1) **Key Vocabulary:** Informal definitions are allowed so long as they are accurate.

Function: *Set of ordered pairs where each member of the domain (x values) is used exactly once. See illustration I example 1 below. Graphically the vertical line test is used to determine if a graph represents a function.*

One – to – one: *A function where **every** member of the range (y value) has been mapped from (came from) exactly one member of the domain (x values). See illustration I example 2 below. Graphically a horizontal line test is used to determine if the graph of a function is one-to-one. One-to-one functions are necessary to determine if the inverse of a function will itself be a function.*

Intercepts: *Points where a graph crosses the x-axis (x intercept) or y-axis (y intercept).*

Roots or Zeros: *Solutions to polynomial equations. We say x = r is a root or zero of a polynomial if P(r) = 0. Typically found by factoring polynomials however some polynomials may require special techniques to find the solutions as is the case with quadratic polynomials when the quadratic formula may be required.*

Odd/Even functions: *If $f(-x) = f(x)$ the function is said to be even as is the case with $f(x) = x^2$ or $f(x) = \cos(x)$. If $f(-x) = -f(x)$ then the function is said to be odd as is the case with $f(x) = x^3$ or $f(x) = \sin(x)$.*

Maximum/Minimum (informal): *A maximum is the highest point on a graph over an interval (see point "b" in illustration II). A minimum is the lowest (point "c" in illustration II). There is a distinction between absolute and relative(local) maximums and minimums. The idea of an interval is important in understanding this distinction. This will be discussed more during the semester.*

Increasing/Decreasing (informal): *In an interval if the x values increase with the corresponding y – values then the function is said to be increasing over that interval (see the intervals (a,b) or (c,d) below). If the x values are increasing but the corresponding y – values are decreasing then the function is said to be decreasing over that interval. (see the interval (b , c) below.)*

Inverse of a Function: *The inverse of a function is derived by switch the x and y variables then solving for y. The graph of the inverse of a function will reflect the graph of the function about the line y = x (see illustration III). Every function has an inverse but the inverse of a function may not be a function itself. See one-to-one function definition above.*

Illustration I

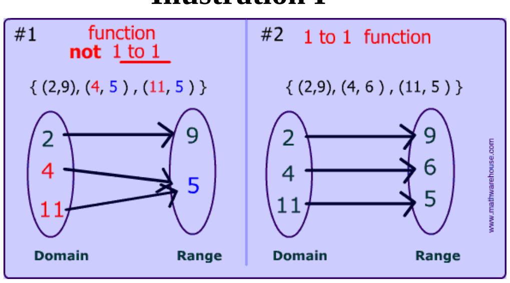


Illustration II

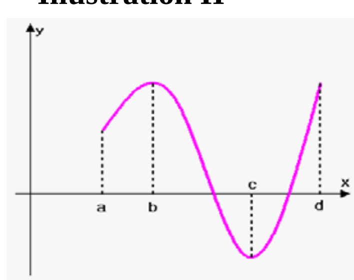
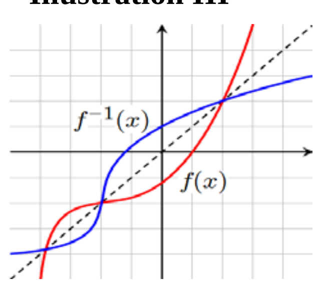
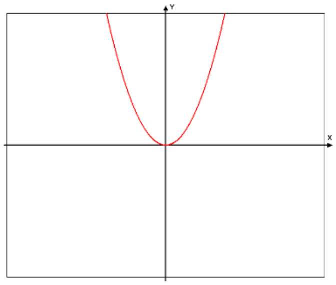


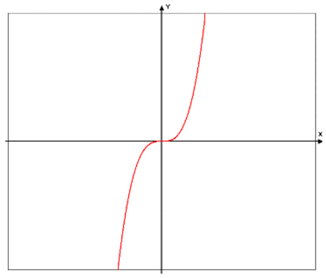
Illustration III



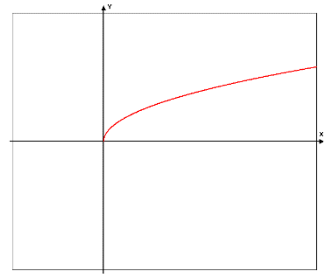
2) Write the function for each graph: If the graph is exponential or logarithmic, base '***e***' must be used.



