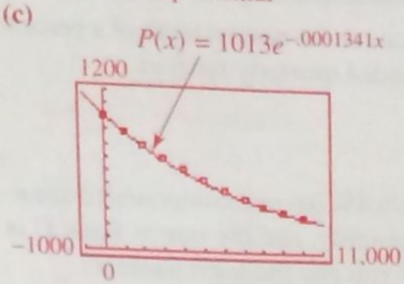
73. **(a)** See the answer graph for part (c). **(b)** exponential
(c)



(d) $P(1500) \approx 828$ mb; $P(11{,}000) \approx 232$ mb **74.** **(a)** The function gives approximately 6965 million, which differs by 886 million from the actual value. **(b)** 7458 million **(c)** 8554 million **75.** **(a)** about 63,000 **(b)** about 42,000 **(c)** about 21,000 **76.** **(a)** about 207 **(b)** about 235 **(c)** about 249 **(d)** The number of symbols approaches 250.

75. ***Deer Population*** The exponential growth of the deer population in Massachusetts can be calculated using the model

$$T = 50{,}000(1 + .06)^n,$$

where 50,000 is the initial deer population and .06 is the rate of growth. T is the total population after n years have passed.

(a) Predict the total population after 4 yr.

(b) If the initial population was 30,000 and the growth rate was .12, approximately how many deer would be present after 3 yr?

(c) How many additional deer can we expect in 5 yr if the initial population is 45,000 and the current growth rate is .08?

76. ***Employee Training*** A person learning certain skills involving repetition tends to learn quickly at first. Then learning tapers off and approaches some upper limit. Suppose the number of symbols per minute that a person using a word processor can type is given by

$$p(t) = 250 - 120(2.8)^{-.5t},$$

where t is the number of months the operator has been in training. Find each value.

(a) $p(2)$ **(b)** $p(4)$ **(c)** $p(10)$

(d) What happens to the number of symbols per minute after several months of training?

9.2 Logarithmic Functions

Logarithms ▪ **Logarithmic Equations** ▪ **Logarithmic Functions** ▪ **Properties of Logarithms**

Logarithms The previous section dealt with exponential functions of the form $y = a^x$ for all positive values of a, where $a \neq 1$. The horizontal line test shows that exponential functions are one-to-one, and thus have inverse functions. The equation defining the inverse of a function is found by interchanging x and y in the equation that defines the function. Doing so with

$$y = a^x \quad \text{gives} \quad x = a^y \quad \text{(Section 6.1)}$$

as the equation of the inverse function of the exponential function defined by $y = a^x$. This equation can be solved for y by using the following definition.

Logarithm

For all real numbers y, and all positive numbers a and x, where $a \neq 1$:

$$\mathbf{y = \log_a x} \quad \text{if and only if} \quad \mathbf{x = a^y}.$$

Exponent
↓
Logarithmic form: $y = \log_a x$
↑
Base

The "log" in the definition above is an abbreviation for **logarithm.** Read $\log_a x$ as "the logarithm to the base a of x."

By the definition of logarithm, if $y = \log_a x$, then the power to which a must be raised to obtain x is y, or $x = a^y$. To remember the location of the base and the exponent in each form, refer to the diagrams in the margin.

Exponent
↓
Exponential form: $a^y = x$
↑
Base

Meaning of $\log_a x$

A logarithm is an exponent. The expression $\log_a x$ is the exponent to which the base a must be raised to obtain x.

The table shows several pairs of equivalent statements, written in both logarithmic and exponential forms.

Teaching Tip Have students verbalize the statements in the left column of the table. For instance, "$\log_2 8$ is the exponent to which 2 must be raised to obtain 8."

Logarithmic Form	Exponential Form
$\log_2 8 = 3$	$2^3 = 8$
$\log_{1/2} 16 = -4$	$\left(\frac{1}{2}\right)^{-4} = 16$
$\log_{10} 100{,}000 = 5$	$10^5 = 100{,}000$
$\log_3 \frac{1}{81} = -4$	$3^{-4} = \frac{1}{81}$
$\log_5 5 = 1$	$5^1 = 5$
$\log_{3/4} 1 = 0$	$\left(\frac{3}{4}\right)^0 = 1$

Now try Exercises 3, 5, 7, and 9.

Logarithmic Equations The definition of logarithm can be used to solve a **logarithmic equation,** an equation with a logarithm in at least one term, by first writing the equation in exponential form.

EXAMPLE 1 Solving Logarithmic Equations

Solve each equation.

(a) $\log_x \frac{8}{27} = 3$ **(b)** $\log_4 x = \frac{5}{2}$ **(c)** $\log_{49} \sqrt[3]{7} = x$

Solution

(a)

$$\log_x \frac{8}{27} = 3$$

$$x^3 = \frac{8}{27} \quad \text{Write in exponential form.}$$

$$x^3 = \left(\frac{2}{3}\right)^3 \quad \frac{8}{27} = \left(\frac{2}{3}\right)^3$$

$$x = \frac{2}{3} \quad \text{Take cube roots.}$$

The solution set is $\left\{\frac{2}{3}\right\}$.

