3-4-Contd

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3/4 3D Equilibrium
STATICS

J. Morrow

Folsom Lake College

October 19, 2015

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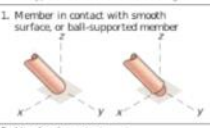
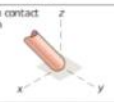
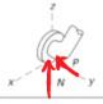
Contact Forces

MODELING THE ACTION OF FORCES IN THREE-DIMENSIONAL ANALYSIS

Type of Contact and Force Origin	Action on Body to Be Isolated
1. Member in contact with smooth surface, or ball-supported member 	Force must be normal to the surface and directed toward the member.
2. Member in contact with rough surface 	The possibility exists for a force F tangent to the surface (friction force) to act on the member, as well as a normal force N.
3. Roller or wheel support with lateral constraint	 A lateral force P exerted by the guide on the wheel can exist, in addition to the normal force N.

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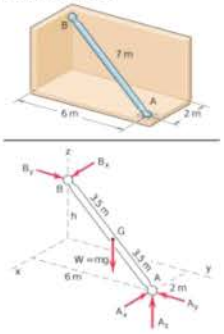
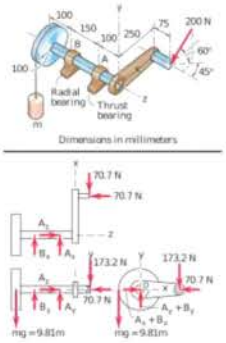
3D FBDs

Pictoral View



Orthographic Projections



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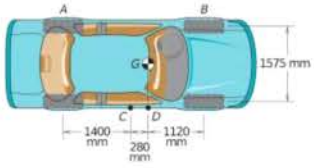
Problem 3/71

An overhead view of a car is shown in the figure. Two different locations C and D are considered for a single jack. In each case, the entire right side of the car is lifted just off the ground. Determine the normal reaction forces at A and B and the vertical jacking force required for the case of each jacking location. Consider the 1600-kg car to be rigid. The mass center G is on the midline of the car.



4 / 1

Problem 3/71

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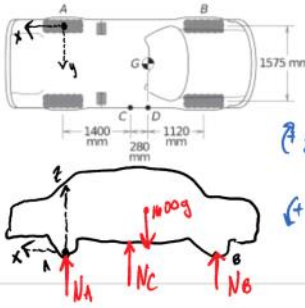
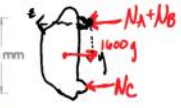




$\Sigma F_z = 0 \Rightarrow N_A, N_B, N_C$

$\Sigma M_x = 0 \Rightarrow N_C$

$\Sigma M_y = 0 \Rightarrow N_B, N_C$

$(+\circlearrowleft) \Sigma M_x = -1600(9.81)\left[\frac{1.96}{2}\right] + N_C[1.96] = 0$

$N_C = 800(9.81) = 7850\ N$

$(+\circlearrowleft) \Sigma M_y = N_C[1.4] - 1600(9.81)[1.68] + N_B[2.8] = 0$

$N_B = \frac{\ast}{2.8} = 5490\ N$

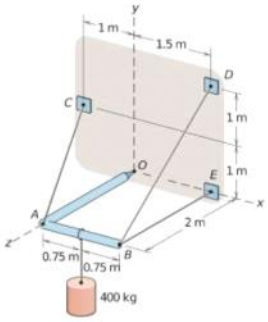
$\Sigma F_z = N_A + N_B + N_C - 1600(9.81)$

$N_A = 2350\ N \;/\; 2360\ N$

ON YOUR OWN, TRY @ D

Problem 3/73

The light right-angle boom which supports the 400-kg cylinder is supported by three cables and a ball-and-socket joint at O attached to the vertical x-y surface. Determine the reactions at O and the cable tensions.



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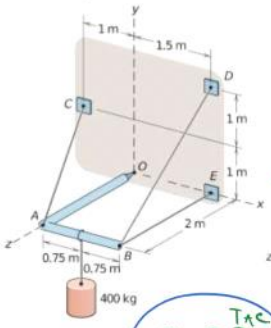

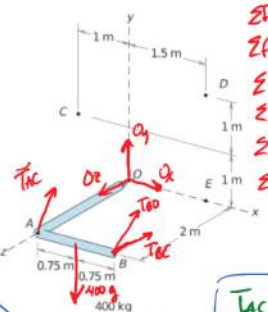

$T_{ACx}(T_{AC})$

YIELD WHAT VARIABLES $\vec{T}_{AC} = T_{AC}\frac{\langle -1,1,-2\rangle}{\sqrt{1^2+1^2+2^2}}$

$\Sigma F_x = 0 \Rightarrow O_x, T_{AC}$

$\Sigma F_y = 0 \Rightarrow O_y, T_{AC}, T_{BD}$ $\quad T_{ACx} = \frac{T_{AC}}{\sqrt{6}}(-1)$

$\Sigma F_z = 0 \Rightarrow O_z, T_{AC}, T_{BE}, T_{BD}$

$\Sigma M_x = 0 \Rightarrow T_{AC}, T_{BD}$

$\Sigma M_y = 0 \Rightarrow T_{BD}, T_{BE}, T_{AC}$

$\Sigma M_z = 0 \Rightarrow T_{BD}$ ✓

$\Sigma F_x = 0 \quad O_x = \frac{T_{AC}}{\sqrt{6}}$

$\Sigma F_y = 0 \quad O_y - 400g + T_{AC}\frac{1}{\sqrt{6}} + T_{BD}\frac{1}{\sqrt{2}}$

$\Sigma F_z = 0 \quad O_z = \frac{2T_{AC}}{\sqrt{6}} + T_{BE} + T_{BD}\frac{1}{\sqrt{2}}$

$\Sigma M_x = 0 \quad \frac{T_{AC}}{\sqrt{6}} + \frac{T_{BD}}{\sqrt{2}} = 3924\,N$

$\Sigma M_y = 0 \quad T_{BE}\cdot 2m + \frac{T_{BD}}{\sqrt{2}} - \frac{T_{AC}}{\sqrt{6}} = 0$

$T_{AC}\left(\frac{2}{\sqrt{6}}\right) + T_{BD}\left(\frac{-2}{\sqrt{8}}\right)(1.5) + T_{BD}(1.5) + mg(1.5) - T_{AC}\frac{2}{\sqrt{6}} = 0$

$\Sigma M_z = 0 \quad 2770\,N = T_{BD}$

ON YOUR OWN

TRY USING ALTERNATE EQNS

$\Sigma M_{AB}, \Sigma M_{BE}, \Sigma M_z, \Sigma F_x, \Sigma M_y, \Sigma F_z$

$T_{BD} = 2770\,N$

$T_{AC} = 4810\,N$

$T_{BE} = 654\,N$

$O_x = 1962\,N$

$O_y = 0\,N$

$O_z = 6540\,N$

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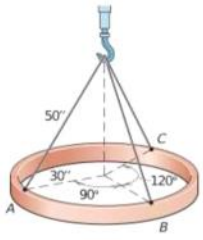
Problem 3/77

A uniform steel ring 60 in. in diameter and weighing 600 lb is lifted by the three cables, each 50 in. long, attached at points A, B, and C as shown. Compute the tension in each cable.



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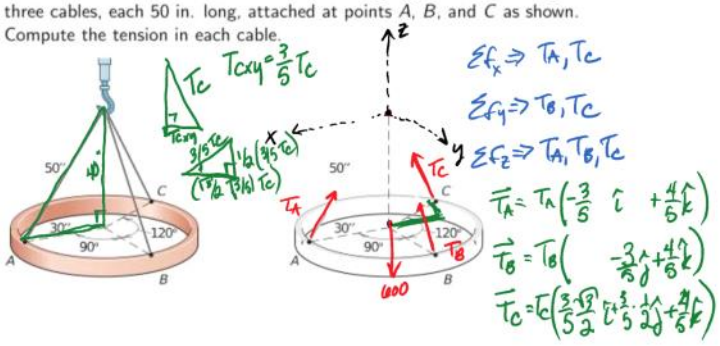

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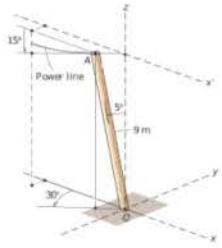
Problem 3/82

Because of a combination of soil support conditions and the tension in the single power cable, the utility pole has developed the indicated 5° lean. The 9-m uniform pole has a mass per unit length of 25 kg/m, and the tension in the power cable is 900 N. Determine the reactions at the base O. Note that the power cable lies in a vertical plane parallel to the x-z plane.



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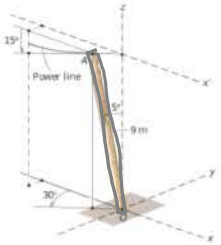
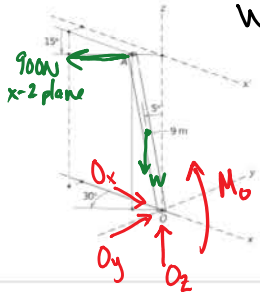




$W = 25\,\frac{kg}{m} \cdot 9\,m \cdot g$

$W = 2207.25\,N$

$\Sigma F_y = 0 = O_y$

$\Sigma F_x = 0 = O_x - 900\cos 15 \Rightarrow O_x = 900\cos 15 = 869N$

$\Sigma F_z = 0 = O_z - 2207.25 - 900\sin 15 \Rightarrow O_z = 2440N$

$\Sigma M_o = 0 = M_o + M_w + M_T \Rightarrow M_o = -(M_w + M_T)$

$\vec{M} = \vec{r} \times \vec{F}$

$\hat{r} = \langle -\sin 5 \cdot \cos 30, \sin 5 \sin 30, \cos 5 \rangle = \langle -.07548, -.04358, .9962 \rangle\ m$

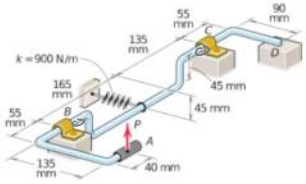
$M_w = 4.5\hat{r} \times \langle 0, 0, -2207.25 \rangle = \langle 432.8, -750, 0 \rangle\ Nm$

$M_T = 9\,\hat{r} \times 900\langle -\cos 15, 0, -\sin 15 \rangle = \langle 91.36, -7952, -341 \rangle\ Nm$

$M_o = -(M_w + M_T) = \langle -524, 8700, 341 \rangle\ Nm$

Problem 3/90

The spring of modulus $k = 900$ N/m is stretched a distance $\delta = 60$ mm when the mechanism is in the position shown. Calculate the force P_{min} required to initiate rotation about the hinge axis BC, and determine the corresponding magnitudes of the bearing forces which are perpendicular to BC. What is the normal reaction force at D if $P = P_{min}/2$?



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$F_s = 900 \frac{N}{m}(.06m) = 54\ N$

Problem 3/90

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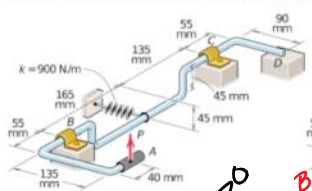

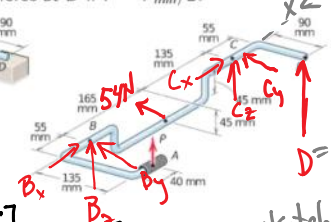

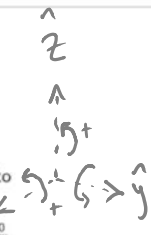

① $\Sigma M_{BC} = 0 = -54[45] + \cancelto{0}{D}[355] + P[135]$

$P = \frac{54[\cancel{45}]}{\cancel{45}\cdot 3} = \frac{54}{3} = \boxed{18\ N = P}$

② $\Sigma M_{By} = 0 = -18[15] + C_z[300]$

$C_z = \frac{18\cdot \cancel{15}}{\cancel{15}\cdot 20} = \frac{18}{20} = \frac{9}{10} = .9N$

Left for Student.

sketch of work

① $\Sigma M_{BC} \Rightarrow P$

$\Sigma F_z \Rightarrow B_z, C_z$

$\Sigma F_x \Rightarrow B_x = -C_x$

$\Sigma F_y \Rightarrow B_y, C_y$

② $\Sigma M_{By} \Rightarrow C_z$

③ $\Sigma M_{Cy} \Rightarrow B_z$

④ $\Sigma M_{Bz} \Rightarrow C_y$

⑤ $\Sigma M_{Cz} \Rightarrow B_y$

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Engr 420
4.1-4.3 n...