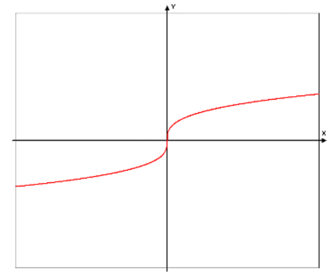
f(x) = x^2



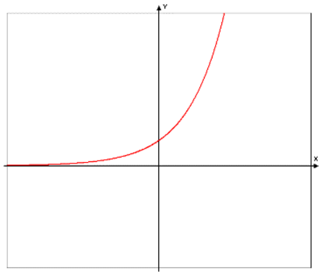
f(x) = x^3



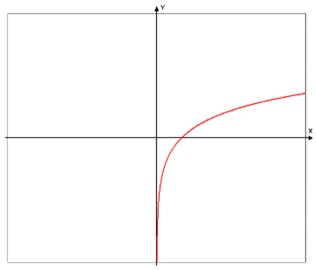
$f(x) = \sqrt{x}$



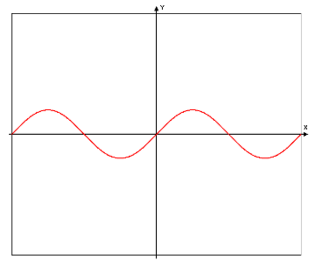
$f(x) = \sqrt[3]{x}$



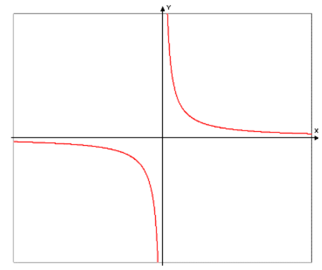
$f(x) = e^x$



$f(x) = \ln(x)$



$f(x) = \sin(x)$



$f(x) = \dfrac{1}{x}$

3) Circle each <u>even</u> function and box each <u>odd</u> function:

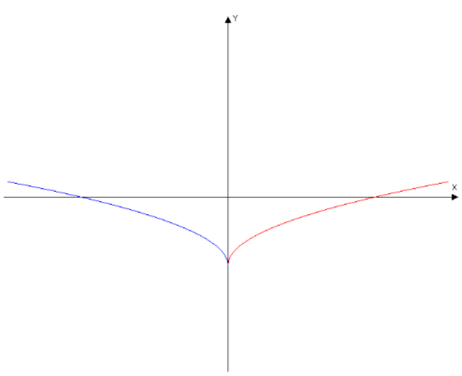
$y = x^2$ $y = x^3$ $y = x^2 - 4x + 8$ $y = 3x^4 + x^2 - 5$ $y = x^3 + 2$

$y = x^3 + x$ $y = \dfrac{x^3 + x}{x^2}$ $y = \dfrac{x^3 + 1}{x^2}$ $y = \dfrac{x^3 + 4x}{x^3}$

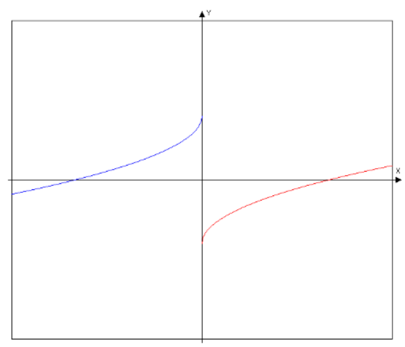
$y = \sin x$ $y = \cos x$ $y = \tan x$ $y = e^x$

4) Below, a partial function graph is given. You will need to complete the graph so that it is an even function in the first grid and an odd function in the second grid.

Finish the 'even' function.