(b)

$$\log_4 x = \frac{5}{2}$$

$$4^{5/2} = x \quad \text{Write in exponential form.}$$

$$(4^{1/2})^5 = x \quad a^{m/n} = (a^{1/n})^m$$

$$2^5 = x \quad 4^{1/2} = (2^2)^{1/2} = 2$$

$$32 = x$$

The solution set is $\{32\}$.

(c)

$$\log_{49} \sqrt[3]{7} = x$$

$$49^x = \sqrt[3]{7} \quad \text{Write in exponential form.}$$

$$(7^2)^x = 7^{1/3} \quad \text{Write with the same base.}$$

$$7^{2x} = 7^{1/3} \quad (a^m)^n = a^{mn}$$

$$2x = \frac{1}{3} \quad \text{Set exponents equal.}$$

$$x = \frac{1}{6} \quad \text{Divide by 2. (Appendix A)}$$

The solution set is $\left\{\frac{1}{6}\right\}$.

Now try Exercises 13, 25, and 27.

Logarithmic Functions We define the logarithmic function with base a as follows.

Logarithmic Function

If $a > 0$, $a \neq 1$, and $x > 0$, then

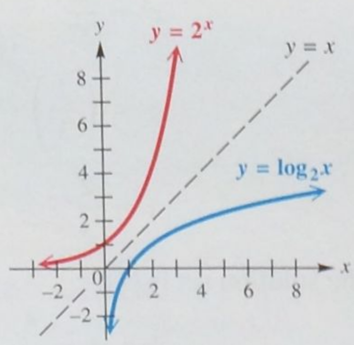
$$f(x) = \log_a x$$

defines the **logarithmic function** with base a.

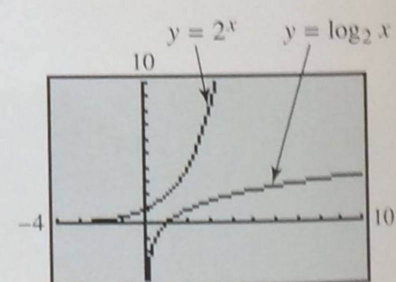
Exponential and logarithmic functions are inverses of each other. The graph of $y = 2^x$ is shown in red in Figure 13(a). The graph of its inverse is found by reflecting the graph of $y = 2^x$ across the line $y = x$. The graph of the inverse function, defined by $y = \log_2 x$, shown in blue, has the y-axis as a vertical asymptote. Figure 13(b) shows a calculator graph of the two functions.

x	2^x
−2	.25
−1	.5
0	1
1	2
2	4

x	$\log_2 x$
.25	−2
.5	−1
1	0
2	1
4	2



(a)



The graph of $y = \log_2 x$ can be obtained by drawing the inverse of $y = 2^x$.

(b)

Figure 13

Since the domain of an exponential function is the set of all real numbers, the range of a logarithmic function also will be the set of all real numbers. In the same way, both the range of an exponential function and the domain of a logarithmic function are the set of all positive real numbers, so ***logarithms can be found for positive numbers only.***

We now summarize information about the graphs of logarithmic functions.

LOGARITHMIC FUNCTION $f(x) = \log_a x$

Domain: $(0, \infty)$ Range: $(-\infty, \infty)$

For $f(x) = \log_2 x$:

x	$f(x)$
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2
8	3

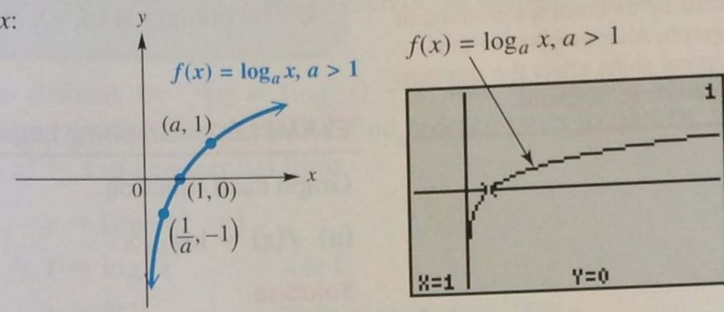


Figure 14

- $f(x) = \log_a x$, $a > 1$, is increasing and continuous on its entire domain, $(0, \infty)$.
- The y-axis is a vertical asymptote as $x \to 0$ from the right.
- The graph goes through the points $(\frac{1}{a}, -1)$, $(1, 0)$, and $(a, 1)$.

