4.1-4.3 10.27.14

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Engr 420
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Inserted from: <file://H:\My Documents\Folsom Lake College\ENGR 420 Statics\Notes\Engr 420 4.1-4.3 notes (intro, plane trusses, method of joints).docx>

Chapter 4: Structures

- Section 4.1: Introduction
- Section 4.2: Plane Trusses
- Section 4.3: Method of Joints
- Section 4.4: Method of Sections
- Section 4.6: Frames and Machines

4/1 Introduction

In chapter 3 we looked at FBD's and practiced analyzing forces acting on a body. In doing so under equilibrium conditions all internal forces canceled out and we never attempted to analyze these forces. Now in chapter 4 we focus our gaze on determining the forces internal to a body or structure. We will again only consider statically determinate structures which do not have more supporting constraints than are necessary to maintain equilibrium.

In sections 4.3 and 4.4 we will focus on trusses. In section 4.6 we will look at frames and then move on to machines.

4/2 Plane Trusses

A framework composed of members joined at their ends to form a rigid structure is called a *truss*. Examples of a truss are seen in bridges, roof supports, derricks, and supports for machines, signs, buildings, etc.

When the members of the truss lie essentially in a single plane, the truss is called a *plane truss*.

For bridges and similar structures, plane trusses are commonly utilized in pairs with one truss assembly placed on each side of the structure. A section of a typical bridge structure is shown in Fig. 4/1. The combined weight of the roadway and vehicles is transferred to the longitudinal stringers, then to the cross beams, and finally, with the weights of the stringers and cross beams accounted for, to the upper joints of the two plane trusses which form the vertical sides of the structure. A simplified model of the truss structure is indicated at the left side of the illustration; the forces L represent the joint loadings.

Several examples of commonly used trusses which can be analyzed as plane trusses are shown in Fig. 4/2.