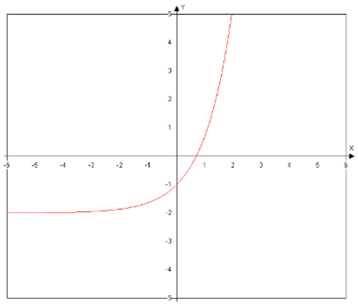


Finish the 'odd' function.

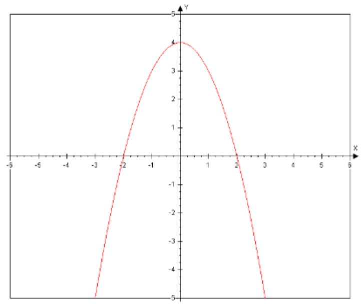


5) Sketch each graph on the grids provided. For all trigonometric functions, label the scale on both the *x* and *y* axis.

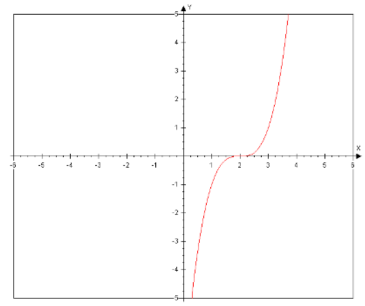
$y = e^x - 2$



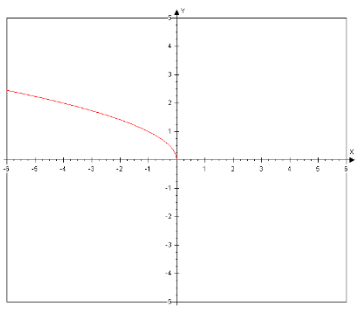
$y = -x^2 + 4$



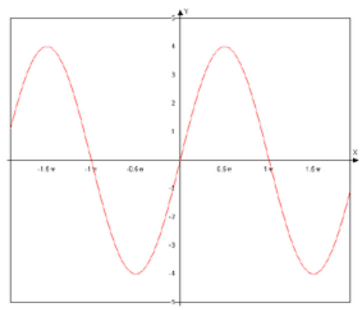
$y = (x-2)^3$



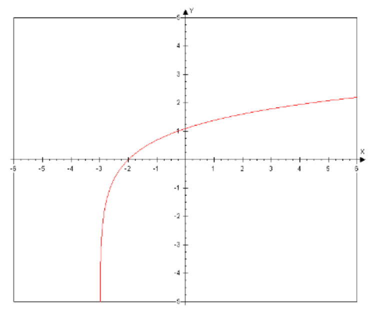
$y = \sqrt{-x}$



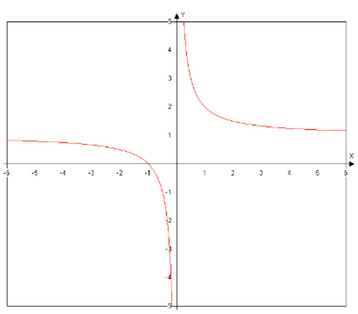
$y = 4\sin x$



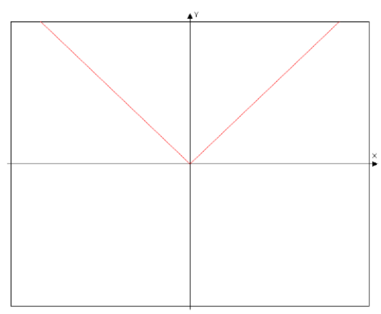
$y = \ln(x+3)$



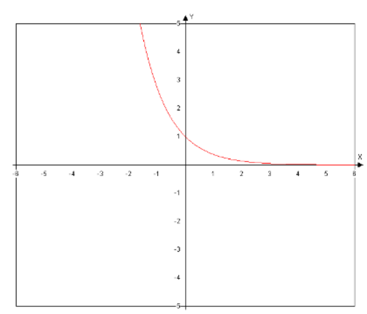
$y = \frac{1}{x} + 1$



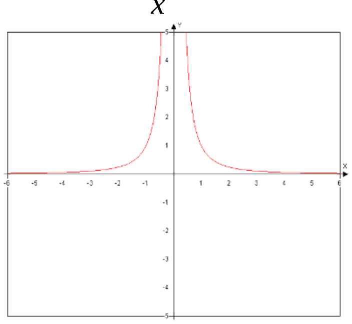
$y = |\,x\,|$



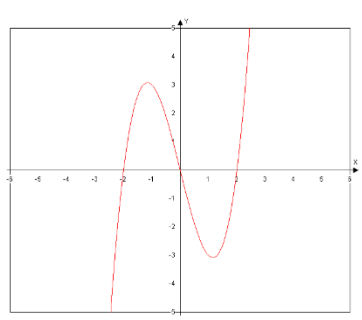
$y = e^{-x}$