Inserted from: <file://H:\My Documents\Folsom Lake College\ENGR 420 Statics\Notes\Engr 420 4.1-4.3 notes (intro, plane trusses, method of joints).docx>

Chapter 4: Structures

- Section 4.1: Introduction
- Section 4.2: Plane Trusses
- Section 4.3: Method of Joints
- Section 4.4: Method of Sections
- Section 4.6: Frames and Machines

4/1 Introduction

In chapter 3 we looked at FBD's and practiced analyzing forces acting on a body. In doing so under equilibrium conditions all internal forces canceled out and we never attempted to analyze these forces. Now in chapter 4 we focus our gaze on determining the forces internal to a body or structure. We will again only consider statically determinate structures which do not have more supporting constraints than are necessary to maintain equilibrium.

In sections 4.3 and 4.4 we will focus on trusses. In section 4.6 we will look at frames and then move on to machines.

4/2 Plane Trusses

A framework composed of members joined at their ends to form a rigid structure is called a *truss*. Examples of a truss are seen in bridges, roof supports, derricks, and supports for machines, signs, buildings, etc.

When the members of the truss lie essentially in a single plane, the truss is called a *plane truss*.

For bridges and similar structures, plane trusses are commonly utilized in pairs with one truss assembly placed on each side of the structure. A section of a typical bridge structure is shown in Fig. 4/1. The combined weight of the roadway and vehicles is transferred to the longitudinal stringers, then to the cross beams, and finally, with the weights of the stringers and cross beams accounted for, to the upper joints of the two plane trusses which form the vertical sides of the structure. A simplified model of the truss structure is indicated at the left side of the illustration; the forces L represent the joint loadings.

Several examples of commonly used trusses which can be analyzed as plane trusses are shown in Fig. 4/2.

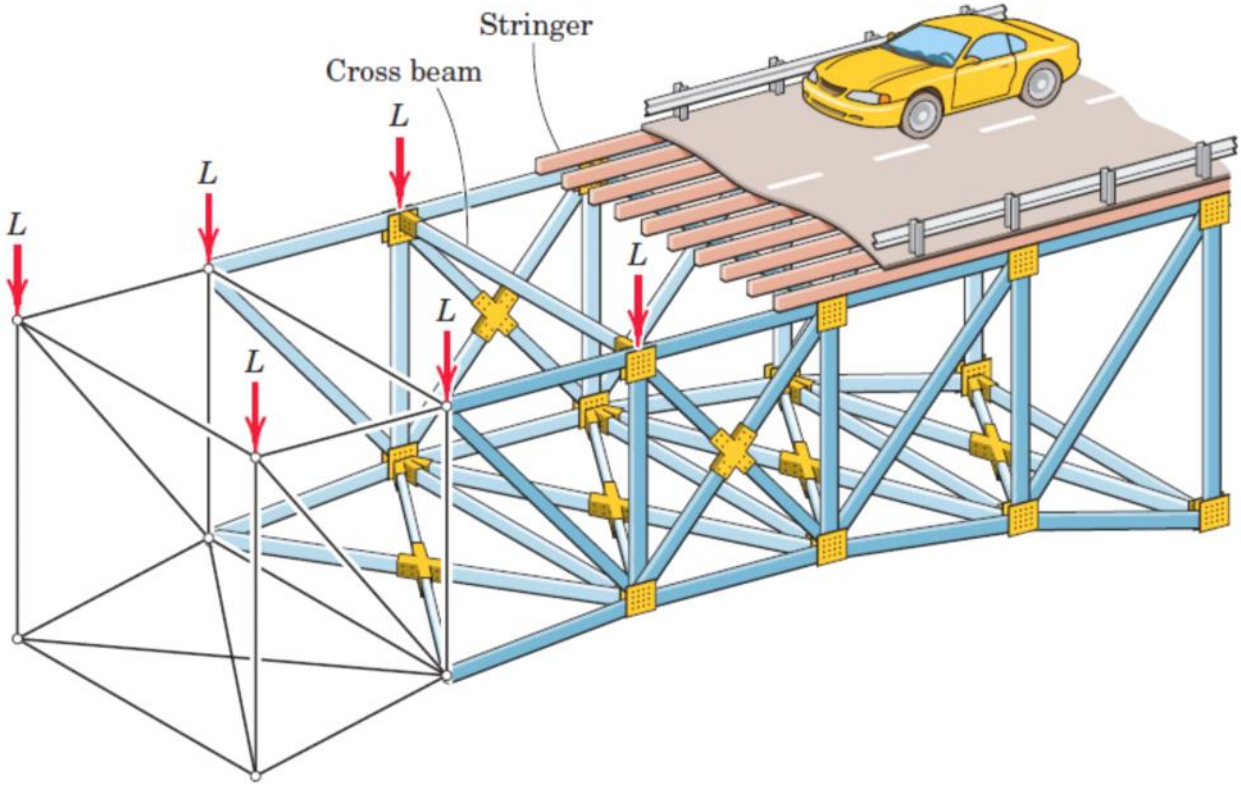
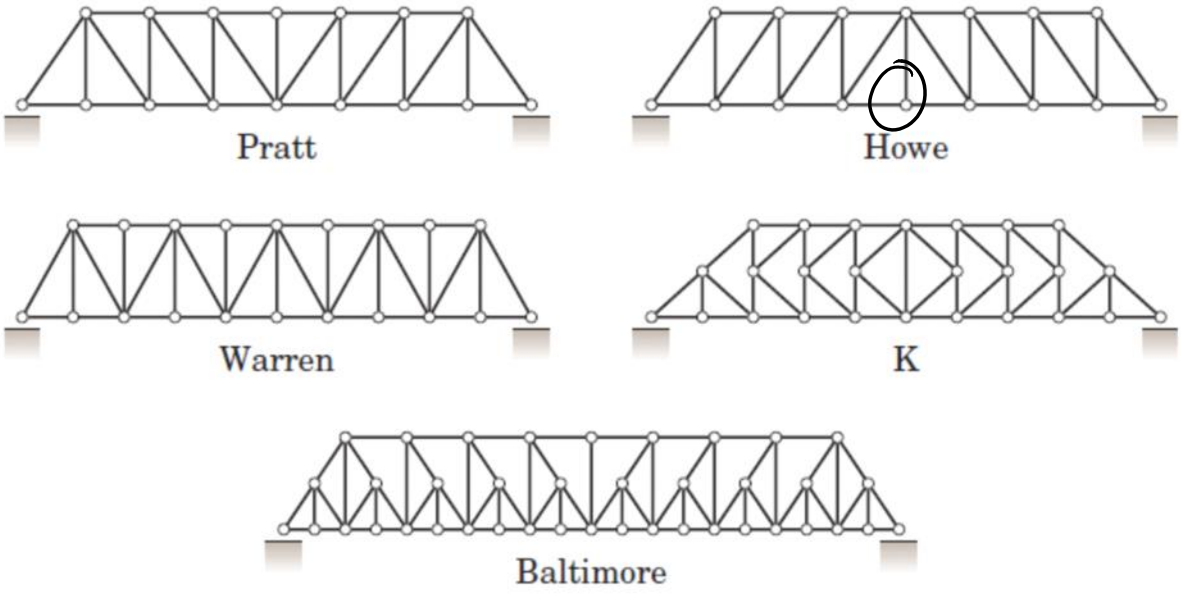
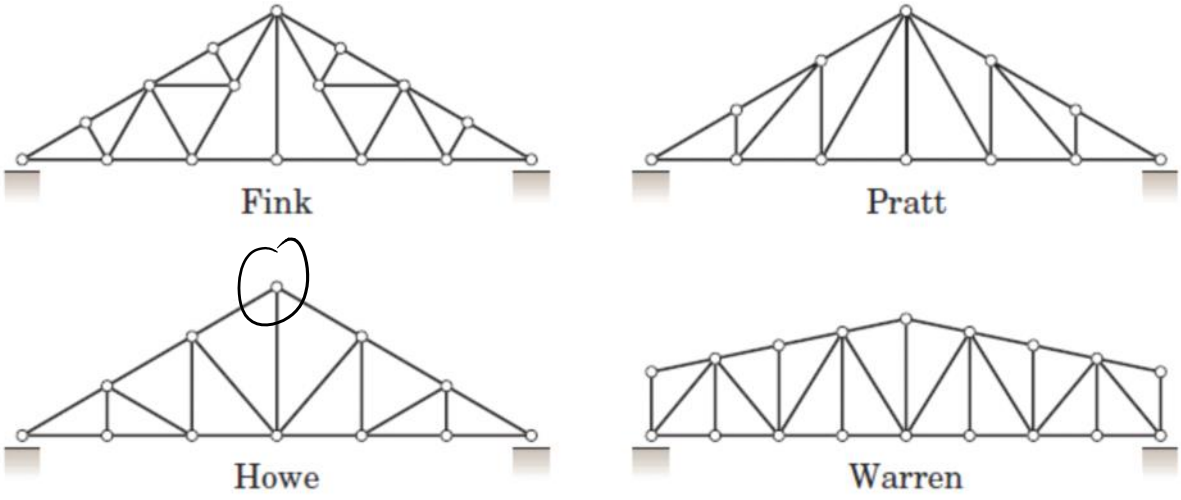


Figure 4/1



Commonly Used Bridge Trusses



Commonly Used Roof Trusses

Figure 4/2

Simple Trusses

The most basic building block or element of a truss is the triangle. A structure pined at each connection in the shape of a triangle is termed *rigid*, and implies that it will not collapse or deform (negligibly) under the load of its own weight or of an external load (unless the load exceeds the capacity of the truss). Quadrilateral shapes are *non-rigid* when pined because they are free to move at their joints and are configured in such a way as to not support most loads. The way to turn a non-rigid truss shape into a rigid one is to add a minimum number of supports such that the structure is self-supporting (this often means creating lots of triangles).

Structures built from one or more basic triangles are called *simple trusses*. When there are more members present than are essential to make the truss rigid, the truss is statically indeterminate, and as such cannot be analyzed by the equations of equilibrium alone. This is due to the presence of more unknowns than available equilibrium equations. The additional members above and beyond the ones absolutely necessary to create a rigid truss are called *redundant* members.

The proper design of a truss requires selecting appropriate sizes, materials, thicknesses, etc. Determining these characteristics requires one to know the expected forces in each member of the truss.

A Two-force member is a member in equilibrium under the action of two forces only. As such, both forces must be equal and opposite in the direction of the member itself for the member to remain in equilibrium. Because the forces in a two force member are collinear with the line connecting its points of contact, all forces will be directed either in or out, or the member will be loaded in tension or in compression.

4/3 Method of Joints

This method is used for finding the forces in the members of a truss. The method involves analyzing the forces present at each joint, and using the two independent equilibrium equations, finding the forces in each member.

To use it a few parameters must be satisfied. When analyzing any particular joint:

- At least one force must be known.
- No more than two *unknown* forces may be present.
- Because we can specify the equilibrium of each joint by two scalar force equations, there are in all $2j$ such equations for a truss with ***j* number of joints**. For the entire truss composed of ***m* number of two-force members** and having the maximum of three unknown support reactions (support reactions for the entire truss), there are in all $m + 3$ unknowns in the system. Thus, for any plane truss, the equation $\mathbf{2}\boldsymbol{j} = \boldsymbol{m} +$ will be satisfied if the truss is statically determinate internally.