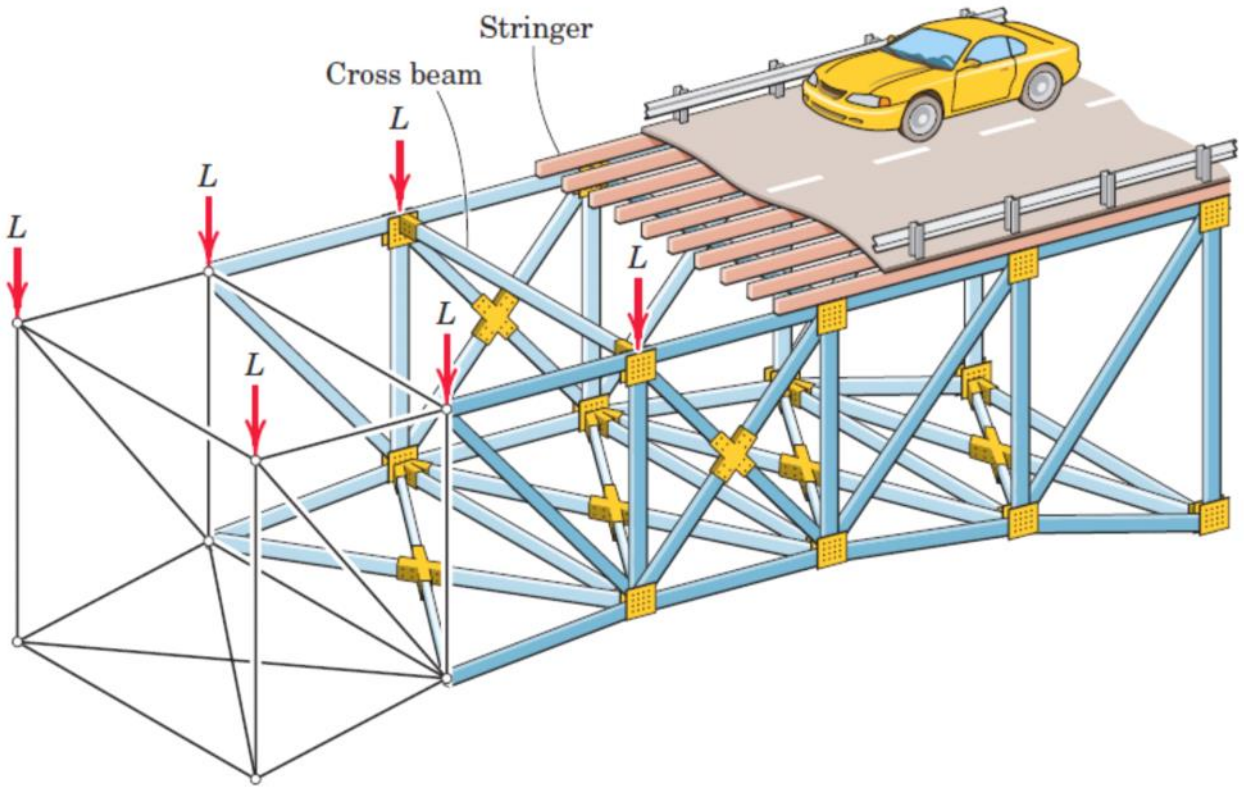
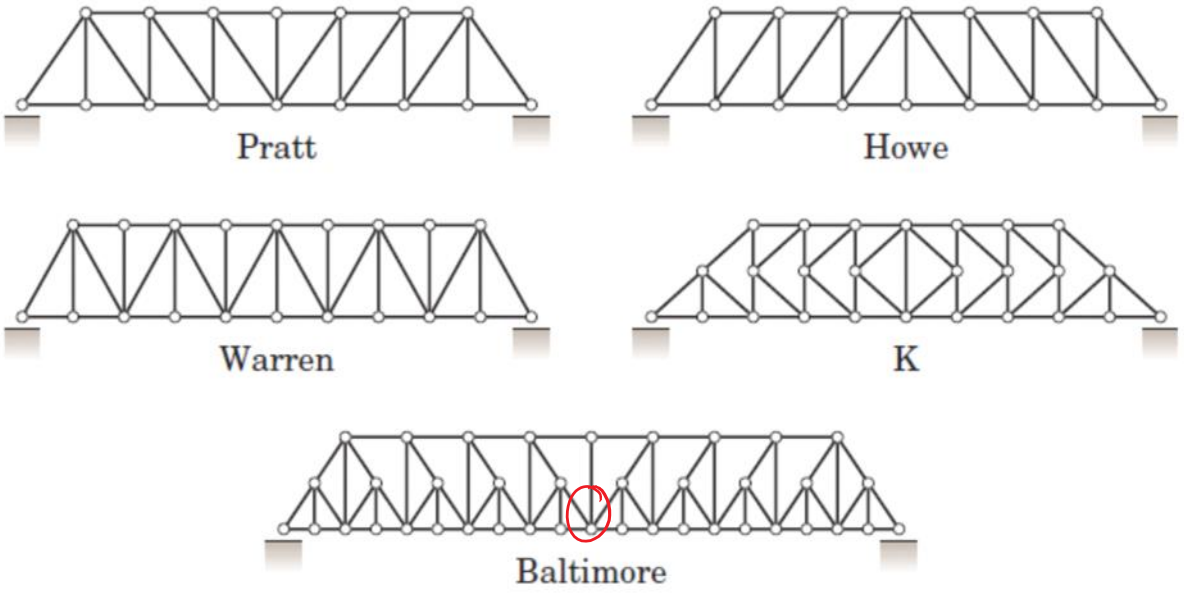
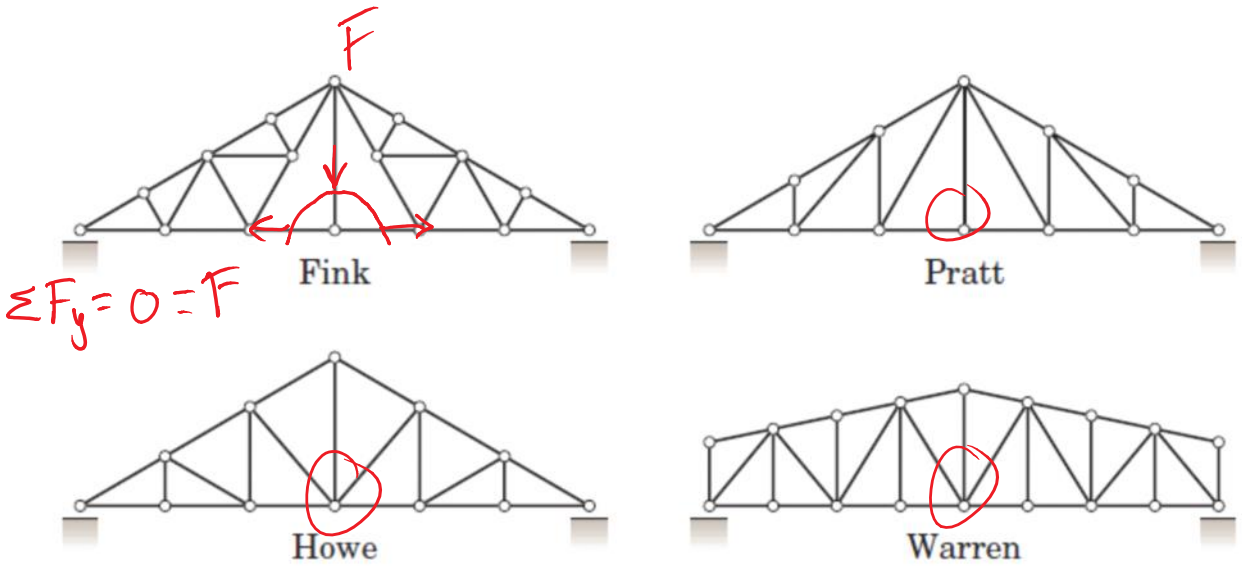