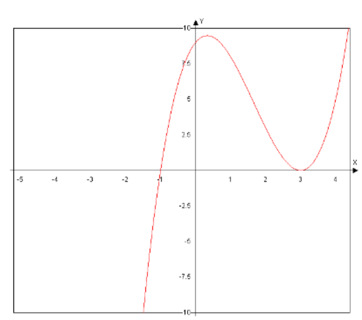
$y = \frac{1}{x^2}$



$y = x(x+2)(x-2)$

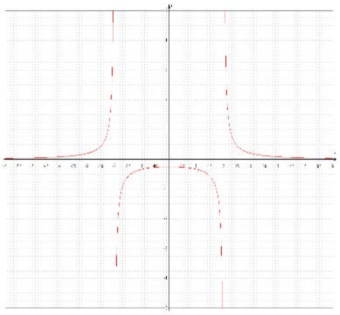


$y = (x+1)(x-3)^2$

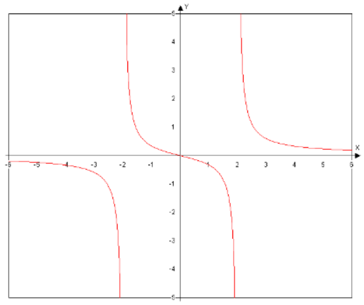


5) continued

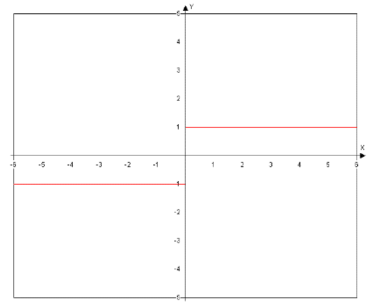
$y = \dfrac{1}{x^2 - 4}$



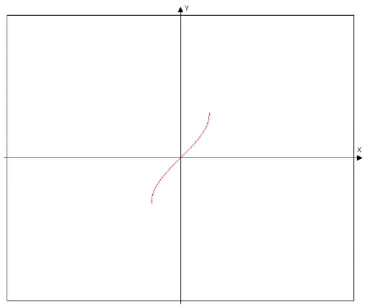
$y = \dfrac{x}{x^2 - 4}$



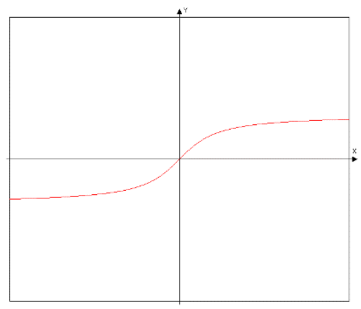
$y = \dfrac{|x|}{x}$



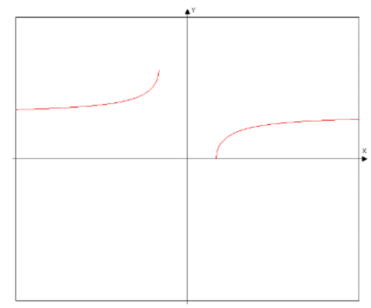
$y = \arcsin x$



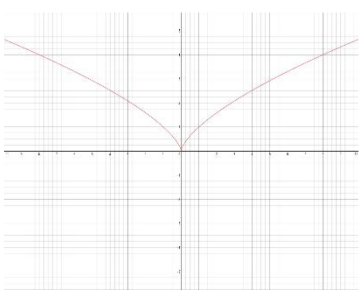
$y = \arctan x$



$y = \operatorname{arcsec} x$

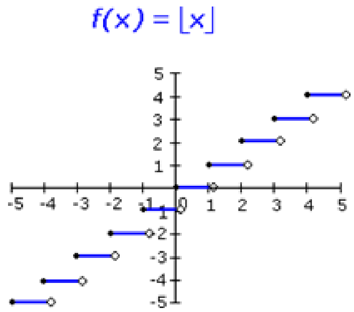


$y = x^{2/3}$

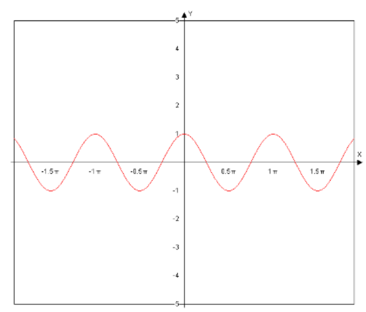


$y = Int(x)$ ***or*** $y = [x]$

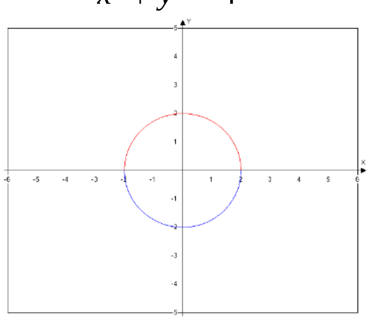
Greatest Integer function



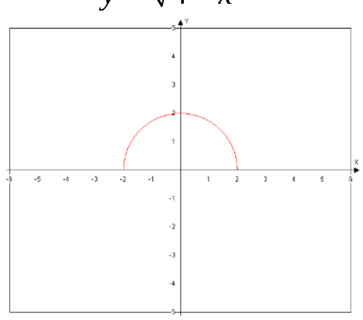