For $f(x) = \log_{1/2} x$:

x	$f(x)$
$\frac{1}{4}$	2
$\frac{1}{2}$	1
1	0
2	-1
4	-2
8	-3

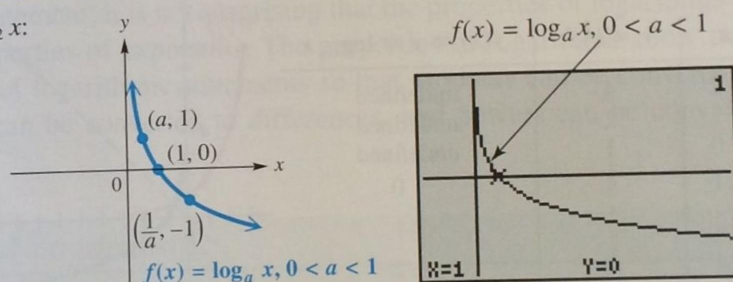


Figure 15

- $f(x) = \log_a x$, $0 < a < 1$, is decreasing and continuous on its entire domain, $(0, \infty)$.
- The y-axis is a vertical asymptote as $x \to 0$ from the right.
- The graph goes through the points $(\frac{1}{a}, -1)$, $(1, 0)$, and $(a, 1)$.

Calculator graphs of logarithmic functions do not, in general, give an accurate picture of the behavior of the graphs near the vertical asymptotes. While it may seem as if the graph has an endpoint, this is not the case. The resolution of the calculator screen is not precise enough to indicate that the graph approaches the vertical asymptote as the value of x gets closer to it. Do not draw incorrect conclusions just because the calculator does not show this behavior. ■

The graphs in Figures 14 and 15 and the information with them suggest the following generalizations about the graphs of logarithmic functions of the form $f(x) = \log_a x$.

TEACHING TIP Encourage students to memorize the characteristics of the graph of $f(x) = \log_a x$. Ask them how the coordinates $\left(\frac{1}{a}, -1\right)$, $(1, 0)$, and $(a, 1)$ are affected by different translations of the graph. Also, emphasize that horizontal shifts affect the domains of logarithmic functions.

Characteristics of the Graph of $f(x) = \log_a x$

1. The points $\left(\frac{1}{a}, -1\right)$, $(1, 0)$, and $(a, 1)$ are on the graph.
2. If $a > 1$, then f is an increasing function; if $0 < a < 1$, then f is a decreasing function.
3. The y-axis is a vertical asymptote.
4. The domain is $(0, \infty)$, and the range is $(-\infty, \infty)$.

EXAMPLE 2 Graphing Logarithmic Functions

Graph each function.

(a) $f(x) = \log_{1/2} x$ **(b)** $f(x) = \log_3 x$

Solution

(a) First graph $y = \left(\frac{1}{2}\right)^x$, which defines the inverse function of f, by plotting points. Some ordered pairs are given in the table with the graph shown in red in Figure 16. The graph of $f(x) = \log_{1/2} x$ is the reflection of the graph of $y = \left(\frac{1}{2}\right)^x$ across the line $y = x$. The ordered pairs for $y = \log_{1/2} x$ are found by interchanging the x- and y-values in the ordered pairs for $y = \left(\frac{1}{2}\right)^x$. See the graph in blue in Figure 16.

x	$y = \left(\frac{1}{2}\right)^x$	$f(x) = y = \log_{1/2} x$
-2	4	undefined
-1	2	undefined
0	1	undefined
1	$\frac{1}{2}$	0
2	$\frac{1}{4}$	-1
4	$\frac{1}{16}$	-2

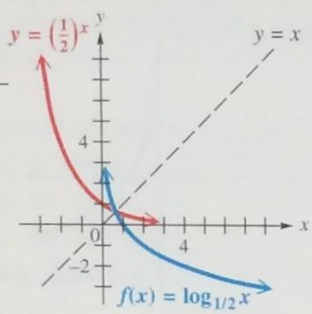


Figure 16

x	$y = \log_3 x$
$\frac{1}{3}$	-1
1	0
3	1
9	2

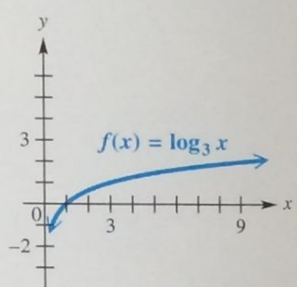


Figure 17

(b) Another way to graph a logarithmic function is to write $f(x) = y = \log_3 x$ in exponential form as $x = 3^y$. Now find some ordered pairs; several are shown in the table with the graph in Figure 17. Be careful to get the ordered pairs in the correct order.

Now try Exercise 45.

EXAMPLE 3 Graphing Translated Logarithmic Functions

Graph each function.