Figure 4/1



Commonly Used Bridge Trusses



$\Sigma F_y = 0 = F$

Commonly Used Roof Trusses

Figure 4/2

Simple Trusses

The most basic building block or element of a truss is the triangle. A structure pined at each connection in the shape of a triangle is termed *rigid*, and implies that it will not collapse or deform (negligibly) under the load of its own weight or of an external load (unless the load exceeds the capacity of the truss). Quadrilateral shapes are *non-rigid* when pined because they are free to move at their joints and are configured in such a way as to not support most loads. The way to turn a non-rigid truss shape into a rigid one is to add a minimum number of supports such that the structure is self-supporting (this often means creating lots of triangles).

Structures build from one or more basic triangles are called *simple trusses*. When there are more members present than are essential to make the truss rigid, the truss is statically indeterminate, and as such cannot be analyzed by the equations of equilibrium alone. This is due to the presence of more unknowns than available equilibrium equations. The additional members above and beyond the ones absolutely necessary to create a rigid truss are called *redundant* members.

The proper design of a truss requires selecting appropriate sizes, materials, thicknesses, etc. Determining these characteristics requires one to know the expected forces in each member of the truss.

A Two-force member is a member in equilibrium under the action of two forces only. As such, both forces must be equal and opposite in the direction of the member itself for the member to remain in equilibrium. Because the forces in a two force member are collinear with the line connecting its points of contact, all forces will be directed either in or out, or the member will be loaded in tension or in compression.

4/3 Method of Joints

This method is used for finding the forces in the members of a truss. The method involves analyzing the forces present at each joint, and using the two independent equilibrium equations, finding the forces in each member.

To use it a few parameters must be satisfied. When analyzing any particular joint:

- At least one force must be known.
- No more than two *unknown* forces may be present.
- Because we can specify the equilibrium of each joint by two scalar force equations, there are in all $2j$ such equations for a truss with ***j* number of joints**. For the entire truss composed of ***m* number of two-force members** and having the maximum of three unknown support reactions (support reactions for the entire truss), there are in all $m + 3$ unknowns in the system. Thus, for any plane truss, the equation $\mathbf{2}\boldsymbol{j} = \boldsymbol{m} +$ will be satisfied if the truss is statically determinate internally.

Convention of Force Directions:

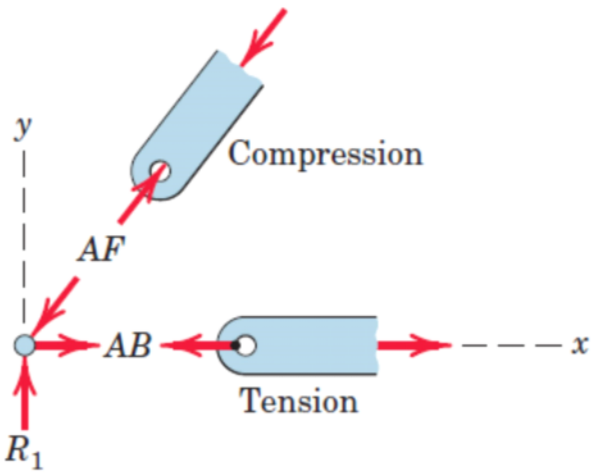
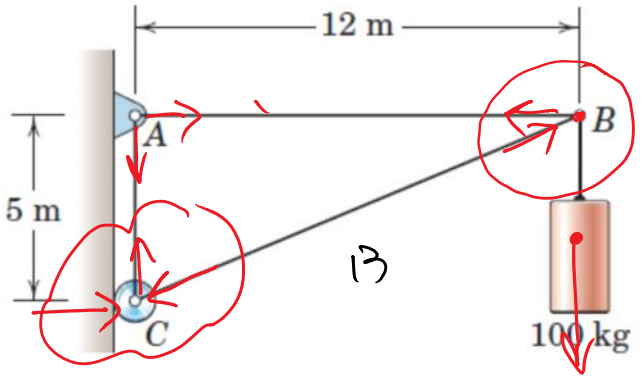


Figure 4/7

In this text and as customary, we will indicate tension forces at a joint by a force pulling away from the joint, and compression forces by a force pushing into the joint.