$y = \cos(2x)$



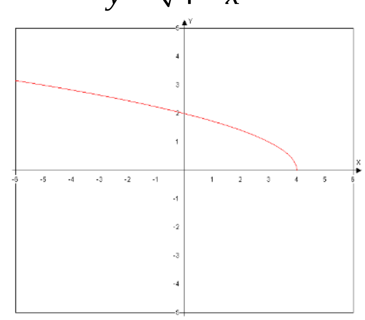
$x^2 + y^2 = 4$



$y = \sqrt{4 - x^2}$



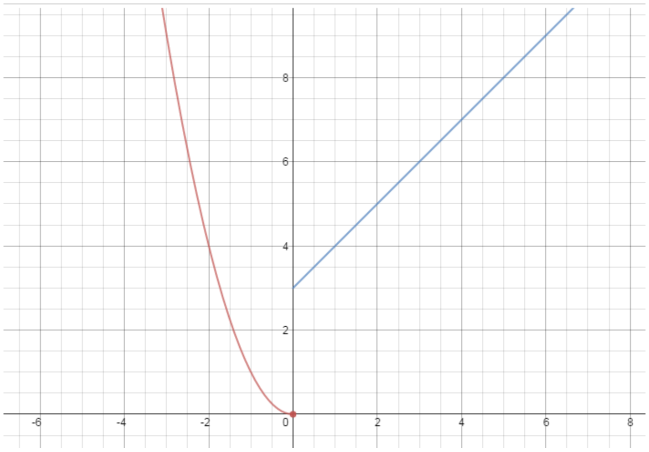
$y = \sqrt{4 - x}$

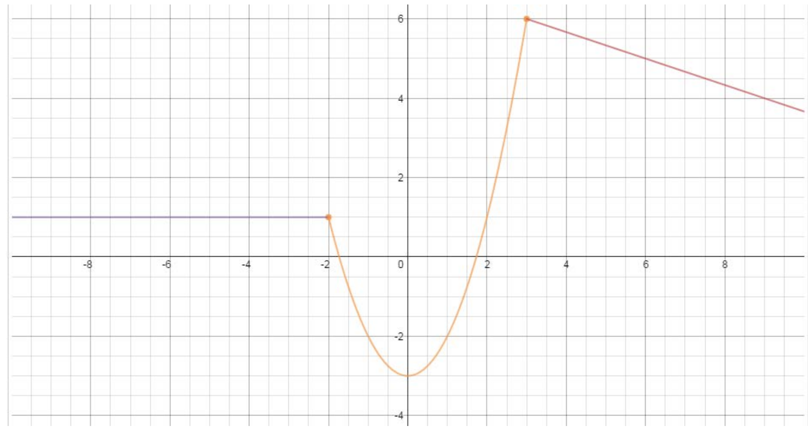


6) Graph each piecewise function.

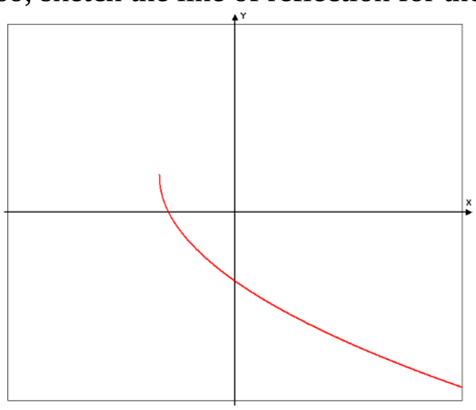
$$f(x)=\begin{cases} x^2 : x \le 0 \\ x+3 : x > 0 \end{cases}$$

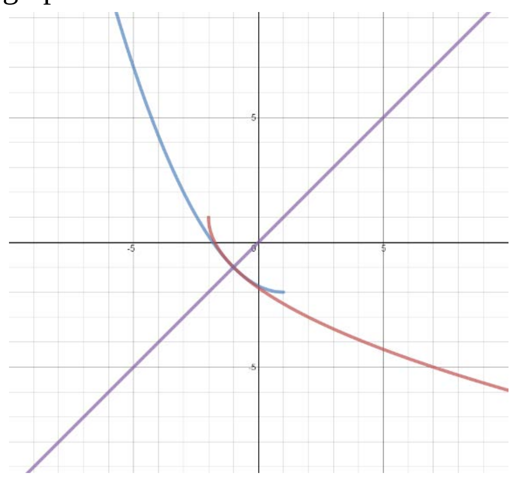
$$g(x)=\begin{cases} 1 \quad : x < -2 \\ x^2-3 : -2 \le x \le 3 \\ -\frac{1}{3}x+7 : x > 3 \end{cases}$$





7) Sketch the Inverse of the given function on the same graph.
Also, sketch the line of reflection for the inverses.





Note on solution for number 7: we lost the original function used to create the original graph so we were unable to get an exact machine generated solution. The solution given above gives the general idea using a radical function.

8) Write each of the following expressions in the form x^p where 'p' is a real number.

Examples: $\frac{1}{x^3} = x^{-3}$ $\quad \frac{1}{\sqrt{x}} = x^{-1/2}$ $\quad \frac{x}{\sqrt[3]{x}} = \frac{x}{x^{1/3}} = x^{1-1/3} = x^{2/3}$

$x^{12} \cdot x^{-4} = x^8$ $\quad \frac{1}{x^{15}} = x^{-15}$ $\quad \frac{x}{x^5} = x^{-4}$

$\frac{\sqrt{x}}{x} = x^{-1/2}$ $\quad \frac{1}{\sqrt[4]{x^3}} = x^{-3/4}$ $\quad x^4 \cdot \sqrt{x} = x^{9/2}$

$\frac{x^2}{\sqrt[4]{x}} = x^{7/4}$ $\quad \sqrt{x} \cdot \sqrt[3]{x} = x^{5/6}$ $\quad \frac{\sqrt{x}}{\sqrt[3]{x}} = x^{-1/6}$

9) Given the list of functions, answer the following. Use interval notation for the domain and range problems.