Convention of Force Directions:

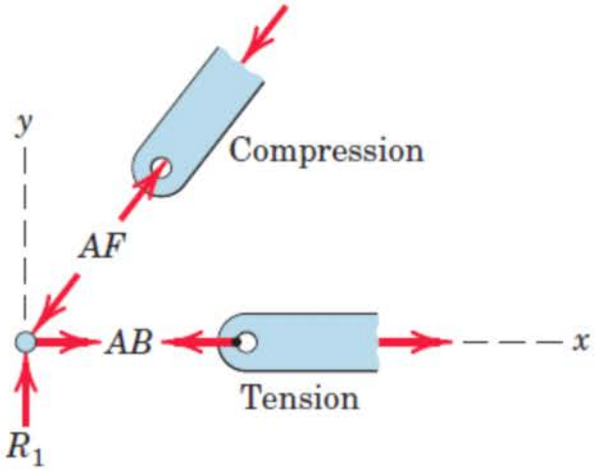
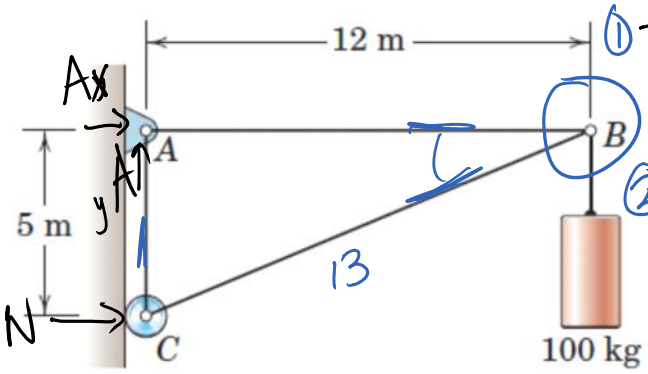


Figure 4/7

In this text and as customary, we will indicate tension forces at a joint by a force pulling away from the joint, and compression forces by a force pushing into the joint.

Examples:

4/1 Determine the force in each member of the loaded truss.



Sketch

① Joint B: $\Sigma F_y = 0 \Rightarrow BC$

$\Sigma F_x = 0 \Rightarrow AB$

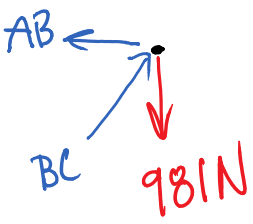
② $\Sigma M_A \Rightarrow N$

$\Sigma F_x \Rightarrow A_x = -N$

$\Sigma F_y \Rightarrow A_y = 981\ N$

③ Joint C: $\Sigma F_y \Rightarrow AC$

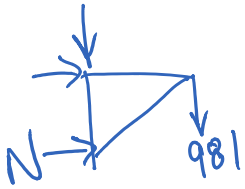
Joint B



$\Sigma F_y = +BC\frac{5}{13} - 981 = 0$

$\Rightarrow BC = \frac{981 \cdot 13}{5} = 2550.6 = \boxed{2550N\ C = BC}$

$\Sigma F_x = -AB + BC\frac{12}{13} = 0 \Rightarrow AB = 2354 = \boxed{2350\ NT = AB}$

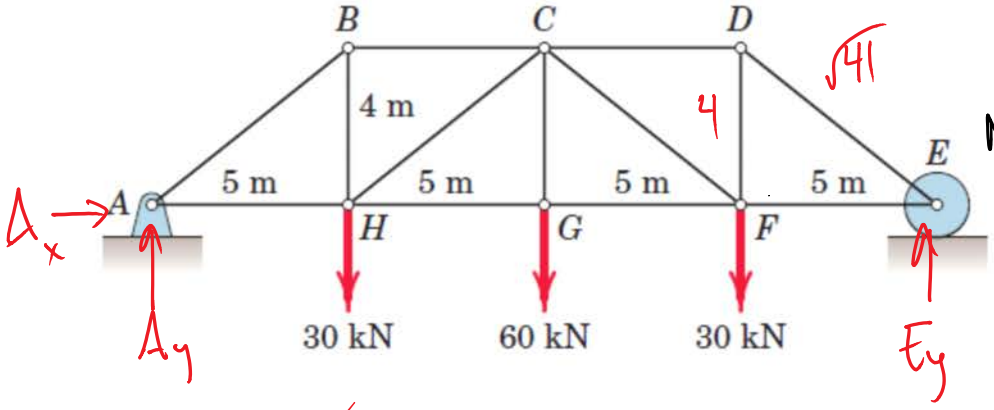


$\Sigma M_A = N(5) - 981(12) = 0 \Rightarrow N = \frac{981(12)}{5} = 2354.4\ N = \boxed{2350N = N}$

Joint C:

$\Sigma F_y = AC - BC\frac{5}{13} = 0 \Rightarrow \boxed{AC = 2550.6\left(\frac{5}{13}\right) = 981\ NT}$

4/7 Determine the force in each member of the loaded truss. Make use of the symmetry of the truss and of the loading.



sketch
Entire Truss

$\Sigma F_x \Rightarrow A_x = 0$

ΣF_y & $\Sigma M_A \Rightarrow A_y = E_y$

$A_y = 60\text{kN} = E_y$

Method of Joints

E -> DE & EF

D -> DF & CD

F -> CF & FG

G -> CG

Symmetry => all the Rest.

$\Sigma F_x = 0 : A_x$ ✓

$\Sigma M_A = 30(20) + 60(10) - E_y(20) = 0$

$E_y = \frac{600 + 600}{20} = \frac{1200}{20} = 60\text{kN}$

$\Sigma F_y \Rightarrow A_y = E_y = 60\text{kN}$

$\Sigma F_y = 0 = A_y + 60 - 120$

Joint E:

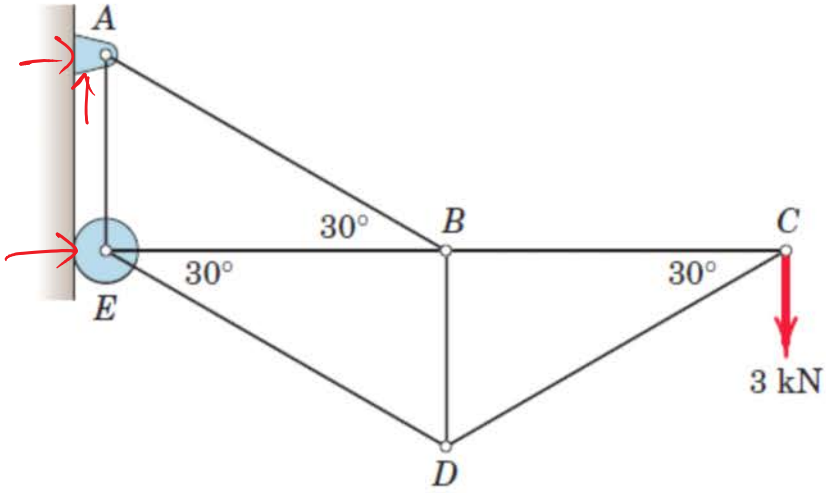
DE
EF
60kN

$\Sigma F_y = 0 = -DE\frac{4}{\sqrt{41}} + 60 \Rightarrow DE = 60\frac{\sqrt{41}}{4} = 15\sqrt{41} = 96.0\text{kN}$

DE = 96.0kN C

Finish on your own.

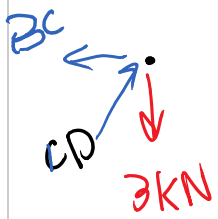
4.3.9 Determine the forces in members BC & CD.



what if I wanted to find forces in all Members

C: → BC & CD
D: → DE & BD
B: → AB & BE
E: → ΣF_y ⇒ AE

Joint C



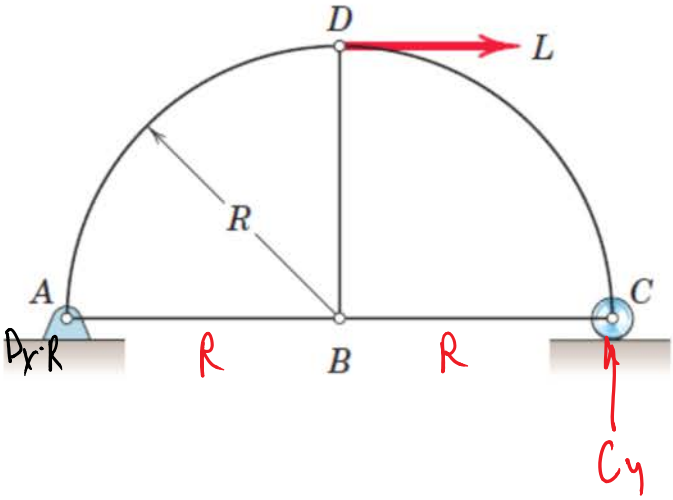
$\Sigma F_y = -3kN + CD \sin 30 = 0$

$CD = \frac{3kN}{\sin 30} = 6\,kN\ C$

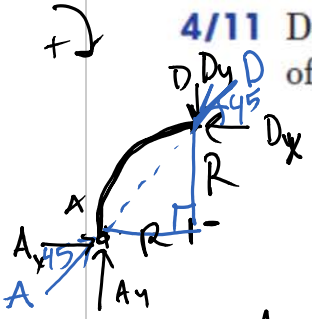
$\Sigma F_x = -BC + CD\cos 30 = 0$

$BC = 6\cos 30 = 3\sqrt{3} = 5.20kN$ T

4/11 Determine the forces in members AB, BC, and BD of the loaded truss.



$\Sigma M_A = 0$



$\Sigma F_y = 0 \Rightarrow D_y = A_y$

$\Sigma F_x = 0 \Rightarrow D_x = A_x$

$\Sigma M_A = 0 \Rightarrow D_y \cdot R - D_x \cdot R$

$\Rightarrow D_y \cdot R = D_x \cdot R$

$\frac{D_y}{R} \quad \frac{D_x}{R}$

$D_y = D_x$

C_y

$AB = DE = BC = CD = 3.35\,kNC$

$AH = EF = 3kNT$

$BH = DF = 1kNC$

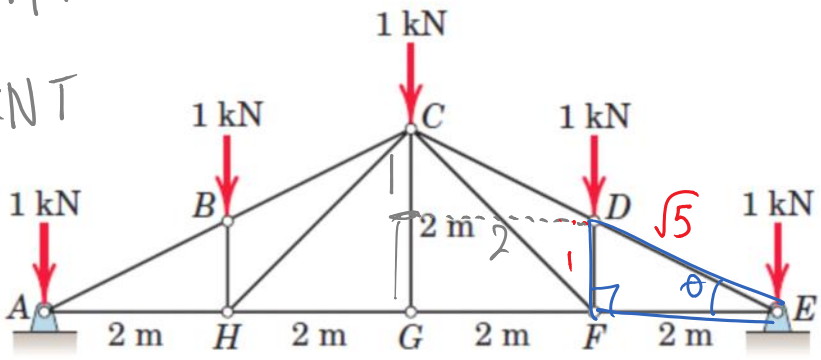
$CF = CH = 1.414\,kNT$

$CG = 0$

$FG = GH = 2kNT$

4/21 A snow load transfers the forces shown to the upper joints of a Pratt roof truss. Neglect any horizontal reactions at the supports and solve for the forces in all members.