Examples:

4/1 Determine the force in each member of the loaded truss.



$\Sigma F_y \Rightarrow A_y$

$A_y = 981\ N$

$\Sigma M_A = 0 = N(5) - 981(12$

$N = \frac{981(12)}{5} =$

$= 2354.4\ N$

$|BA| = A_x = 2354\ N\ C$

$|CA| = 981\ N\ C$

$981N \quad \Sigma F_x \Rightarrow A_x = N$

Joint B: AB, CB, 981

$\Sigma F_y = 0 = -981 - CB\frac{5}{13} \Rightarrow CB = -981 \cdot \frac{13}{5}$

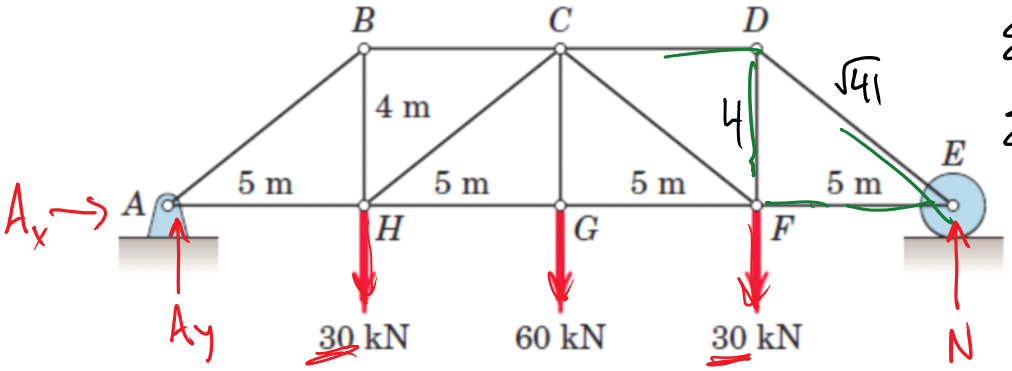
$CB = -2550.6N$ C

$\Sigma F_x = -AB - CB\frac{12}{13} = 0$

$AB = -CB\frac{12}{13} = +2354.4N$

$AB = 2354\ N\ T$

4/7 Determine the force in each member of the loaded truss. Make use of the symmetry of the truss and of the loading.



Entire Bridge: ↺

$\Sigma M_A \Rightarrow N$

$\Sigma F_y \Rightarrow A_y$

$\Sigma F_x \Rightarrow A_x = 0$

$\Sigma M_A = 0 = N(20) - 30(15) - 60(10) - 30(5) \rightarrow N = 60\ kN$

$\Sigma F_y = A_y + N - 30 - 60 - 30 = 0 \Rightarrow A_y = 60\ kN$

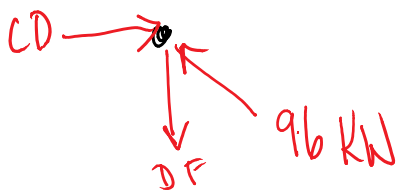
(60)

Joint E:

$\Sigma F_y = 0 = 60 - DE\frac{4}{\sqrt{41}} = 0 \Rightarrow DE = 96.04 = 96.0\ kN$ C

$\Sigma F_x = 0 = DE\frac{5}{\sqrt{41}} - EF = 0 \Rightarrow EF = 75.0\ KNT$

Joint D:

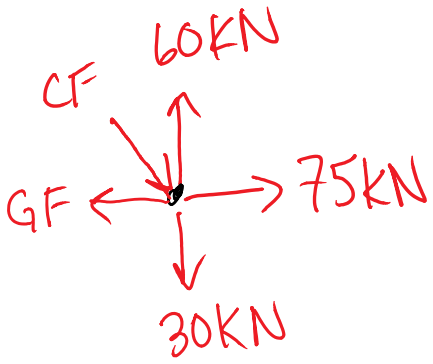


$\Sigma F_y = 0 = 96\left(\frac{4}{\sqrt{41}}\right) - DF \Rightarrow DF = 60\ KNT$