(a) $f(x) = \log_2(x - 1)$ **(b)** $f(x) = (\log_3 x) - 1$

Solution

(a) The graph of $f(x) = \log_2(x - 1)$ is the graph of $f(x) = \log_2 x$ translated 1 unit to the right. The vertical asymptote is $x = 1$. The domain of this function is $(1, \infty)$ since logarithms can be found only for positive numbers.

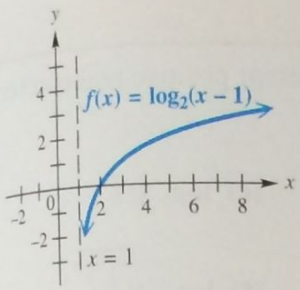


Figure 18

To find some ordered pairs to plot, use the equivalent exponential form of the equation $y = \log_2(x - 1)$.

$$y = \log_2(x - 1)$$

$$x - 1 = 2^y \quad \text{Write in exponential form.}$$

$$x = 2^y + 1 \quad \text{Add 1.}$$

We choose values for y and then calculate each of the corresponding x-values. See Figure 18.

(b) The function defined by $f(x) = (\log_3 x) - 1$ has the same graph as $g(x) = \log_3 x$ translated 1 unit down. We find ordered pairs to plot by writing $y = (\log_3 x) - 1$ in exponential form.

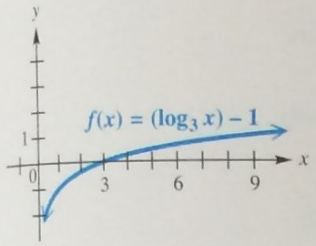


Figure 19

$$y = (\log_3 x) - 1$$

$$y + 1 = \log_3 x \quad \text{Add 1.}$$

$$x = 3^{y+1} \quad \text{Write in exponential form.}$$

Again, choose y-values and calculate the corresponding x-values. The graph is shown in Figure 19.

Now try Exercises 33 and 37.

Properties of Logarithms Since a logarithmic statement can be written as an exponential statement, it is not surprising that the properties of logarithms are based on the properties of exponents. The properties of logarithms allow us to change the form of logarithmic statements so that products can be converted to sums, quotients can be converted to differences, and powers can be converted to products.

Teaching Tip Encourage students to memorize the properties of logarithms.

Looking Ahead to Calculus

A technique called *logarithmic differentiation,* which uses the properties of logarithms, can often be used to differentiate complicated functions.

Properties of Logarithms

For $x > 0$, $y > 0$, $a > 0$, $a \neq 1$, and any real number r:

Property	Description
Product Property $\log_a xy = \log_a x + \log_a y$	The logarithm of the product of two numbers is equal to the sum of the logarithms of the numbers.
Quotient Property $\log_a \frac{x}{y} = \log_a x - \log_a y$	The logarithm of the quotient of two numbers is equal to the difference between the logarithms of the numbers.
Power Property $\log_a x^r = r \log_a x$	The logarithm of a number raised to a power is equal to the exponent multiplied by the logarithm of the number.

Two additional properties of logarithms follow directly from the definition of $\log_a x$ since $a^0 = 1$ and $a^1 = a$.

$$\log_a 1 = 0 \quad \text{and} \quad \log_a a = 1$$

TEACHING TIP Point out that the phrase "quotients can be converted to differences" refers to the quotient property. It does not say that

$$\frac{\log_a x}{\log_a y} = \log_a x - \log_a y$$

or

$$\frac{\log_a x}{\log_a y} = \log_a(x - y),$$

which are common errors that students make. A quotient or product of logarithms cannot be simplified by using the properties of logarithms given in the box.

TEACHING TIP Point out that in Example 4(d), the product and quotient properties of logarithms were applied before the power property. In Example 4(f), the power property was applied first. In general, properties of logarithms should follow the order of operations with respect to exponents, multiplication, and division.

Also point out that the properties are frequently used from right to left.

EXAMPLE 4 Using the Properties of Logarithms

Rewrite each expression. Assume all variables represent positive real numbers, with $a \neq 1$ and $b \neq 1$.

(a) $\log_6(7 \cdot 9)$ **(b)** $\log_9 \frac{15}{7}$ **(c)** $\log_5 \sqrt{8}$

(d) $\log_a \frac{mnq}{p^2}$ **(e)** $\log_a \sqrt[3]{m^2}$ **(f)** $\log_b \sqrt[n]{\frac{x^3y^5}{z^m}}$

Solution

(a) $\log_6(7 \cdot 9) = \log_6 7 + \log_6 9$ Product property

(b) $\log_9 \frac{15}{7} = \log_9 15 - \log_9 7$ Quotient property

(c) $\log_5 \sqrt{8} = \log_5(8^{1/2}) = \frac{1}{2}\log_5 8$ Power property

(d) $\log_a \frac{mnq}{p^2} = \log_a m + \log_a n + \log_a q - \log_a p^2$

$= \log_a m + \log_a n + \log_a q - 2\log_a p$

(e) $\log_a \sqrt[3]{m^2} = \log_a m^{2/3} = \frac{2}{3}\log_a m$

(f) $\log_b \sqrt[n]{\frac{x^3y^5}{z^m}} = \log_b \left(\frac{x^3y^5}{z^m}\right)^{1/n}$ $\sqrt[n]{a} = a^{1/n}$

