Goals:

- To lean about Modular Systems of arithmetic
- To understand the correlation of these systems and our own understanding of clocks and remainders as being the same ideas.

Modular Systems, Clocks, and Remainders

Since very early in our history, it has been known that things in our world have a notion of cycles. That the length of time from one day to the next was constant, that the length of time between the solstice was constant, that the calendar was essentially cyclic, our days are cyclic, our seasons, and our lives.

Because of this, many early cultures began to formalize their recognition that in some sense, each day of the solstice, or, the first day of their calendar in a year, or the time when the sun is highest, is essentially equivalent from one day or year to the next. This is the idea in modular arithmetic. That 1pm is 14 hours past midnight, and tomorrow 38 hours past midnight it will be the same time, as will the time 62 hours past midnight, it will be 1pm.

Clock Arithmetic (12hr and 24 hr. time keeping)

Question: If the time is 2pm, then what time will it be in 11 hours?

Ans: 2am+11hrs = 13 hrs so by the 24 hr clock it will be 13:00 or on the 12 hr. clock it will be 1pm.

In the 12 hour clock, the time at the top of the clock is always one of these numbers:

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

In the 24 hour clock, the time at the top of the clock is always one of these numbers:

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12,13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24

After these times, the clock returns to the beginning and begins to count again.

Question: If the time is 8pm, what time will it be in 90 hrs? (use the 24 hour clock) Ans: 2pm

The mathematics of this question (and others of their type) are exactly where Modular arithmetic was derived.

It came from the cyclic nature of our experiences and our ability to generalize that all "Mondays" are in some sense, the same.

Example: If today is Monday, then what day of the week will it be in 31 days?

In our examples we are acknowledging that after counting some number of things (12 hours, 24 hours, 7 days) that the next hour, day, etc. will be equivalent to the first.
In mathematics we would say that
8 PM is equivalent to 8am Modulo 12
5 AM is equivalent to 29 modulo 24
And that Wednesday is equivalent to the Wednesday of next week when you think of it modulo 7.

Example:
In 12 hr clock arithmetic, find the sum 9+13.

Method 1:
9+13=22

22/12=1.8333333…
This means 22 has one 12 hour period in it then it start to go around again, but not all the way, in fact only .83333… times the way.
So what number is .83333…times 12?
.83333… * 12 = 10 (which is the remainder so 22/12=**1**r10)

So we say that 22 hours is equivalent to 10 o'clock in 12 hr clock arithmetic.

In grade school we would have said 1 r 10

But in math these mean the same thing as when we say that
$22 \equiv 10 \ mod \ 12$ (we say this as 22 is equivalent to 10 mod 12)

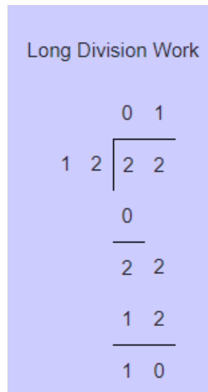
Method 2:

9+13=22 what is the time at 22 hours?

Visually it looks like this

Hour:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	
Time:	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10	

But to write it out in a shorter way, it looks like this



long division calculator

So 22 is the same as 1- 12 hour period plus another 10 more hours after that.

In grade school we would say 22/12 = 1 remainder 10
Which is to say one time through the 12 hour clock and then another 10 hours.

And in math it again means the same thing as when we say $22 \equiv 10 \ mod \ 12$

Example: Find 4 numbers that are equivalent to 5 mod 12.
(Another way to ask this is what number of hours are the same time as 5 in 12 hr clock arithmetic [find 4 more numbers that are equal to some number with remainder 5 when divided by 12] or find 4 more numbers that are a multiple of 12, then plus 5)

Modular Systems:

In a more generalized way, modular arithmetic reduces the infinite set of the whole numbers (0,1,2,3,4,5,…) to a finite subset, which are based upon remainders.
Any two numbers when divided by the same number, n, are said to be equivalent modulo n if they have the same remainder.

Example: 1,3,5,7,9 are all equivalent modulo 2 because when divided by 2 ($\equiv 1 \; mod \; 2$), they all have the same remainder, 1. Likewise, 0,2,4,6,8 are also all equivalent modulo 2 ($\equiv 0 \; mod \; 2$)because they all have a remainder of 0.
So we can think of the entire set of the whole numbers as being reduced to either even (remainder 0 when divided by 2) or odd (remainder 1 when divided by 2).

Congruence Modulo n

The integers a and b are congruent modulo n (where n is a natural number greater than 1, called the modulus) if and only if the difference $a - b$ is divisible by n. Symbolically, this congruence is written as

$$a \equiv b \; mod \; n$$

What $a \equiv b \; mod \; n$ tells us is that they both have the same remainders when divided by the modulus, n.

Ex: is $31 \equiv 47 \; mod \; 16$?
Method 1: Do they have the same remainders when divided by the modulus?

Method 2: Is their difference divisible by the modulus?

Ex: is $20 \equiv 39 \; mod \; 19$?

If today is a Wednesday, then what day of the week will it be one year from now (365 days from now)? What day will it be if this coming year is a leap year (366 days)?

To find the sum difference or product of numbers mod n, the resulting equivalent number mod n will be the same if you mod each number first and add/subtract/multiply their results mod n or add/subtract/multiply the numbers first then mod the result by n.

Examples: Find each sum difference or product as specified.

$a)\ (36+44)(mod\ 7)$ $b)\ (50-37)(mod\ 6)$ $c)\ (7\cdot 8)(mod\ 9)$

Ans: a) 80 mod 7 $\equiv 3\ mod\ 7$ b) 13 mod 6 $\equiv 1\ mod\ 6$ c) 56 mod 9 $\equiv 2\ mod\ 9$

Example: if today is Wednesday, lets say Christmas is in 5 days and new years is 7 days after that, what day of the week will you get your new years day off from work?

Ans: in 7+5 days it will be 12 days from today, which is 12 days from Wednesday, or 12 mod 7 $\equiv 5\ mod\ 7$ five days of the week past next Wednesday: Monday.