$\Sigma F_x = 0 = CD - 96\left(\frac{5}{\sqrt{41}}\right) \Rightarrow CD = 75\ kNC$

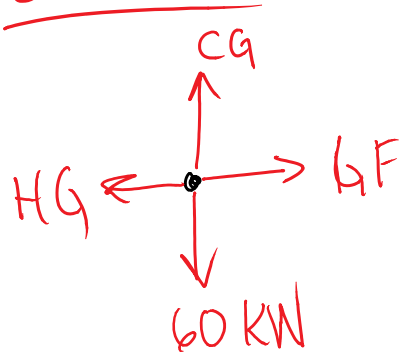
Joint F:

$\Sigma F_y = 0 = 60\text{KN} - 30\text{kN} - CF\frac{4}{\sqrt{41}} \Rightarrow CF = 48.0\text{KNC}$



$\Sigma F_x = 0 = 75 + 48\frac{5}{\sqrt{41}} - GF \Rightarrow GF = 112.5\text{kN}\overset{T}{}$

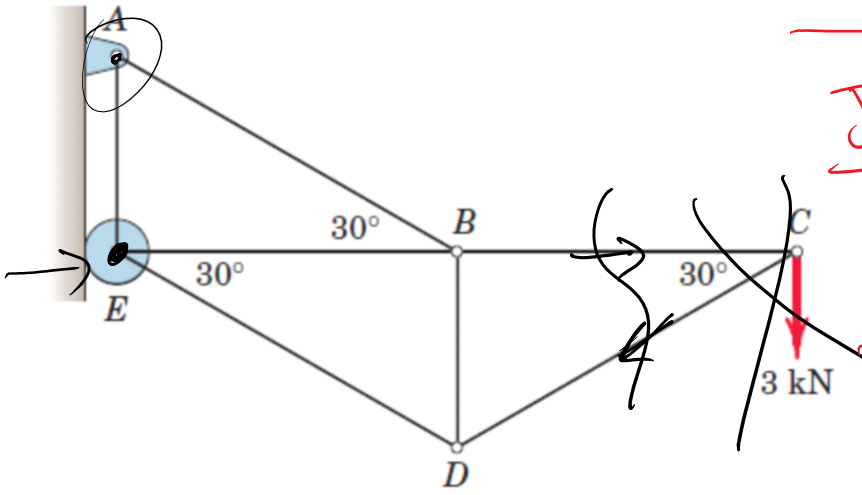
Joint G:



$\Sigma F_y \Rightarrow CG = 60\text{KNT}$

Go directly to C do not Pass Go

4.3.9 Determine the forces in members BC & CD.



Joint C:

$\Sigma F_y = DC\sin 30 - 3\text{kN} = 0$

$DC = 3\text{kN} \cdot 2 = 6\text{KNC}$

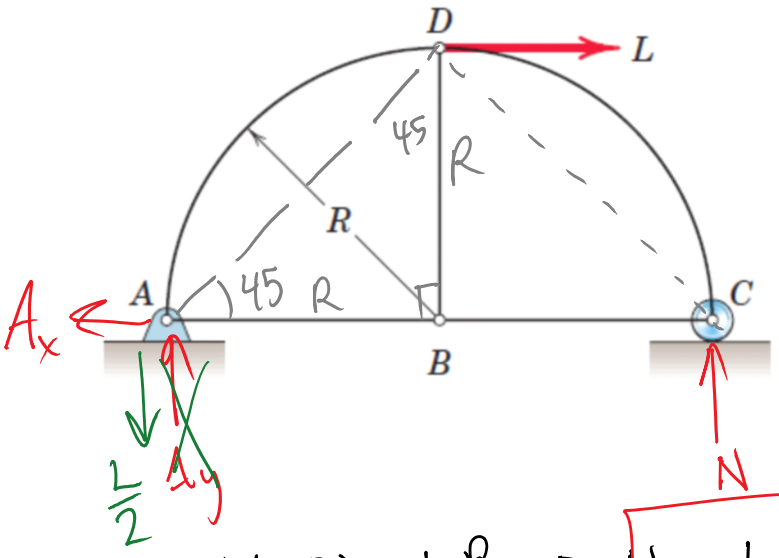
$\Sigma F = DC\cos 30 - BC = 0 \Rightarrow BC = 1.4\text{kN} \cdot \sqrt{3} = 3\sqrt{3}$

$DC = 3kN \cdot 2 = 6kN\,C$

$\Sigma F_x = DC\cos 30 - BC = 0 \Rightarrow BC = 6kN \cdot \frac{\sqrt{3}}{2} = 3\sqrt{3}$

$BC = 3\sqrt{3}\,kN\,T$

4/11 Determine the forces in members *AB*, *BC*, and *BD* of the loaded truss.