$\sin\theta = \frac{1}{\sqrt{5}}$



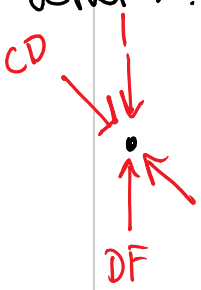
Joint E

$\Sigma F_y = 0 = 2.5 - DE\left(\frac{1}{\sqrt{5}}\right) - 1$

$\Rightarrow DE = 1.5\cdot\sqrt{5} = 3.35\,kNC$

$\Sigma F_x = 0 = -EF + DE\left(\frac{2}{\sqrt{5}}\right) \Rightarrow EF = 1.5\cdot\sqrt{5}\left(\frac{2}{\sqrt{5}}\right) = 3\,kNT$

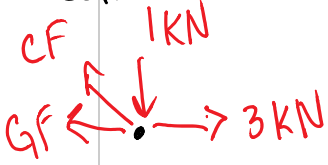
Joint D:



$\Sigma F_x = 0 = -1.5\sqrt{5}\left(\frac{2}{\sqrt{5}}\right) + CD\left(\frac{2}{\sqrt{5}}\right) \Rightarrow CD = 1.5\sqrt{5} = 3.35\,kNC$

$\Sigma F_y = 0 \Rightarrow DF = 1\,kNC$

Joint F



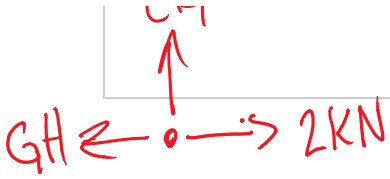
$\Sigma F_y = 0 = -1 + CF\frac{1}{\sqrt{2}} \Rightarrow CF = 1\cdot\sqrt{2} = 1.414\,kNT$

$\Sigma F_x = 0 = -GF - \sqrt{2}\cdot\frac{1}{\sqrt{2}} + 3 \Rightarrow GF = 3 - 1 = 2kNT$

Joint G:

$\Sigma F_y = \boxed{0 = CG}$

$\Sigma F_y = 2kN - GH = 0 \Rightarrow GH = 2kNT$



$\Sigma F_y = 2kN - GH = 0 \Rightarrow GH = 2kN$!

Engr 420 4.4 notes (method of sections)

Wednesday, October 28, 2015 4:01 PM

Engr 420 4.4 notes (method of sections)

Chapter 4: Structures

- ~~Section 4.1: Introduction~~
- ~~Section 4.2: Plane Trusses~~
- ~~Section 4.3: Method of Joints~~
- Section 4.4: Method of Sections
- Section 4.6: Frames and Machines

4/4 Method of Sections or Method of Cuts

When analyzing plane trusses by the method of joints, we need only two of the three equilibrium equations because the procedures involve concurrent forces at each joint. We can take advantage of the third or moment equation of equilibrium by selecting an entire section of the truss for the free body in equilibrium under the action of a nonconcurrent system of forces. This *method of sections* has the basic advantage that the force in almost any desired member may be found directly from an analysis of a section which has cut that member. Thus, it is not necessary to proceed with the calculation from joint to joint until the member in question has been reached. In choosing a section of the truss, we note that, in general, not more than three members whose forces are unknown should be cut, since there are only three available independent equilibrium relations.

The main idea behind this method is that the entire body is in equilibrium. All forces internal to the body have equal and opposite reactions allowing the internal forces to sum to zero. The method of cuts creates an imaginary cut in the body. This cut removes the equal and opposite reaction force from the other half of the body which was cut away, therefore, in order to keep this section of the body in equilibrium, the missing reactive forces needs to be reapplied to the cut member(s). This way a new body will be considered.

With convenient selections for where to sum moments and or summing forces in a particular direction, you will be able to determine the forces in your three unknown members.

The most important rule is that you cannot cut more than 3 unknown members. You may cut more, but only if you know the forces in all but 3 members. This is because we have three independent equations available, $\sum \boldsymbol{F}_x$, $\sum \boldsymbol{F}_y$, $\sum \boldsymbol{M}$.

It is important to remember if you cut a two-force member which is in tension you will need to reapply a force which pulls the cut member back into tension (away from the joint it originates from. When you cut a member in compression, you will need to apply a force pushing into the member to recompress it again. This force will point into the joint it originates from.

In some cases the method of sections can be applied twice or combined with the method of joints. In a case where the member you are searching for requires you to cut members where there are more than 3 unknown, you must first make a cut with 3 or less unknowns which will allow you to find enough forces in your other cut to allow you no more than 3.

Convention of Force Directions:

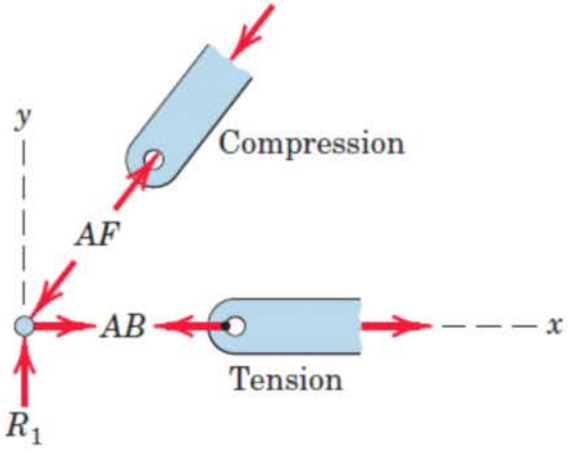
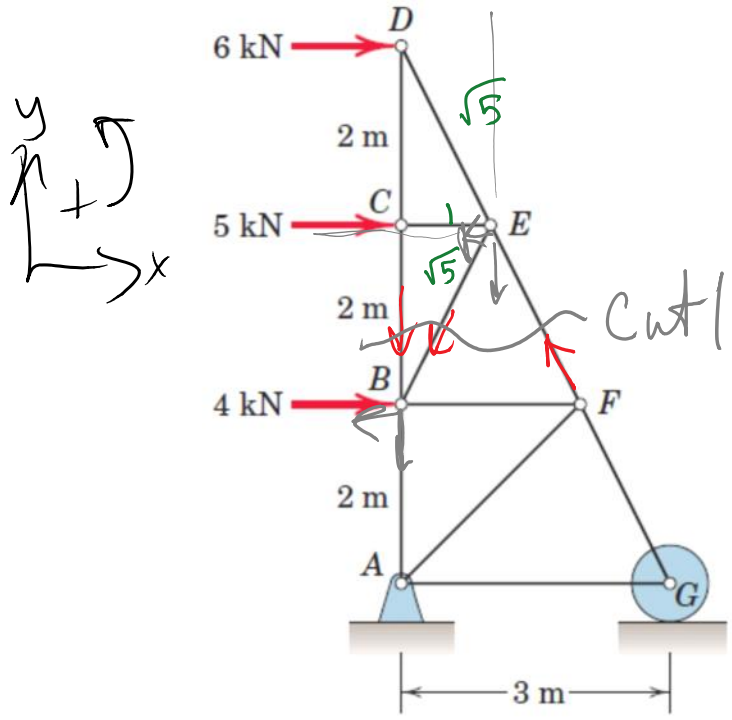


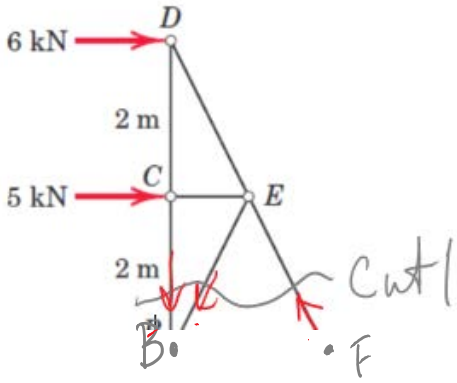
Figure 4/7

In this text and as customary, we will indicate tension forces at a joint by a force pulling away from the joint, and compression forces by a force pushing into the joint.

Examples:

4/33 Determine the force in member BE of the loaded truss.





$\Sigma M_B \Rightarrow EF$

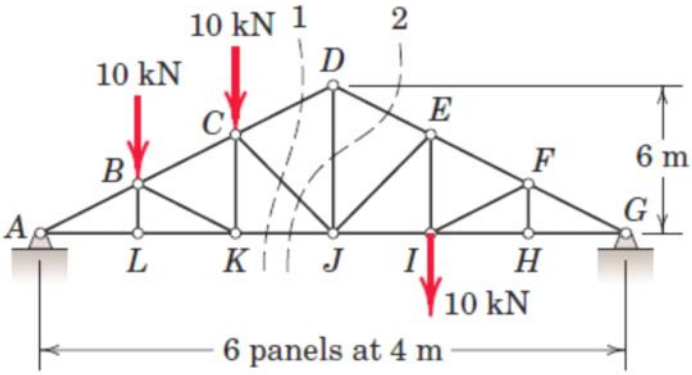
$\Sigma M_D \Rightarrow BE$

$\Sigma M_D = 0 : +5(2) - BE\,\frac{1}{\sqrt{5}}(4) = 0 \Rightarrow BE = \frac{10\cdot\sqrt{5}}{4} = \frac{5\sqrt{5}}{2}$

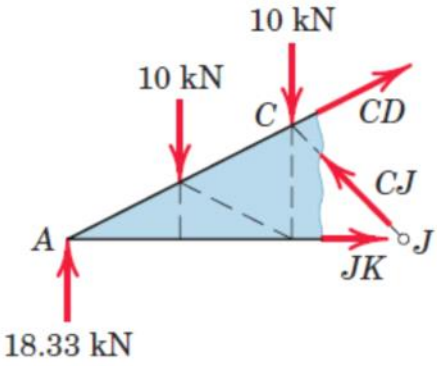
$BE = 5.59\ \text{KN T}$

SAMPLE PROBLEM 4/4

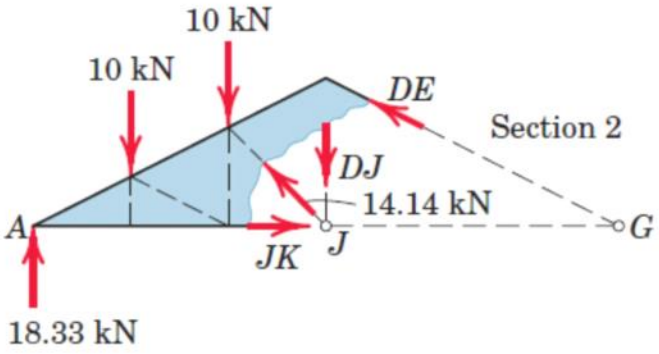
Calculate the force in member DJ of the Howe roof truss illustrated. Neglect any horizontal components of force at the supports.



DJ = 16.67 KNT

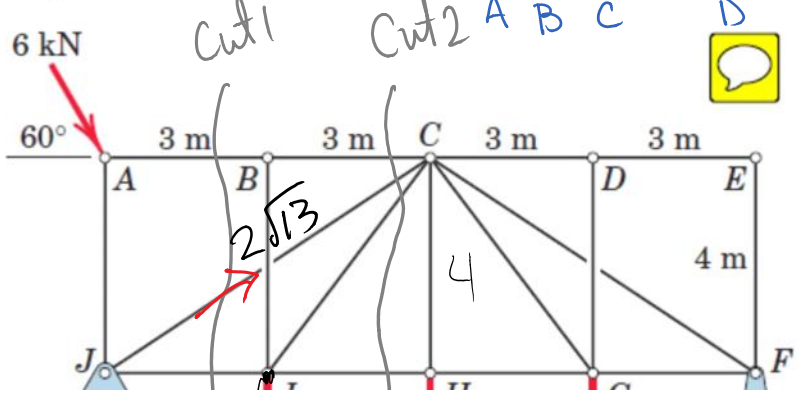


Section 1



Section 2

4/43 The members CJ and CF of the loaded truss cross but are not connected to members BI and DG. Compute the forces in members BC, CJ, CI, and HI.