$= \frac{1}{n}\log_b \frac{x^3y^5}{z^m}$ Power property

$= \frac{1}{n}(\log_b x^3 + \log_b y^5 - \log_b z^m)$ Product and quotient properties

$= \frac{1}{n}(3\log_b x + 5\log_b y - m\log_b z)$ Power property

$= \frac{3}{n}\log_b x + \frac{5}{n}\log_b y - \frac{m}{n}\log_b z$ Distributive property

Notice the use of parentheses in the second and third steps. The factor $\frac{1}{n}$ applies to each term.

Now try Exercises 53, 55, and 59.

EXAMPLE 5 Using the Properties of Logarithms

Write each expression as a single logarithm with coefficient 1. Assume all variables represent positive real numbers, with $a \neq 1$ and $b \neq 1$.

(a) $\log_3(x + 2) + \log_3 x - \log_3 2$ **(b)** $2\log_a m - 3\log_a n$

(c) $\frac{1}{2}\log_b m + \frac{3}{2}\log_b 2n - \log_b m^2 n$

Solution

(a) $\log_3(x + 2) + \log_3 x - \log_3 2 = \log_3 \dfrac{(x + 2)x}{2}$ Product and quotient properties

(b) $2 \log_a m - 3 \log_a n = \log_a m^2 - \log_a n^3$ Power property

$= \log_a \dfrac{m^2}{n^3}$ Quotient property

(c) $\dfrac{1}{2} \log_b m + \dfrac{3}{2} \log_b 2n - \log_b m^2 n$

$= \log_b m^{1/2} + \log_b (2n)^{3/2} - \log_b m^2 n$ Power property

$= \log_b \dfrac{m^{1/2}(2n)^{3/2}}{m^2 n}$ Product and quotient properties

$= \log_b \dfrac{2^{3/2} n^{1/2}}{m^{3/2}}$ Rules for exponents

$= \log_b \left(\dfrac{2^3 n}{m^3}\right)^{1/2}$ Rules for exponents

$= \log_b \sqrt{\dfrac{8n}{m^3}}$ Definition of $a^{1/n}$

Now try Exercises 61, 63, and 65.

CAUTION There is no property of logarithms to rewrite a logarithm of a *sum* or *difference*. That is why, in Example 5(a), $\log_3(x + 2)$ was not written as $\log_3 x + \log_3 2$. Remember, $\log_3 x + \log_3 2 = \log_3(x \cdot 2)$.

The distributive property does not apply in a situation like this because $\log_3(x + y)$ is one term; "log" is a function name, not a factor.

EXAMPLE 6 Using the Properties of Logarithms with Numerical Values

Assume that $\log_{10} 2 = .3010$. Find each logarithm.

(a) $\log_{10} 4$

(b) $\log_{10} 5$

Solution

(a) $\log_{10} 4 = \log_{10} 2^2 = 2 \log_{10} 2 = 2(.3010) = .6020$

(b) $\log_{10} 5 = \log_{10} \dfrac{10}{2} = \log_{10} 10 - \log_{10} 2 = 1 - .3010 = .6990.$

Now try Exercise 69.

Recall from algebra that if $f(x)$ and $g(x)$ are inverse functions, then

$$f[g(x)] = x \qquad \text{and} \qquad g[f(x)] = x.$$

Since $f(x) = a^x$ and $g(x) = \log_a x$ are inverses, the next theorem follows.

TEACHING TIP Point out that the theorem on inverses really follows from the definition of logarithm. For instance, since $\log_a x$ is the power to which a must be raised to obtain x, a raised to that power will be x. In practice, x will often be an algebraic expression.

Theorem on Inverses

For $a > 0$, $a \neq 1$:

$$a^{\log_a x} = x \qquad \text{and} \qquad \log_a a^x = x.$$

By the results of this theorem,

$$7^{\log_7 10} = 10, \qquad \log_5 5^3 = 3, \qquad \text{and} \qquad \log_r r^{k+1} = k + 1.$$

The second statement in the theorem will be useful in Section 9.3 when we solve other logarithmic and exponential equations.