Entire Body:

$\Sigma M_A \Rightarrow N$

$\Sigma F_y \Rightarrow A_y$

$\Sigma F_x \Rightarrow A_x$

$+\circlearrowleft$

$$\Sigma M_A = 0 = N(2R) - LR \Rightarrow N = \frac{L \cdot R}{2R} = \frac{L}{2}$$

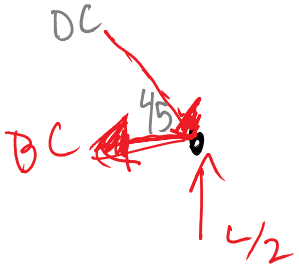
$$\Sigma F_y = -A_y + N = 0 \Rightarrow A_y = +N = +\frac{L}{2}$$

$$\Sigma F_x = 0 = L - A_x \Rightarrow A_x = L$$

Joint B: $\Rightarrow DB = 0$

$$\Sigma F_y = BD + 0 = 0 \Rightarrow DB = 0$$

Joint C:

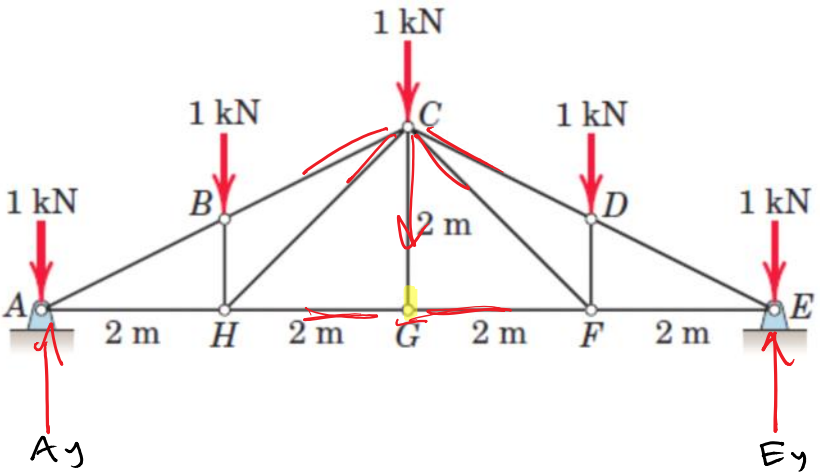


$$\Sigma F_y = \frac{L}{2} - DC\frac{\sqrt{2}}{2} = 0 \Rightarrow DC = \frac{L}{2}\cdot\sqrt{2}\,C$$

$$\Sigma F_x = \frac{L}{2}\frac{\sqrt{2}}{\sqrt{2}} - BC = 0 \Rightarrow BC = \frac{L}{2}\,T$$

Revisiting Joint B we find that $\boxed{AB = BC = \frac{L}{2}T}$

4/21 A snow load transfers the forces shown to the upper joints of a Pratt roof truss. Neglect any horizontal reactions at the supports and solve for the forces in all members.



$\Sigma M_A \Rightarrow G_y$

$\Sigma F_y \Rightarrow A_y$

Joint E:

$\Sigma F_y \Rightarrow DE$

$\Sigma F_x \Rightarrow FE$

Joint D:

$\Sigma F_x \Rightarrow CD$

$\Sigma F_y \Rightarrow DF$

Joint F:

$\Sigma F_y \Rightarrow CF$

$\Sigma F_x \Rightarrow GF$

Joint G

$\Sigma F_y \rightarrow CG = 0$

4.4 10.29

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Engr 420
4.4 notes ...
Inserted from: <file://H:\My Documents\Folsom Lake College\ENGR 420 Statics\Notes\Engr 420 4.4 notes (method of sections).docx>

Chapter 4: Structures

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- Section 4.4: Method of Sections
- Section 4.6: Frames and Machines

4/3 Method of Sections or Method of Cuts

When analyzing plane trusses by the method of joints, we need only two of the three equilibrium equations because the procedures involve concurrent forces at each joint. We can take advantage of the third or moment equation of equilibrium by selecting an entire section of the truss for the free body in equilibrium under the action of a nonconcurrent system of forces. This *method of sections* has the basic advantage that the force in almost any desired member may be found directly from an analysis of a section which has cut that member. Thus, it is not necessary to proceed with the calculation from joint to joint until the member in question has been reached. In choosing a section of the truss, we note that, in general, not more than three members whose forces are unknown should be cut, since there are only three available independent equilibrium relations.