Sketch

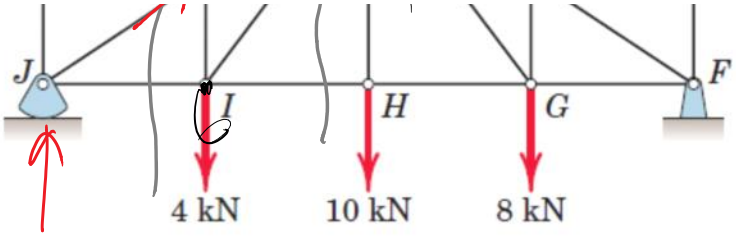
Entire Truss

$\Sigma M_F \rightarrow R$

Cut 1

$\Sigma F_y \Rightarrow CJ$ B

Cut 2



Cut 2

$\Sigma M_I \Rightarrow BC$ A

$\Sigma F_y \Rightarrow CI$ C

$\Sigma F_x \Rightarrow HI$

$R = 14.20 kN$

or

$\Sigma M_C \Rightarrow HI$

$\Sigma F_y \Rightarrow CI$

$\Sigma F_x \Rightarrow BC$

$\Sigma F_y = 14.20 - 3\sqrt{3} + JC\,\frac{4}{2\sqrt{13}}$

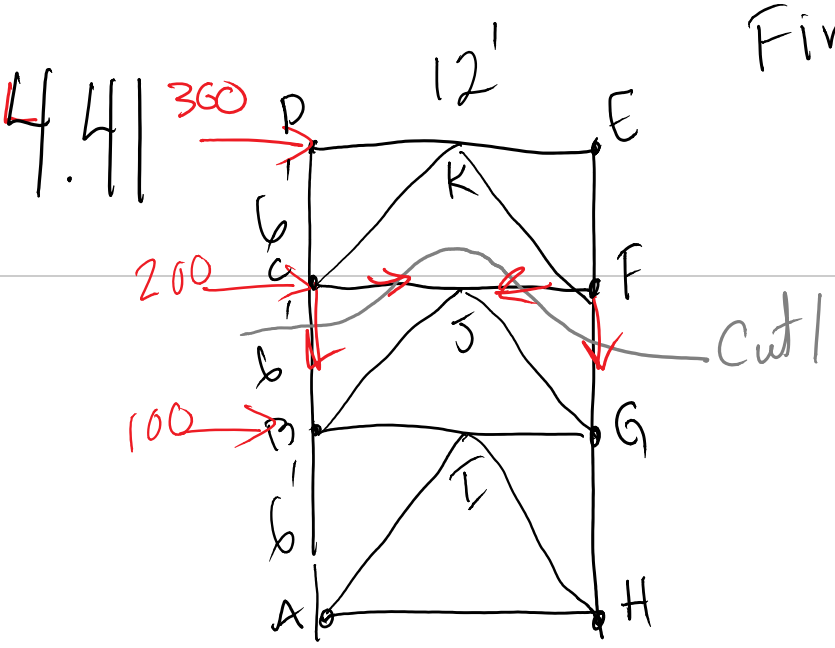
$JC = (3\sqrt{3} - 14.2)\,\frac{2\sqrt{13}}{4}$

$JC = -16.23\ KN$

$= 16.23\ KNC$

Find FG & BC

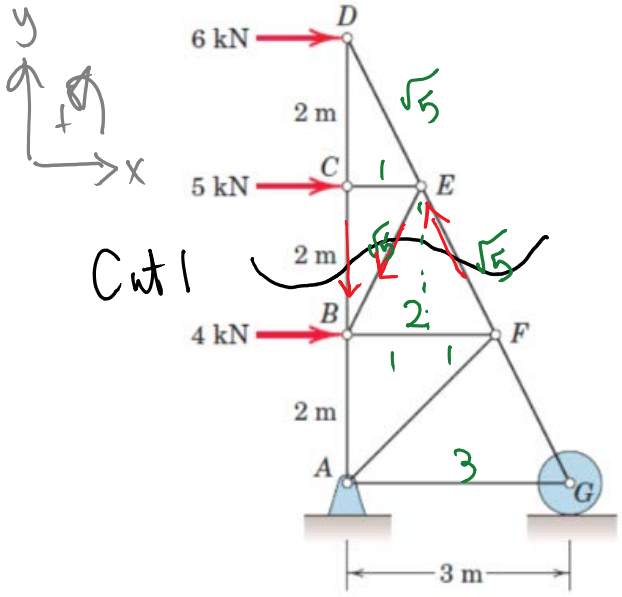
4.41



4.4 example 4/33

Monday, November 02, 2015 4:50 PM

4/33 Determine the force in member *BE* of the loaded truss.



Cut 1:

$\Sigma M_D \Rightarrow BE$!

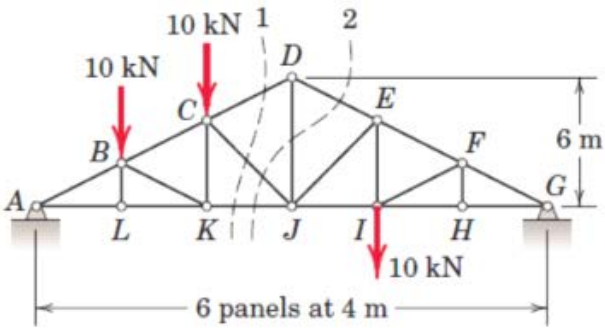
$\Sigma M_D = 0 = -5(2) - BE\left(\frac{1}{\sqrt{5}}\right)(4)$

$\Rightarrow BE = -\frac{5 \cdot 2\sqrt{5}}{4} = -\frac{5\sqrt{5}}{2}$ kN

$BE = -5.59\text{ kN} = 5.59\text{ kNC}$

4.4 Sample Prob 4/4

Monday, November 02, 2015 5:03 PM

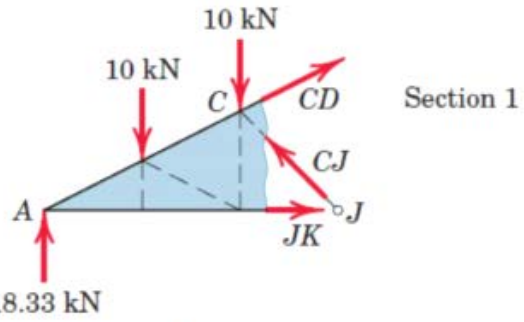


Sketch

Entire Truss $\Sigma M_G \Rightarrow A_y$

Cut 1: $\Sigma M_A \Rightarrow CJ$

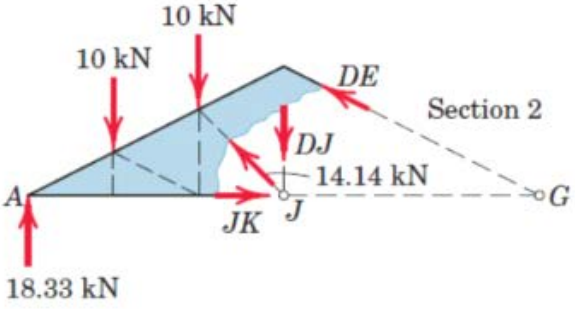
$\Sigma M_C \Rightarrow JK$



Cut 2: Joint J

$\Sigma F_y \Rightarrow DJ$

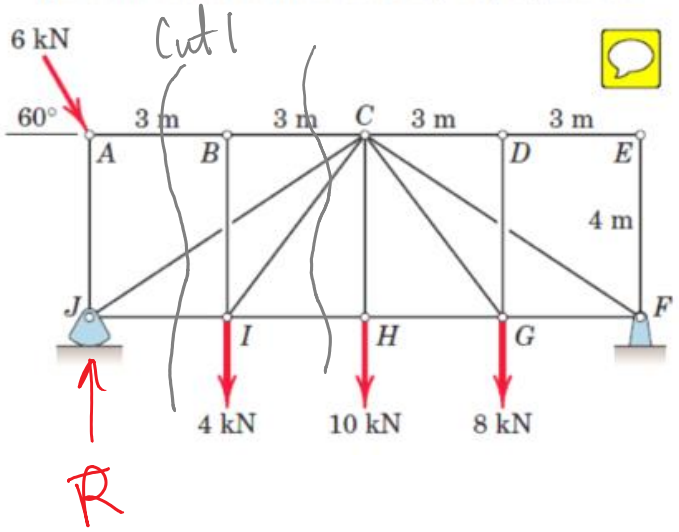
* Note JK not needed for this



4.4 Prob 4.43

Monday, November 02, 2015 5:11 PM

4/43 The members CJ and CF of the loaded truss cross but are not connected to members BI and DG. Compute the forces in members BC, CJ, CI, and HI.

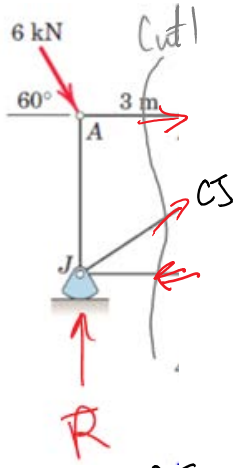


Sketch:

Entire Truss:

$\Sigma M_F \Rightarrow R$

Cut 1

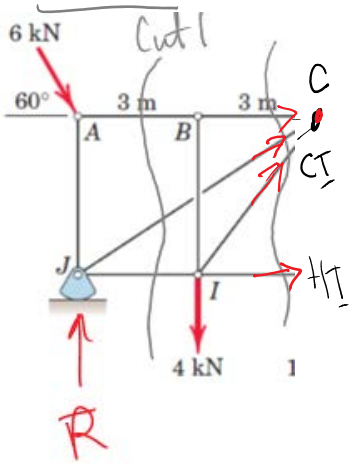


$\Sigma F_y \Rightarrow CJ$

$\Sigma M_J \Rightarrow AB$

$\Sigma F_x \Rightarrow JI$

Cut 2



$\Sigma M_C \Rightarrow HI$

$\Sigma M_I \Rightarrow BC$ or $\Sigma F_x \Rightarrow BC$

$\Sigma F_y \Rightarrow CI$

Engr 420 4.6 notes (Frames and Machines)

Monday, November 02, 2015 3:43 PM

Engr 420 4.6 notes (Frames and Machines)

Chapter 4: Structures

- ~~Section 4.1: Introduction~~
- ~~Section 4.2: Plane Trusses~~
- ~~Section 4.3: Method of Joints~~
- ~~Section 4.4: Method of Sections~~
- Section 4.6: Frames and Machines

4/6 Frames and Machines

Definition: A *multiforce member* is one with three or more forces acting on it, or one with two or more forces and one or more couples acting on it.

Definition: A structure is called a *frame or machine* if at lease one of its individual members is a multiforce member.

Frames are structrues which are designed to support appled loads are are usually fixed in position whereas machines are structures which contain moving parts and are designed to transmit input forces or couples to out put forces or couples. Because these generally contain multiforce members, it is typical that the forces in these members will likely not be in the directions of the members.

Interconnected Rigid Bodies with Multiforce Members

In Chapter 3 we discussed the equilibrium of multiforce bodies, but we concentrated on the equilibrium of a *single* rigid body. In the present article we focus on the equilibrium of *interconnected* rigid bodies which include multiforce members. Although most such bodies may be analyzed as two-dimensional systems, there are numerous examples of frames and machines which are three-dimensional.

The forces acting on each member of a connected system are found by isolating the member with a free-body diagram and applying the equations of equilibrium. The *principle of action and reaction* must be carefully observed when we represent the forces of interaction on the separate free-body diagrams. If the structure contains more members or supports than are necessary to prevent collapse, then, as in the case of trusses, the problem is statically indeterminate, and the principles of equilibrium, although necessary, are not sufficient for solution. Although many frames and machines are statically indeterminate, we will consider in this article only those which are statically determinate.