$f(x) = x^2 + 4$ $\quad$ $g(x) = \sqrt{3-5x}$ $\quad$ $h(x) = x^2 - 4x + 3$

Domain of *f*. $(-\infty, \infty)$ $\quad$ Range of *f*. $[4, \infty)$

Domain of *g*. $\left(-\infty, \frac{3}{5}\right]$ $\quad$ Range of *g*. $[0, \infty)$

Domain of *h*. $(-\infty, \infty)$ $\quad$ Range of *h*. $[-1, \infty)$

$(f \circ g)(x) =$ $7 - 5x$ $\quad$ $(g \circ h)(x) = \sqrt{-5x^2 + 20x - 12}$

$\frac{f(x) - f(3)}{x - 3} =$ $x + 3$

Evaluate and simplify

$\frac{h(3+\Delta x) - h(3)}{\Delta x} = \Delta x + 2$

Evaluate and simplify

10) Let $f(x)$ represent a function. For each of the following, describe the transformation to the graph.

example $\quad$ $y = f(x+3)$ $\quad$ The graph of the function is translated *3 units to the left*.

$y = f(x) + 1$ $\quad$ Up 1

$y = -f(x)$ $\quad$ Reflected about the x-axis

$y = f(x-4)$ $\quad$ 4 units to the right

$y = f(-x) - 2$ $\quad$ reflected about the y-axis and down 2

10) (continued)

$y = 2f(x)$	vertical stretch/compression by a factor of 2
$y = f(2x)$	horizontal stretch/compression by a factor of 2 (best seen with trig functions)
$y = \lvert f(x) \rvert$	reflects any part of the graph below the x axis above the x axis for example try graphing 3x – 2 then the absolute value of 3x - 2

11) Evaluate each expression. The solutions should not involve logarithmic expressions.

$\ln 1 =$ 0

$\ln \sqrt[4]{e} = \frac{1}{4}$

$\log(.01) =$ -2

$\log(-1000) =$ undefined

$\log_7(7^{45}) = 45$

$\log_{1/4} 64 =$ -3

$(e^{\ln 32}) = 32$

$\log_9 27 = \frac{3}{2}$

12) Solve each equation for x such that there are no logarithms in your final answer.

$\log_5 x = 3$

x = 125

$\log_x 2 = \frac{1}{4}$

x = 16

$\log_5(x-3) = -1$

$x = 3\frac{1}{5}$

13) List the domain and range for each function. (Use interval notation.)

$y = 3^x + 2$

D: $(-\infty, \infty)$

R: $(2, \infty)$

$y = \log_2(x+1)$

D: $(-1, \infty)$

R: $(-\infty, \infty)$

14) Answer each true/false question. (Circle T for true or F for false.) If false, rewrite the right had side to make it a true statement.

T (F) $\log\sqrt{15} = 2\log 15$

$\log\sqrt{15} = \frac{1}{2}\log 15$

(T) F $\log_2 \frac{7x}{w} = \log_2 7 + \log_2 x - \log_2 w$

(T) F $\log 32 - \log z = \log\left(\frac{32}{z}\right)$

T (F) $2\ln x - \ln 24 = \frac{\ln x^2}{\ln 24}$

$2\ln x - \ln 24 = \ln\frac{x^2}{24}$

(T) F $\log_2 5 = \frac{\log 5}{\log 2}$

15) Solve each equation. Use natural logarithms (*ln*) only. Give exact answers (no calculators).

$5^x = 7$ $4\cdot 5^x = 7$ $4\cdot 5^x - 3 = 7$

$x = \frac{\ln 7}{\ln 5}$ $x = \frac{\ln\frac{7}{4}}{\ln 5}$ $x = \frac{\ln\frac{5}{2}}{\ln 5}$

16) Evaluate each trigonometric function. Solutions should be exact (no calculator). You are expected to be able to evaluate these without any help items.