The main idea behind this method is that the entire body is in equilibrium. All forces internal to the body have equal and opposite reactions allowing the internal forces to sum to zero. The method of cuts creates an imaginary cut in the body. This cut removes the equal and opposite reaction force from the other half of the body which was cut away, therefore, in order to keep this section of the body in equilibrium, the missing reactive forces needs to be reapplied to the cut member(s). This way a new body will be considered.

With convenient selections for where to sum moments and or summing forces in a particular direction, you will be able to determine the forces in your three unknown members.

The most important rule is that you cannot cut more than 3 unknown members. You may cut more, but only if you know the forces in all but 3 members. This is because we have three independent equations available, $\sum \boldsymbol{F}_x$, $\sum \boldsymbol{F}_y$, $\sum \boldsymbol{M}$.

It is important to remember if you cut a two-force member which is in tension you will need to reapply a force which pulls the cut member back into tension (away from the joint it originates from. When you cut a member in compression, you will need to apply a force pushing into the member to recompress it again. This force will point into the joint it originates from.

In some cases the method of joints can be applied twice or combined with the method of joints. In a case where the member you are searching for requires you to cut members where there are more than 3 unknown, you must first make a cut with 3 or less unknowns which will allow you to find enough forces in your other cut to allow you no more than 3.

Convention of Force Directions:

In this text and as customary, we will indicate tension forces at a joint by a force pulling away from the joint, and compression forces by a force pushing into the joint.

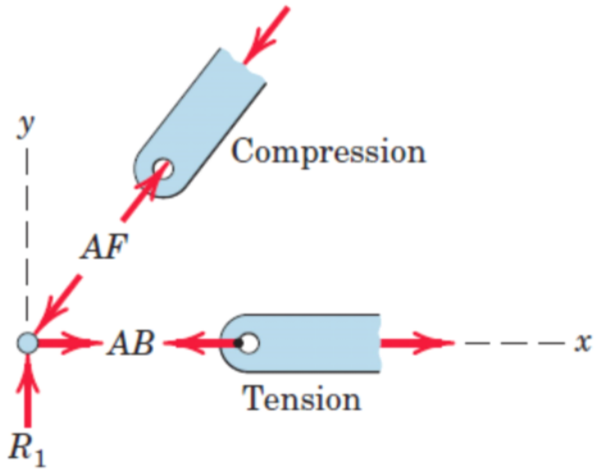
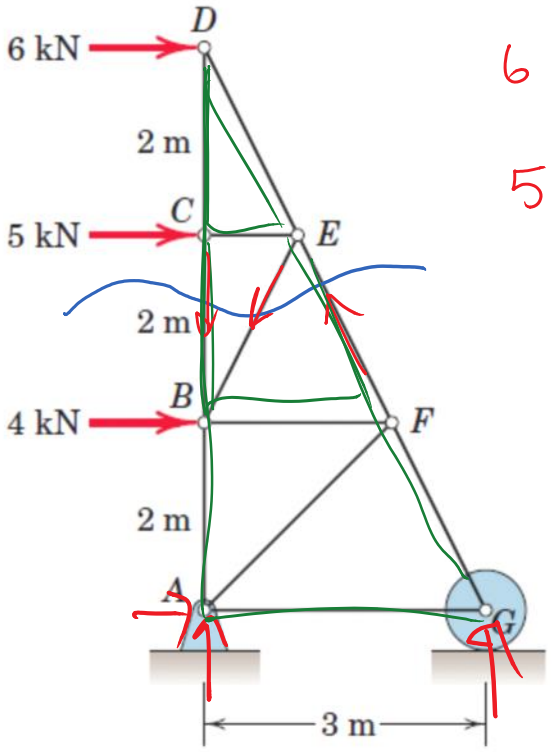
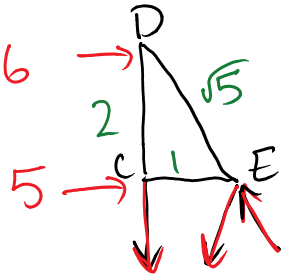


Figure 4/7

Examples:

4/33 Determine the force in member *BE* of the loaded truss.