If the frame or machine constitutes a rigid unit by itself when removed from its supports, like the A-frame in Fig. 4/14*a*, the analysis is best begun by establishing all the forces external to the structure treated as a single rigid body. We then dismember the structure and consider the equilibrium of each part separately. The equilibrium equations for the several parts will be related through the terms involving

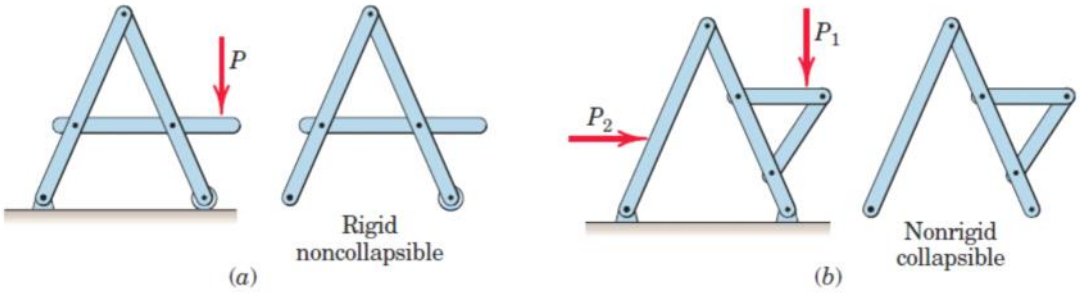


Figure 4/14

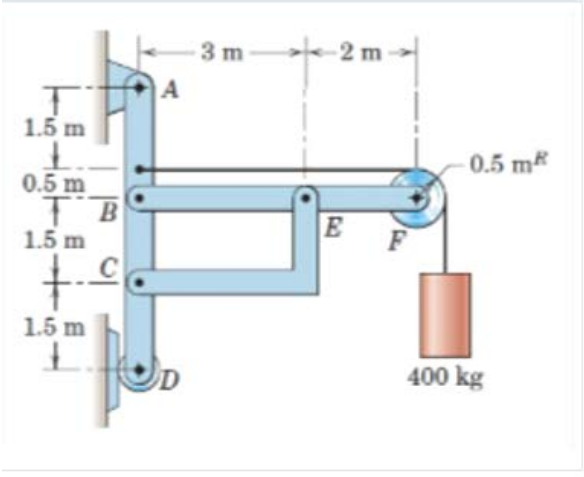
the forces of interaction. If the structure is not a rigid unit by itself but depends on its external supports for rigidity, as illustrated in Fig. 4/14*b*, then the calculation of the external support reactions cannot be completed until the structure is dismembered and the individual parts are analyzed.

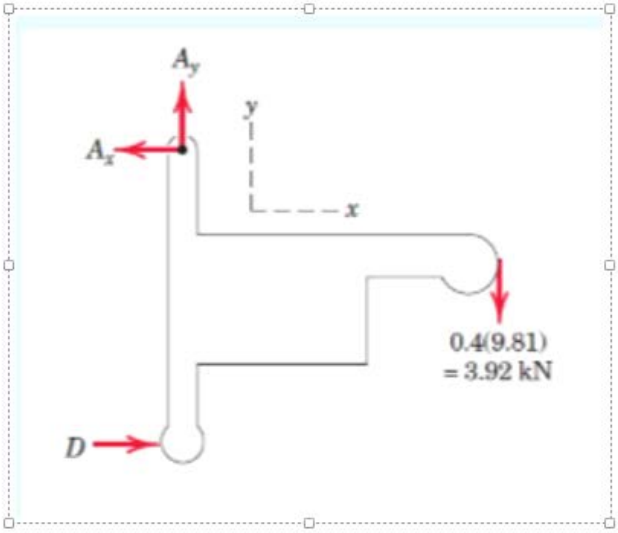
Note:

It is not always possible to assign the proper sense to every force or its components when drawing the free-body diagrams, and it becomes necessary to make an arbitrary assignment. In any event, it is ***absolutely necessary*** that a force be ***consistently*** represented on the diagrams for interacting bodies which involve the force in question. Thus, for two bodies connected by the pin *A*, Fig. 4/15*a*, the force components must be consistently represented in *opposite* directions on the separate free-body diagrams.

SAMPLE PROBLEM 4/6

The frame supports the 400-kg load in the manner shown. Neglect the weights of the members compared with the forces induced by the load and compute the horizontal and vertical components of all forces acting on each of the members.

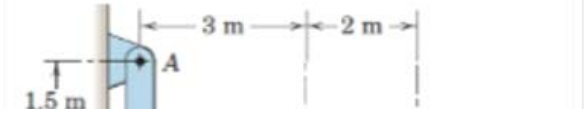


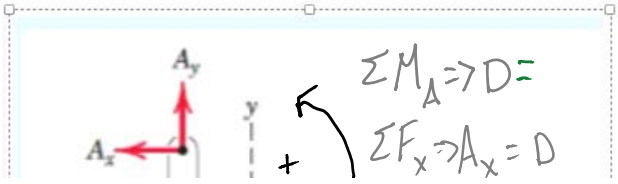


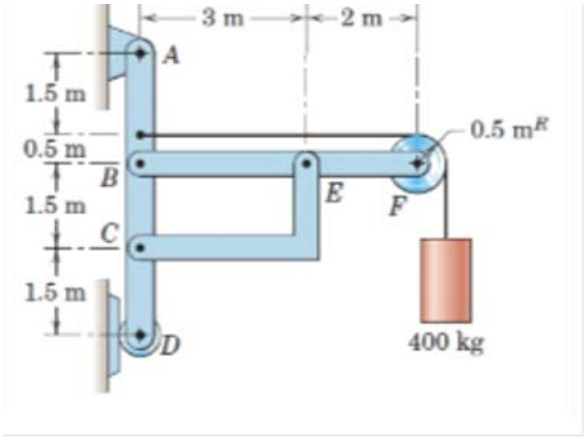
SAMPLE PROBLEM 4/6

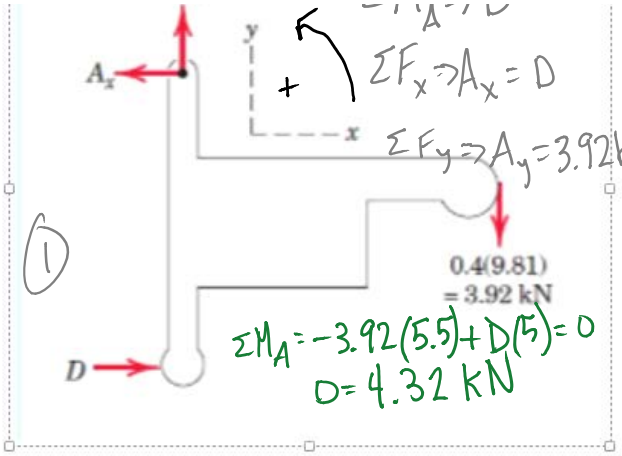
The frame supports the 400-kg load in the manner shown. Neglect the weights of the members compared with the forces induced by the load and compute the horizontal and vertical components of all forces acting on each of the members.

Sketch

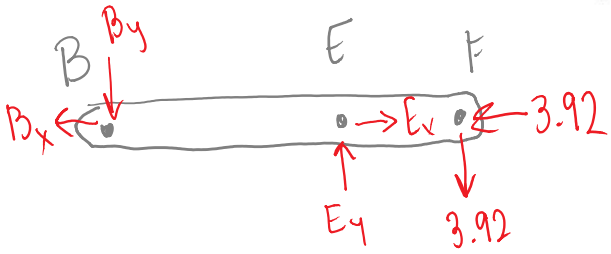




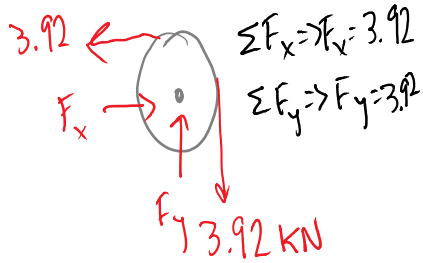




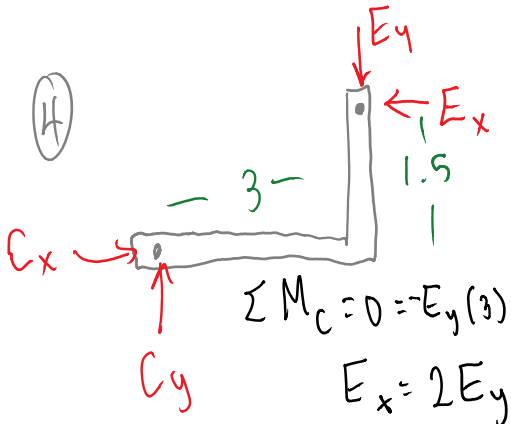
(3)



(2)



(4)



$\Sigma M_C = 0 = -E_y(3) + E_x(1.5)$

$E_x = 2E_y$ *

Now return to (3)

(3) $\Sigma M_B \Rightarrow E_y$; $\Sigma M_E \Rightarrow B_y$

* E_x $\Sigma F_x \Rightarrow B_x$

$\Sigma M_B = 0 = E_y(3) - 3.92(5)$

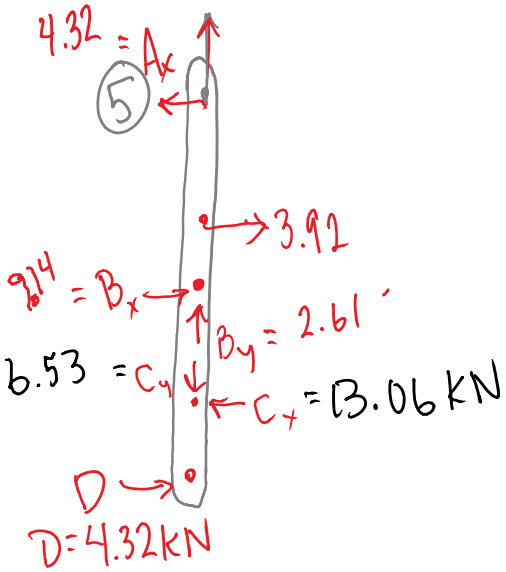
$E_y = \frac{3.92(5)}{3} = 6.53\,kN$

$\Sigma F_y \Rightarrow B_y = 6.53 - 3.92 = 2.61\,kN$

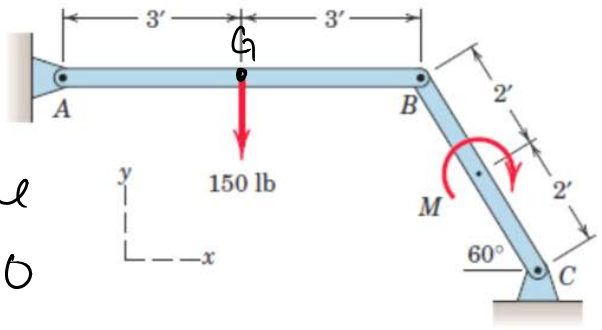
E_x: $E_x = E_y \cdot 2 = 13.06\,kN$

$\Sigma F_x = B_x = 13.06 - 3.92 = 9.14\,kN$

$A_y = 3.92$



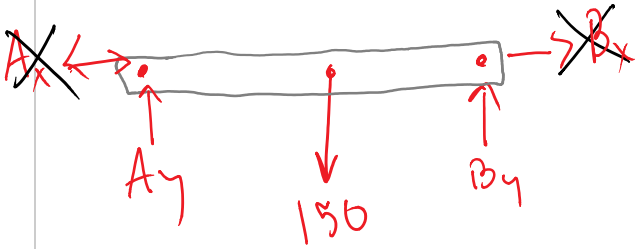
4/74 For what value M of the clockwise couple will the horizontal component A_x of the pin reaction at A be zero? If a couple of that same magnitude M were applied in a counterclockwise direction, what would be the value of A_x?