9.2 Exercises

1. (a) C (b) A (c) E (d) B (e) F (f) D 2. (a) F (b) B (c) A (d) D (e) C (f) E
3. $\log_3 81 = 4$ 4. $\log_2 32 = 5$
5. $\log_{2/3} \frac{27}{8} = -3$
6. $\log_{10} .0001 = -4$
7. $6^2 = 36$ 8. $5^1 = 5$
9. $(\sqrt{3})^8 = 81$ 10. $4^{-3} = \frac{1}{64}$
12. $a^0 = 1$, $(a \neq 0)$ for all real numbers a. 13. $\{-4\}$ 14. $\{-4\}$
15. $\{-3\}$ 16. $\{-3\}$ 17. $\left\{\frac{1}{4}\right\}$
18. $\{4\}$ 19. $\{8\}$ 20. $\{5\}$
21. $\{9\}$ 22. $\{11\}$ 23. $\left\{\frac{1}{5}\right\}$
24. $\{4\}$ 25. $\{64\}$ 26. $\left\{\frac{1}{2}\right\}$
27. $\left\{\frac{2}{3}\right\}$ 28. $\left\{\frac{1}{2}\right\}$
29. $\left\{\frac{1}{3}\right\}$ 30. $(0, 1) \cup (1, \infty)$

Concept Check *In Exercises 1 and 2, match the logarithm in Column I with its value in Column II. Remember that $\log_a x$ is the exponent to which a must be raised in order to obtain x.*

	I	II
1.	(a) $\log_2 16$	A. 0
	(b) $\log_3 1$	B. $\frac{1}{2}$
	(c) $\log_{10} .1$	C. 4
	(d) $\log_2 \sqrt{2}$	D. −3
	(e) $\log_e \frac{1}{e^2}$	E. −1
	(f) $\log_{1/2} 8$	F. −2

	I	II
2.	(a) $\log_3 81$	A. −2
	(b) $\log_3 \frac{1}{3}$	B. −1
	(c) $\log_{10} .01$	C. 0
	(d) $\log_6 \sqrt{6}$	D. $\frac{1}{2}$
	(e) $\log_e 1$	E. $\frac{9}{2}$
	(f) $\log_3 27^{3/2}$	F. 4

For each statement, write an equivalent statement in logarithmic form.

3. $3^4 = 81$
4. $2^5 = 32$
5. $\left(\frac{2}{3}\right)^{-3} = \frac{27}{8}$
6. $10^{-4} = .0001$

For each statement, write an equivalent statement in exponential form.

7. $\log_6 36 = 2$
8. $\log_5 5 = 1$
9. $\log_{\sqrt{3}} 81 = 8$
10. $\log_4 \frac{1}{64} = -3$

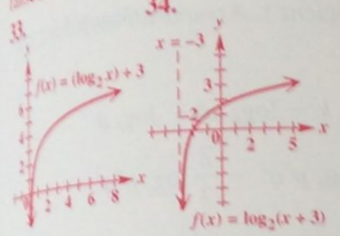
11. Explain why logarithms of negative numbers are not defined.
12. *Concept Check* Why does $\log_a 1$ always equal 0 for any valid base a?

Solve each logarithmic equation. See Example 1.

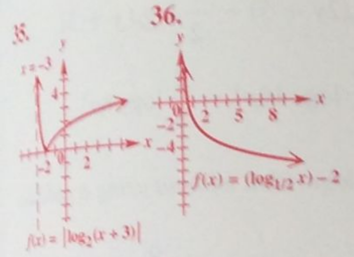
13. $x = \log_5 \frac{1}{625}$
14. $x = \log_3 \frac{1}{81}$
15. $x = \log_{10} .001$
16. $x = \log_6 \frac{1}{216}$
17. $x = \log_8 \sqrt[4]{8}$
18. $x = 8 \log_{100} 10$
19. $x = 3^{\log_3 8}$
20. $x = 12^{\log_{12} 5}$
21. $x = 2^{\log_2 9}$
22. $x = 8^{\log_8 11}$
23. $\log_x 25 = -2$
24. $\log_x \frac{1}{16} = -2$
25. $\log_4 x = 3$
26. $\log_2 x = -1$
27. $x = \log_4 \sqrt[3]{16}$
28. $x = \log_5 \sqrt[4]{25}$
29. $\log_x 3 = -1$
30. $\log_x 1 = 0$

32. y (2.3219281) represents the exponent to which 2 must be raised in order to obtain x (5).

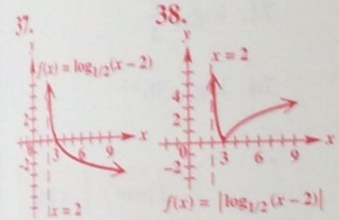
33. 34.



35. 36.

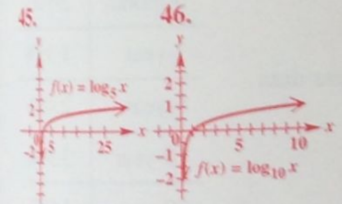


37. 38.