$\cos(0) = 1$ $\sin(0) = 0$ $\tan(0) = 0$ $\csc(0) = \textit{undefined}$

$\cos\left(\frac{\pi}{2}\right) = 0$ $\sin\left(\frac{\pi}{2}\right) = 1$ $\tan\left(\frac{\pi}{2}\right) = \textit{undefined}$ $\cot\left(\frac{3\pi}{4}\right) = -1$

$\cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$ $\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$ $\tan\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{3}$ $\sec\left(\frac{5\pi}{6}\right) = -\frac{2\sqrt{3}}{3}$

$\arcsin(1) = \frac{\pi}{2}$ $\arccos(1) = 0$ $\arctan(1) = \frac{\pi}{4}$

$\arccos\left(-\frac{1}{2}\right)=\frac{2\pi}{3}$ $\arcsin\left(-\frac{1}{2}\right)=-\frac{\pi}{6}$ $\arctan\left(\sqrt{3}\right)=\frac{\pi}{3}$

17) Evaluate each of the given trigonometric expression using the given information and an appropriate trigonometric identity.

If $\sin\theta=\frac{x}{3}$ and θ is in Quadrant I, evaluate:

$\csc\theta=\frac{3}{x}; x\neq 0$

$\cos\theta=\frac{\sqrt{9-x^2}}{3}$

$\tan\theta=\frac{x}{\sqrt{9-x^2}}$

$\sin(2\theta)=\frac{2x\sqrt{9-x^2}}{9}$

$\cos(2\theta)=1-\frac{2x^2}{9}$ other possibilities exist

$\arcsin\left(\frac{x}{3}\right)=\theta$

18) Expand each sum. The first 4 terms and the last term must be shown. *Do Not Simplify*. See the example.

Example: $\sum_{k=1}^{10} k^2+3 = (1^2+3)+(2^2+3)+(3^2+3)+(4^2+3)+\cdots+(10^2+3)$

$$\sum_{k=1}^{74} k^3-1=(1^3-1)+(2^3-1)+(3^3-1)+(4^3-1)+\ldots+(74^3-1)$$

$$\sum_{k=1}^{n}\frac{3}{n}\left(\frac{3k}{n}\right)^2=\frac{3}{n}\left(\frac{3(1)}{n}\right)^2+\frac{3}{n}\left(\frac{3(2)}{n}\right)^2+\frac{3}{n}\left(\frac{3(3)}{n}\right)^2+\frac{3}{n}\left(\frac{3(4)}{n}\right)^2+\cdots\frac{3}{n}\left(\frac{3(n)}{n}\right)^2$$

19) Write the sum using sigma notation.

$$11(1)+11(2)+11(3)+\ldots+11(291)=\sum_{k=1}^{291}11(k)$$

$$\frac{1}{100}\left(\frac{1}{100}\right)^3+\frac{1}{100}\left(\frac{2}{100}\right)^3+\frac{1}{100}\left(\frac{3}{100}\right)^3+\cdots+\frac{1}{100}\left(\frac{100}{100}\right)^3=\sum_{k=1}^{100}\frac{1}{100}\left(\frac{k}{100}\right)^3$$

$$\left[7(1)^2+3\right]+\left[7(2)^2+3\right]+\left[7(3)^2+3\right]+\cdots+\left[7(n)^2+3\right]=\sum_{k=1}^{n}\left[7(k)^2+3\right]$$

20) For each algebra expression, rewrite it as a single fraction in reduced form with no negative numerical exponents and no complex fractions.

$$\frac{x}{4-x^2}-\frac{2}{4-x^2}=-\frac{1}{x+2}$$

$$\sqrt{4-x^2}-\frac{x}{\sqrt{4-x^2}}=-\frac{x^2+x-4}{\sqrt{4-x^2}}$$

I would not rationalize the denominator. In my opinion it results in a more complex answer.

$$\frac{x-(4-x^2)^{-\frac{1}{2}}}{4-x^2}=\frac{x}{4-x^2}-\frac{1}{(4-x^2)(4-x^2)^{\frac{1}{2}}}=\frac{x(4-x^2)^{\frac{1}{2}}-1}{(4-x^2)^{\frac{3}{2}}}$$

Other answers are possible.