Lets Assume

$A_x = B_x = 0$

(1)

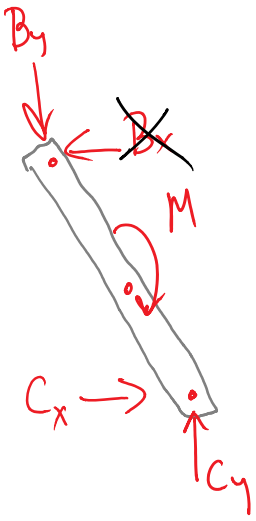


$\Sigma M_G \Rightarrow A_y = B_y$

$\Sigma M_A = 0 = B_y(6) - 150(3)$

$B_y = \frac{1}{2}(150) = 75\text{lb}$

(2)



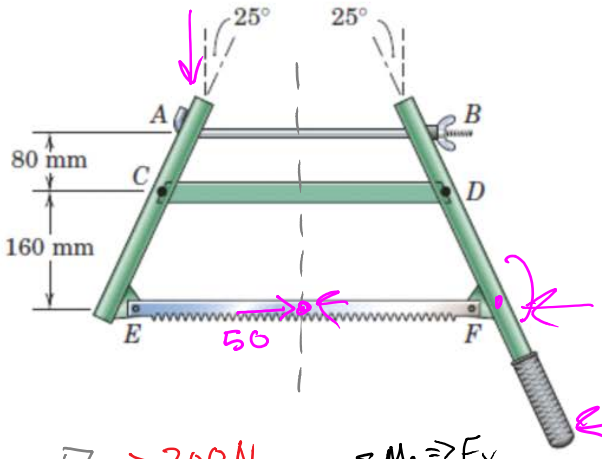
$\Sigma F_y = B_y = C_y = 75$

$\Sigma M_C = 75(4.\frac{1}{2}) - M = 0$

$= M = 150\text{CW}$ lbft

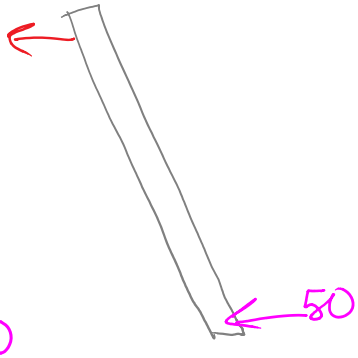
Part b) left to student:

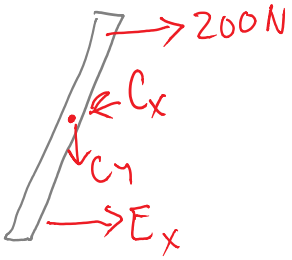
4/79 The wingnut B of the collapsible bucksaw is tightened until the tension in rod AB is 200 N. Determine the force in the saw blade EF and the magnitude F of the force supported by pin C.



Part B: (on your own, consider the following:

If a 50 N horizontal force pointing to
The handle from R to L is applied 160 mm
Below point F, will the reaction at point D
Change?
If yes, Find the new magnitude of the force
At D.





$\Sigma M_C \Rightarrow E_x$

$\Sigma M_C = 0 = -200(80) + E_x(160)$

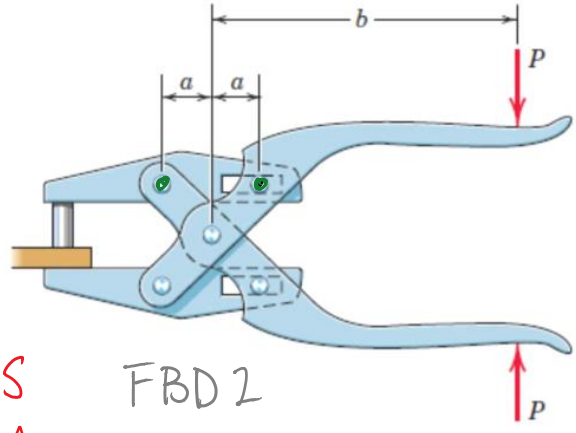
$E_x = 200\left(\frac{1}{2}\right) = 100\,N$

$\Sigma F_y \Rightarrow C_y = 0$

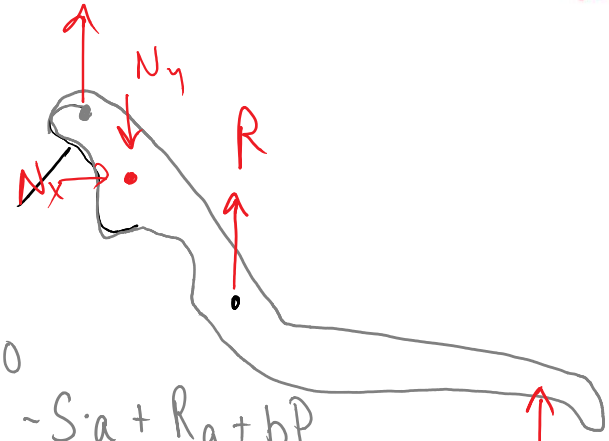
$\Sigma F_x \Rightarrow C_x = 366\,N$

$|C| = 300\,N$

4/89 For the paper punch shown, find the punching force Q corresponding to a hand grip P.



FBD 2



$\Sigma M_N = 0$

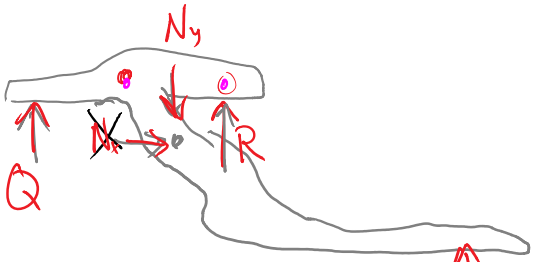
$= -S \cdot a + Ra + bP$

$Sa = Ra + Pb \Rightarrow S = R + P\frac{b}{a}$

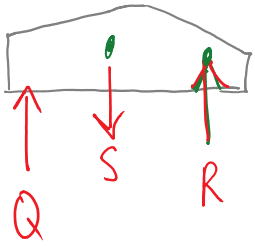
$P = (S - R)\frac{a}{b}$

$\Sigma F_x \Rightarrow N_x = 0$

FBD 3



FBD 1



$\Sigma F_y = R + Q - S = 0$

$S = R + Q$

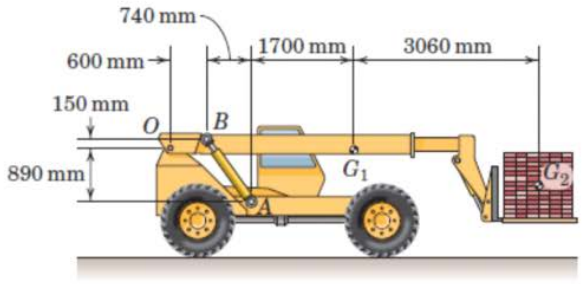
$\Sigma F_y \Rightarrow S = Q + R$

$S = R + P\frac{b}{a} = Q + R$

$Q = P\frac{b}{a}$

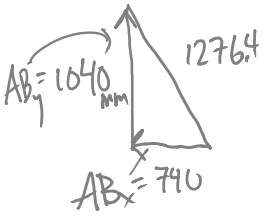
$P = Q\frac{a}{b}$

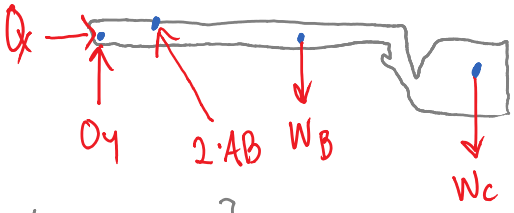
4/103 The machine shown is used for moving heavy items such as pallets of bricks around construction sites. For the horizontal boom position shown, determine the force in each of the two hydraulic cylinders AB. The mass of the boom is 1500 kg with mass center at G_1, and the mass of the cube of bricks is 2000 kg with mass center at G_2.



$W_B = 9.81(1500)$
$= 14715\ N$

$W_C = 9.81(2000)$
$= 19620\ N$





$$\Sigma M_o = 0 = \left[2AB\left(\frac{1040}{1276.4}\right)\right]600 - W_B(3040\text{ mm}) - W_C(6100) + 2AB\left(\frac{740}{1276.4}\right)150$$

Solve for $AB = 142.8\text{KN}$

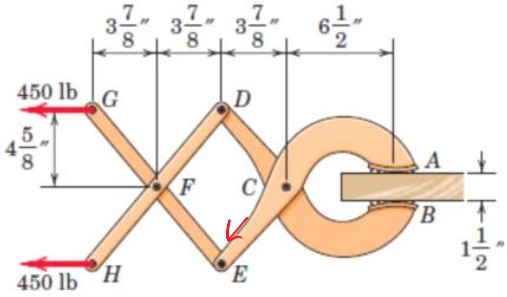
4/109 A modification of the pallet puller of Prob. 4/108 is shown here. For the same net 900-lb force as in Prob. 4/108, determine the magnitude of the force supported by pin C and the normal gripping forces at A and B.

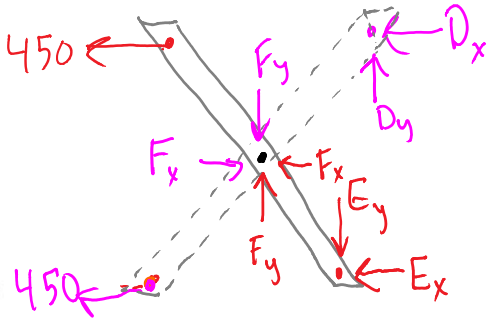
$3\frac{7}{8}''$ $3\frac{7}{8}''$ $3\frac{7}{8}''$ $6\frac{1}{2}''$

-BD!

Note $D_x = E_x$ by symmetry
& $D_y = E_y$

at A and B.





FBD1

$\Sigma M_E = 0 = 450\left(\frac{37}{4}\right) - F_y\left(\frac{31}{8}\right)$

$\Rightarrow F_y = 450\left(\frac{37}{4}\right)\left(\frac{8}{31}\right) = 1074 \text{ lb}$ ✓

$\Sigma F_y = 0 = F_y - D_y \Rightarrow D_y = F_y = 1074 \text{ lb}$ ✓

*Note Joint F &

$\Sigma F_x = 0 = -E_x - F_x - 450$

$\Rightarrow E_x = -F_x - 450$

$\Sigma F_x = 0 = -450 + F_x - D_x$

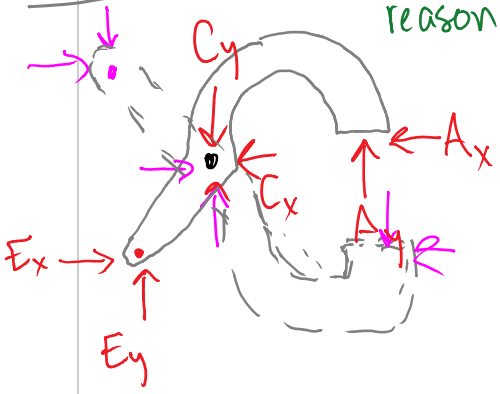
$\Rightarrow D_x = F_x - 450$

So $-F_x - 450 = F_x - 450$

$2F_x = 0 \Rightarrow F_x = 0$

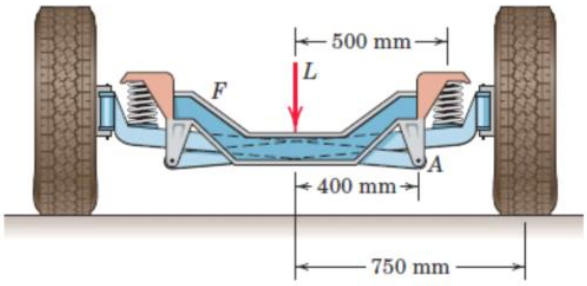
FBD 2

Note: for same reason $C_x = 0$

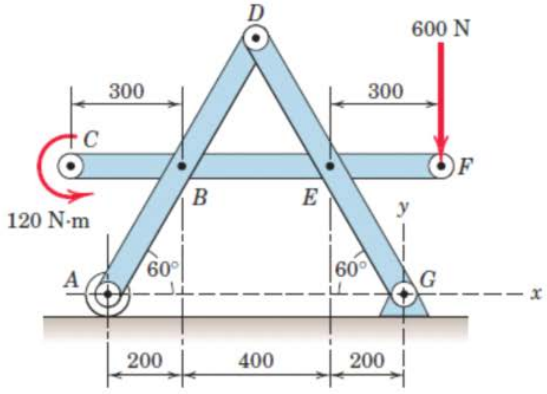


The F_x on both members are mirror images of eachother w/ the exception of F_x which is equall and opp. yet they net to 0.

4/118 A double-axle suspension for use on small trucks is shown in the figure. The mass of the central frame F is 40 kg, and the mass of each wheel and attached link is 35 kg with center of mass 680 mm from the vertical centerline. For a load $L = 12$ kN transmitted to the frame F, compute the total shear force supported by the pin at A.