B

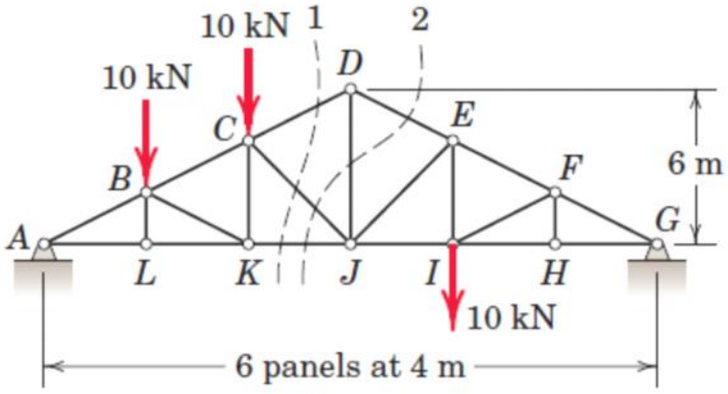
$\Sigma M_E \Rightarrow CB$

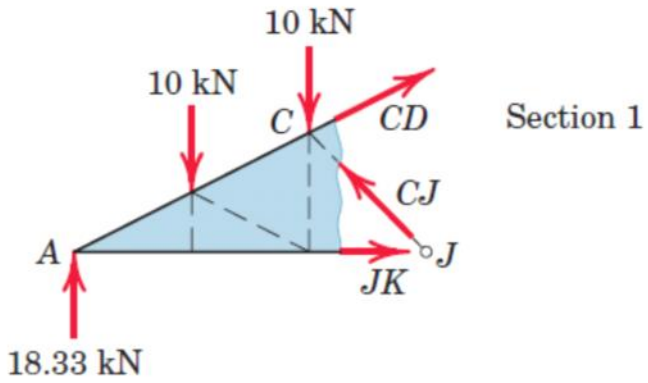
$\Sigma M_D \Rightarrow EB$

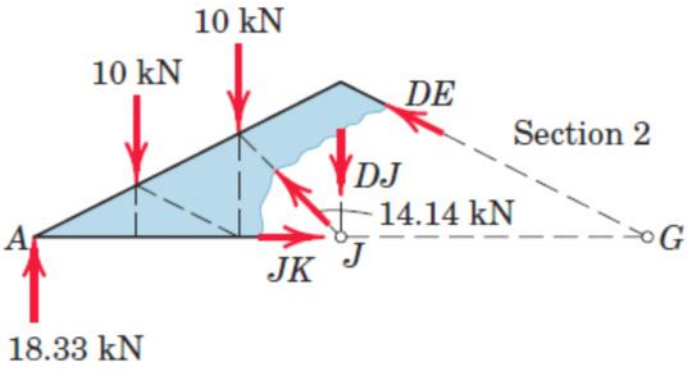
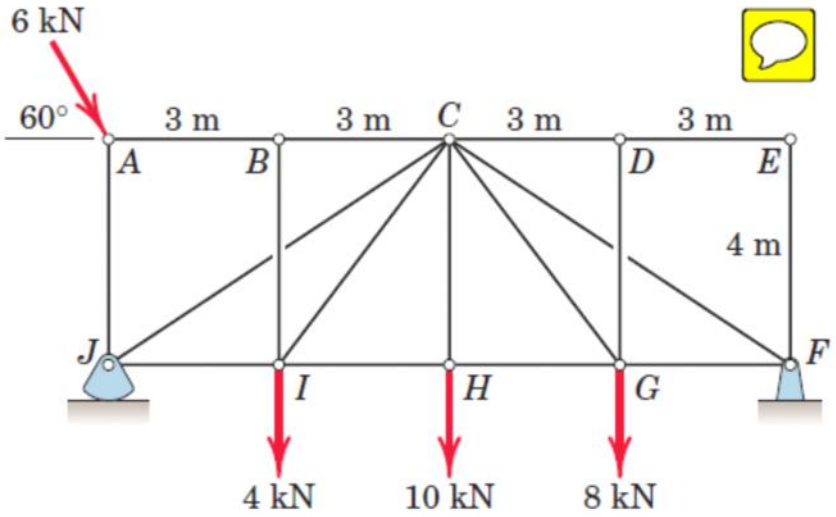
$\Sigma F_x = \Rightarrow EF$

SAMPLE PROBLEM 4/4

Calculate the force in member *DJ* of the Howe roof truss illustrated. Neglect any horizontal components of force at the supports.







4/43 The members CJ and CF of the loaded truss cross but are not connected to members BI and DG. Compute the forces in members BC, CJ, CI, and HI.