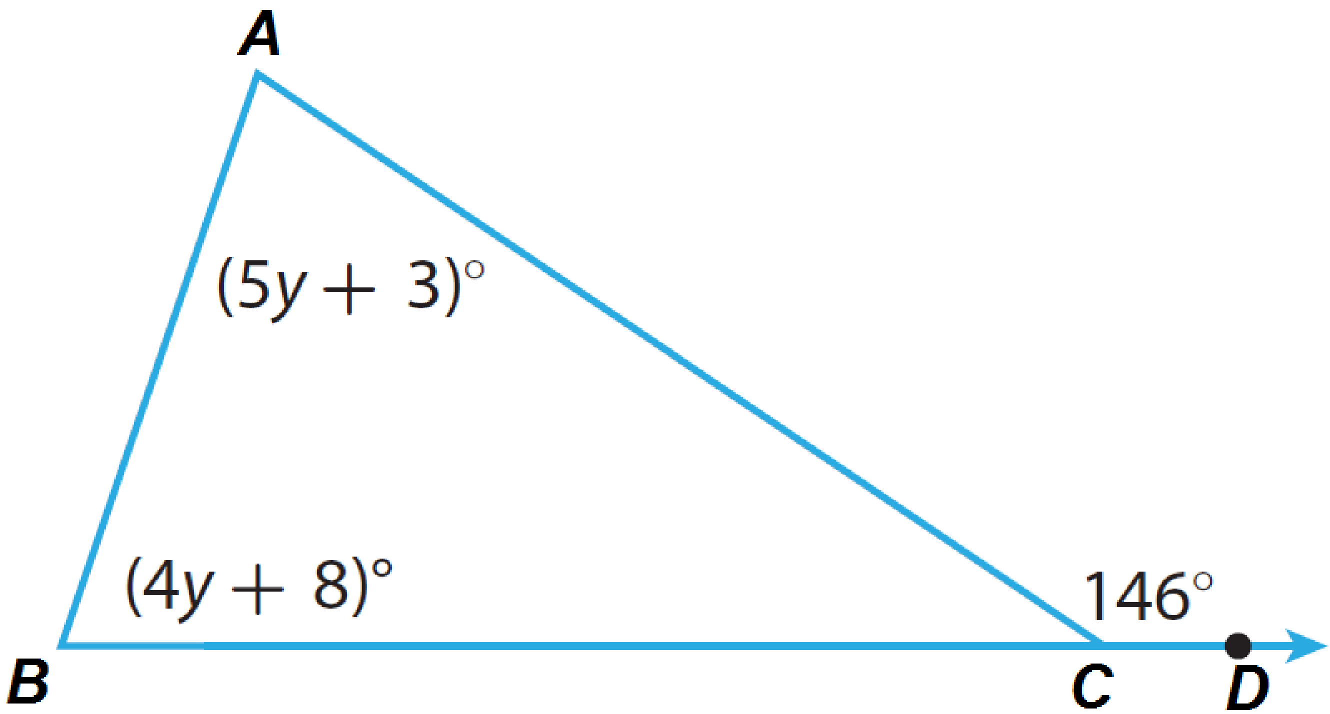
In any triangle, if one of the sides is produced, then the exterior angle equals the sum of the two interior and opposite angles, and the sum of the three interior angles of the triangle equals two right angles.

This theorem has two theorems tied together as one, our book along with most others would bread this theorem down to two smaller theorems.

The first of which is the Angle Sum of a Triangle, which most people already know, that in a Euclidian plan, that the sum of the interior angles of a triangle add to 180°.

The second of which is the Exterior Angle Measure theorem, that says that the sum of any two interior angles in any triangle is equal to the exterior angle formed by the third angle.

Example: Find the value of the unknown variable, y, then fin each interior angle for triangle ABC. Last, verify that the sum of the interior angles of the triangle is 180°.



Ans: y=15, $m\angle BAC = 78^\circ, m\angle ABC = 68^\circ, m\angle ACB = 34^\circ, their\ sum, 78 + 68 + 34 = 180^\circ$

Proving Euclid's Postulate 32 the Sum of the interior angles of any triangle is 180°.:
Lets examine this theorem symbolically.
p=
q=
$p \rightarrow q =$

What we need to do, is what Abraham Lincoln sought to do with the rules of the legal system, prove that something was necessarily true.