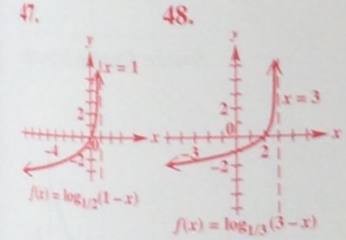


39. E 40. D 41. B 42. C
43. F 44. A

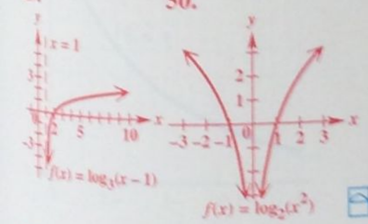
45. 46.



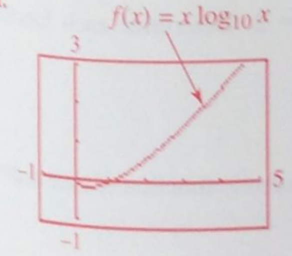
47. 48.



49. 50.

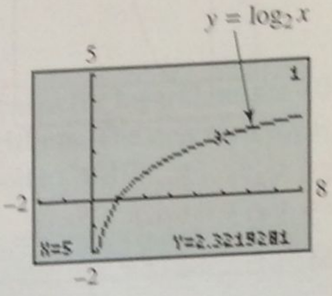


51.



31. Compare the summary of characteristics of the graph of $f(x) = \log_a x$ with the similar summary about the graph of $f(x) = a^x$ in Section 9.1. Make a list of characteristics that reinforce the idea that these are inverse functions.

32. The calculator graph of $y = \log_2 x$ shows the values of the ordered pair with $x = 5$. What does the value of y represent?



Sketch the graph of $f(x) = \log_2 x$. Then refer to it and use graphing techniques to graph each function. See Example 3.

33. $f(x) = (\log_2 x) + 3$ 34. $f(x) = \log_2(x + 3)$ 35. $f(x) = |\log_2(x + 3)|$

Sketch the graph of $f(x) = \log_{1/2} x$. Then refer to it and use graphing techniques to graph each function. See Example 3.

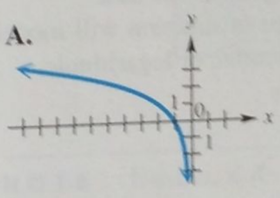
36. $f(x) = (\log_{1/2} x) - 2$ 37. $f(x) = \log_{1/2}(x - 2)$ 38. $f(x) = |\log_{1/2}(x - 2)|$

Concept Check *In Exercises 39–44, match the function with its graph from choices A–F.*

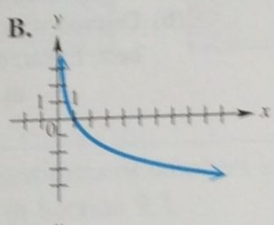
39. $f(x) = \log_2 x$ 40. $f(x) = \log_2 2x$ 41. $f(x) = \log_2 \dfrac{1}{x}$

42. $f(x) = \log_2 \dfrac{x}{2}$ 43. $f(x) = \log_2(x - 1)$ 44. $f(x) = \log_2(-x)$

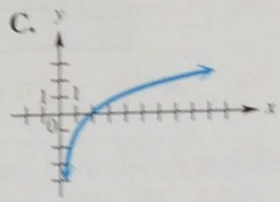
A.



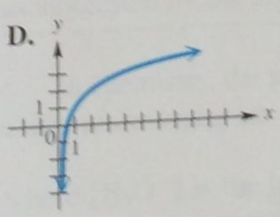
B.



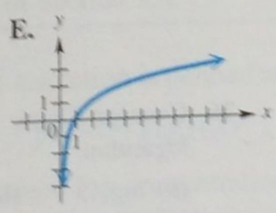
C.



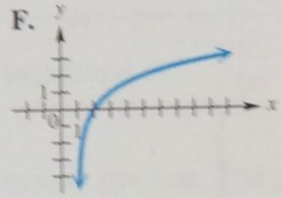
D.



E.



F.



Graph each function. See Examples 2 and 3.

45. $f(x) = \log_5 x$ 46. $f(x) = \log_{10} x$ 47. $f(x) = \log_{1/2}(1 - x)$

48. $f(x) = \log_{1/3}(3 - x)$ 49. $f(x) = \log_3(x - 1)$ 50. $f(x) = \log_2(x^2)$

We write $\log x$ as an abbreviation for $\log_{10} x$. Use the log *key on your graphing calculator to graph each function.*

51. $f(x) = x \log_{10} x$ 52. $f(x) = x^2 \log_{10} x$

Use the properties of logarithms to rewrite each expression. Simplify the result if possible. Assume all variables represent positive real numbers. See Example 4.