We are going to use the logical argument style of modus ponens, but in a less formal way.
This argument will be something like:
Given any triangle, then it has three interior angles.
The sum of two of the three interior angles has a measure that is equal to the exterior angle of the third angle.
Since this exterior angle forms a is supplementary to the third interior angle,
Then the last interior angle must be such that the sum of the three adds to 180°.

It is if this then that, if that then the other, since the other, it must be … and so on.

Lets now try to prove one or both of the parts to postulate 32.

Circles:

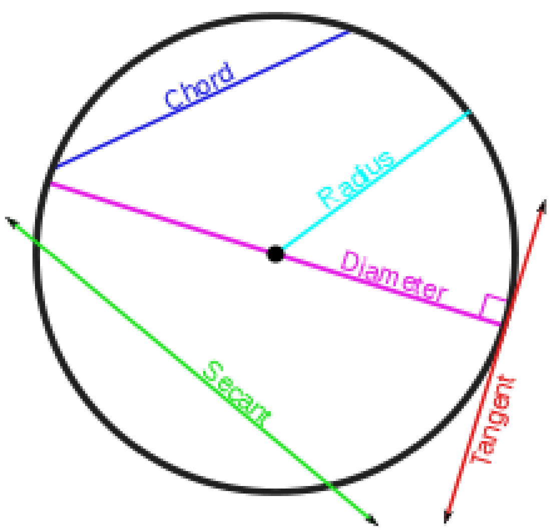
Definition: A **circle** is the set of all points in a plane that are equidistant from a fixed point, called the **center.**

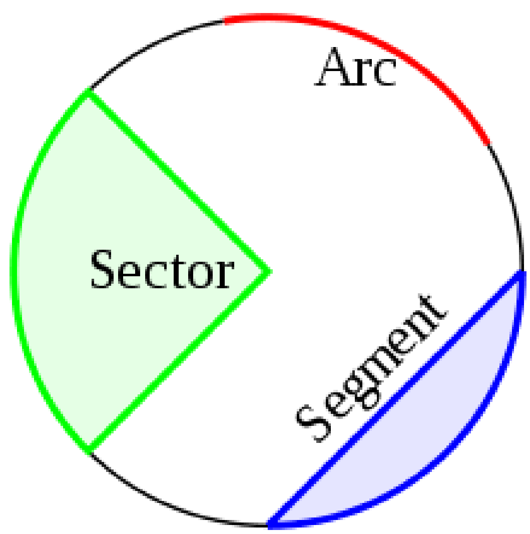
Definition: the distance from the center of a circle to any point on the circle is called the **radius** of the circle. The line segment that passes through the center and touches the circle at two points is called the **diameter** of the circle.

Definition: Any line segment whose endpoints lie on the circle is called a **chord.**
Definition: Any line that intersects a circle exactly once, is said to be a **tangent line** to the circle.

Definition: Any angle formed by two radii are called **central angles**.
Any angle created by two chords with a common intersecting point on the circle are said to be **inscribed angles.**

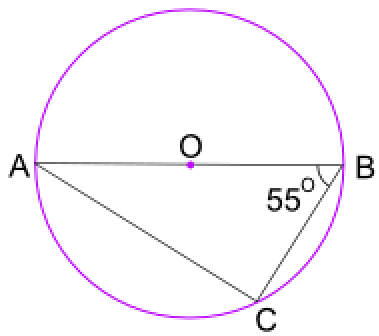
Definition: The **intercepted arc** is a section of the circumference of a circle. It is encased on either side by two different chords or line segments that meet at one point, called a **vertex**, on the other side of the circle or in the middle of the circle. The angle formed by these two chords or line segments is called an **inscribed angle** or a **central angle** depending on where the vertex lies.





Example: Sketch a circle then label its center, radius, diameter, chord, central angle, tangent, inscribed angle, and intercepted arc.

Example: Given the circle with center O, find $m\angle ACB$, $m\angle CAB$, $m\angle OCB, m\angle COB$



Theorem: Any angle inscribed in a circle has a degree measure of half of that of its intercepted arc.

Example, Find the measure of the intercepted arcs: $m\widehat{AB}$, $m\widehat{AC}$, $m\widehat{CB}$

Example: Prove that any angle inscribed in a semi-circle will be a right angle.