53. $\log_2 \dfrac{6x}{y}$ 54. $\log_3 \dfrac{4p}{q}$ 55. $\log_5 \dfrac{5\sqrt{7}}{3}$ 56. $\log_2 \dfrac{2\sqrt{3}}{5}$

57. $\log_4(2x + 5y)$ **58.** $\log_6(7m + 3q)$ **59.** $\log_m \sqrt{\dfrac{5r^3}{z^5}}$ **60.** $\log_p \sqrt[3]{\dfrac{m^5n^4}{t^2}}$

Write each expression as a single logarithm with coefficient 1. Assume all variables represent positive real numbers. See Example 5.

61. $\log_a x + \log_a y - \log_a m$

62. $(\log_b k - \log_b m) - \log_b a$

63. $2 \log_m a - 3 \log_m b^2$

64. $\dfrac{1}{2} \log_y p^3q^4 - \dfrac{2}{3} \log_y p^4q^3$

65. $2 \log_a(z - 1) + \log_a(3z + 2),\quad z > 1$

66. $\log_b(2y + 5) - \dfrac{1}{2} \log_b(y + 3)$

67. $-\dfrac{2}{3} \log_5 5m^2 + \dfrac{1}{2} \log_5 25m^2$

68. $-\dfrac{3}{4} \log_3 16p^4 - \dfrac{2}{3} \log_3 8p^3$

Given $\log_{10} 2 = .3010$ *and* $\log_{10} 3 = .4771$, *find each logarithm without using a calculator. See Example 6.*

69. $\log_{10} 6$ **70.** $\log_{10} 12$ **71.** $\log_{10} \dfrac{9}{4}$

72. $\log_{10} \dfrac{20}{27}$ **73.** $\log_{10} \sqrt{30}$ **74.** $\log_{10} 36^{1/3}$

Solve each problem.

75. *(Modeling) Interest Rates of Treasury Securities* The table gives interest rates for various U.S. Treasury Securities on June 27, 2003.

(a) Make a scatter diagram of the data.

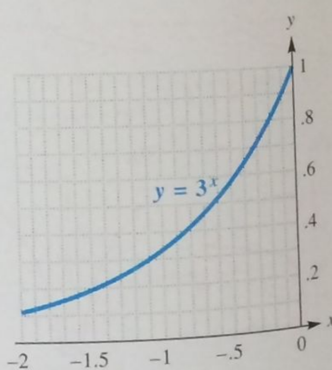
(b) Discuss which type of function will model these data best: linear, exponential, or logarithmic.

Time	Yield
3-month	.83%
6-month	.91%
2-year	1.35%
5-year	2.46%
10-year	3.54%
30-year	4.58%

Source: Charles Schwab.

76. Concept Check Use the graph to estimate each logarithm.

(a) $\log_3 .3$ (b) $\log_3 .8$

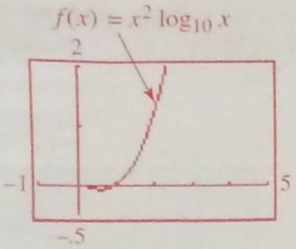


77. Concept Check Suppose $f(x) = \log_a x$ and $f(3) = 2$. Determine each function value.

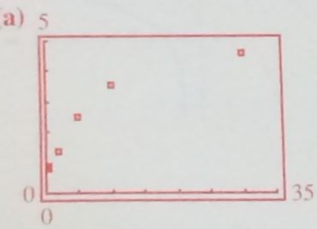
(a) $f\left(\dfrac{1}{9}\right)$ (b) $f(27)$

78. Concept Check If (5, 4) is on the graph of the logarithmic function of base a, does $5 = \log_a 4$ or does $4 = \log_a 5$?

52.



53. $\log_2 6 + \log_2 x - \log_2 y$
54. $\log_3 4 + \log_3 p - \log_3 q$
55. $1 + \dfrac{1}{2}\log_5 7 - \log_5 3$
56. $1 + \dfrac{1}{2}\log_2 3 - \log_2 5$
57. cannot be simplified
58. cannot be simplified
59. $\dfrac{1}{2}(\log_m 5 + 3 \log_m r - 5 \log_m z)$
60. $\dfrac{1}{3}(5 \log_p m + 4 \log_p n - 2 \log_p t)$
61. $\log_a \dfrac{xy}{m}$ **62.** $\log_b \dfrac{k}{ma}$
63. $\log_m \dfrac{a^2}{b^6}$ **64.** $\log_y (p^{-7/6})$
65. $\log_a[(z - 1)^2(3z + 2)]$
66. $\log_b \dfrac{2y + 5}{\sqrt{y + 3}}$
67. $\log_5 \dfrac{5^{1/3}}{m^{1/3}}$ or $\log_5 \sqrt[3]{\dfrac{5}{m}}$
68. $\log_3 \dfrac{1}{32p^5}$ **69.** .7781
70. 1.0791 **71.** .3522
72. −.1303 **73.** .7386
74. .5187
75. (a)



76. (a) −1.1 (b) −.2
77. (a) −4 (b) 6
78. $4 = \log_a 5$