4/115 Calculate the x- and y-components of all forces acting on each member of the loaded frame.



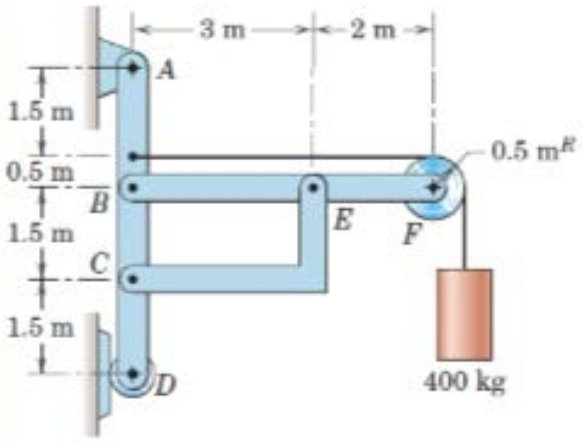
Dimensions in millimeters

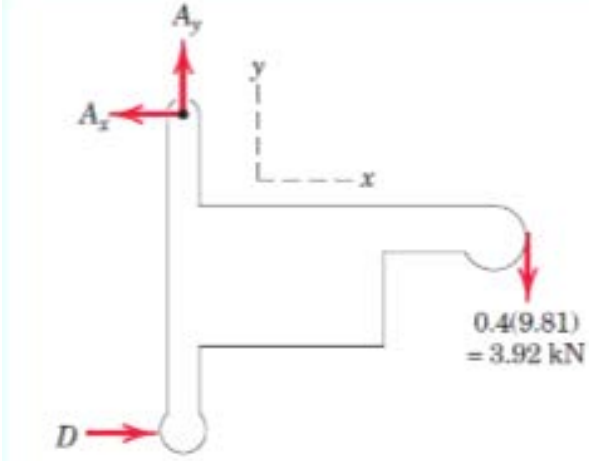
4.6 Solutions to some problems

Monday, November 02, 2015 3:46 PM

SAMPLE PROBLEM 4/6

The frame supports the 400-kg load in the manner shown. Neglect the weights of the members compared with the forces induced by the load and compute the horizontal and vertical components of all forces acting on each of the members.





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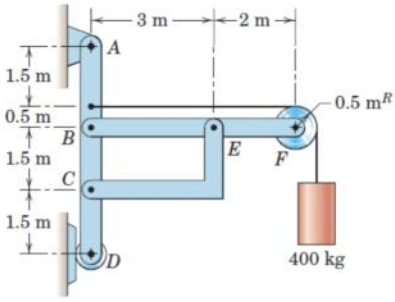


Solution. We observe first that the three supporting members which constitute the frame form a rigid assembly that can be analyzed as a single unit. We also observe that the arrangement of the external supports makes the frame statically determinate.

From the free-body diagram of the entire frame we determine the external reactions. Thus,

$$[\Sigma M_A = 0] \qquad 5.5(0.4)(9.81) - 5D = 0 \qquad D = 4.32 \text{ kN}$$

$$[\Sigma F_x = 0] \qquad A_x - 4.32 = 0 \qquad A_x = 4.32 \text{ kN}$$

$$[\Sigma F_y = 0] \qquad A_y - 3.92 = 0 \qquad A_y = 3.92 \text{ kN}$$

Next we dismember the frame and draw a separate free-body diagram of each member. The diagrams are arranged in their approximate relative positions to aid in keeping track of the common forces of interaction. The external reactions just obtained are entered onto the diagram for AD. Other known forces are the 3.92-kN forces exerted by the shaft of the pulley on the member BF, as obtained from the free-body diagram of the pulley. The cable tension of 3.92 kN is also shown acting on AD at its attachment point.

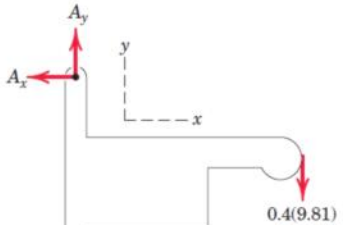
Next, the components of all unknown forces are shown on the diagrams. Here we observe that CE is a two-force member. The force components on CE have equal and opposite reactions, which are shown on BF at E and on AD at C. We may not recognize the actual sense of the components at B at first glance, so they may be arbitrarily but consistently assigned.

The solution may proceed by use of a moment equation about B or E for member BF, followed by the two force equations. Thus,

$$[\Sigma M_B = 0] \qquad 3.92(5) - \tfrac{1}{2}E_x(3) = 0 \qquad E_x = 13.08 \text{ kN} \qquad \textit{Ans.}$$

Helpful Hints

1. We see that the frame corresponds to the category illustrated in Fig. 4/14*a*.
2. Without this observation, the problem solution would be much longer, because the three equilibrium equations for member BF would contain four unknowns: B_x, B_y, E_x, and E_y. Note that the direction of the line joining the two points of force application, and not the shape of the member, determines the direction of the forces acting on a two-force member.



The solution may proceed by use of a moment equation about D or E for member BF, followed by the two force equations. Thus,

$$[\Sigma M_B = 0] \qquad 3.92(5) - \tfrac{1}{2}E_x(3) = 0 \qquad E_x = 13.08 \text{ kN} \qquad \textit{Ans.}$$

$$[\Sigma F_y = 0] \qquad B_y + 3.92 - 13.08/2 = 0 \qquad B_y = 2.62 \text{ kN} \qquad \textit{Ans.}$$

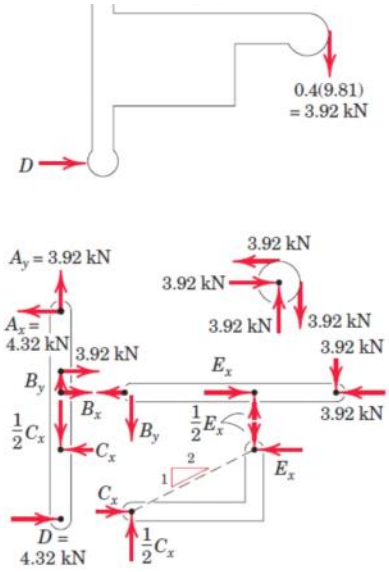
$$[\Sigma F_x = 0] \qquad B_x + 3.92 - 13.08 = 0 \qquad B_x = 9.15 \text{ kN} \qquad \textit{Ans.}$$

Positive numerical values of the unknowns mean that we assumed their directions correctly on the free-body diagrams. The value of $C_x = E_x = 13.08$ kN obtained by inspection of the free-body diagram of CE is now entered onto the diagram for AD, along with the values of B_x and B_y just determined. The equations of equilibrium may now be applied to member AD as a check, since all the forces acting on it have already been computed. The equations give

$$[\Sigma M_C = 0] \qquad 4.32(3.5) + 4.32(1.5) - 3.92(2) - 9.15(1.5) = 0$$

$$[\Sigma F_x = 0] \qquad 4.32 - 13.08 + 9.15 + 3.92 + 4.32 = 0$$

$$[\Sigma F_y = 0] \qquad -13.08/2 + 2.62 + 3.92 = 0$$



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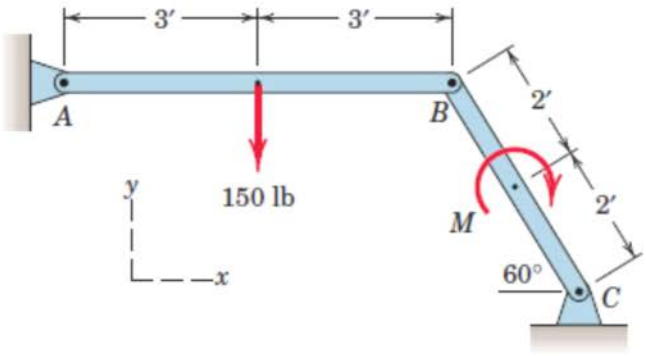

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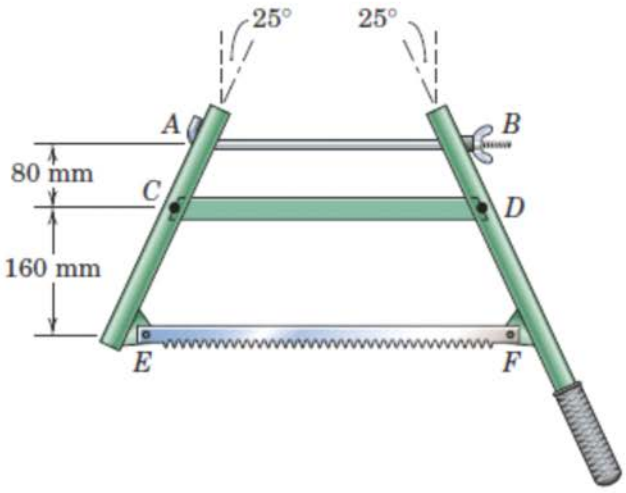

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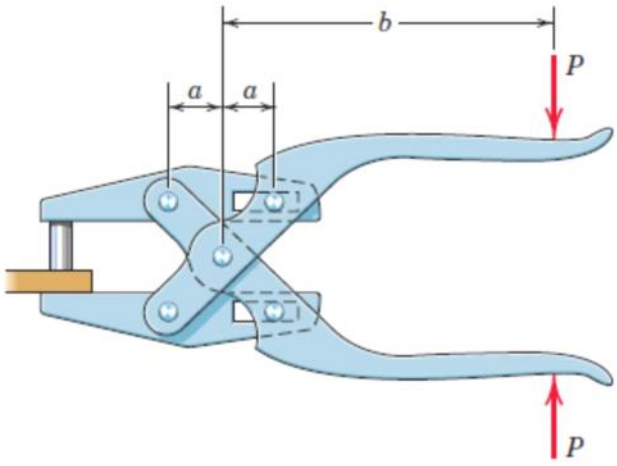

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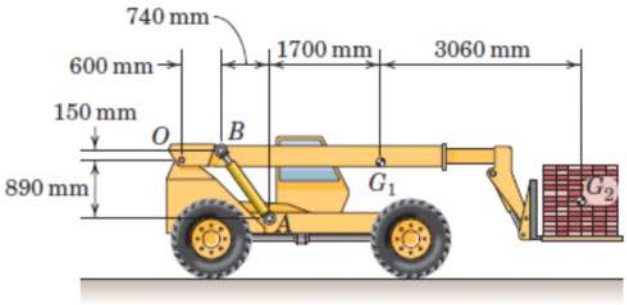

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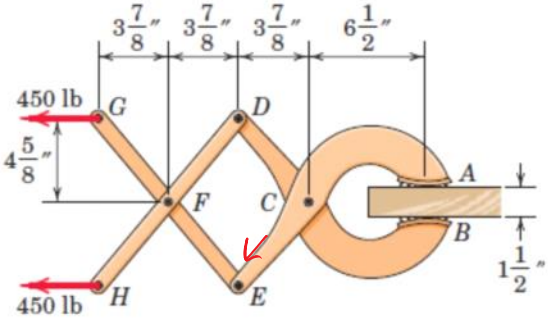
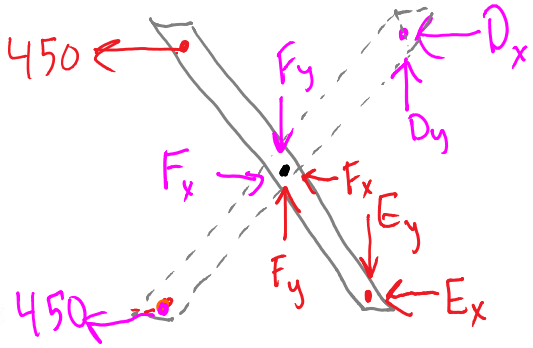


FBD1

Note $D_x = E_x$ by symmetry & $D_y = E_y$



FBD1

$\Sigma M_E = 0 = 450\left(\frac{37}{4}\right) - F_y\left(\frac{31}{2}\right)$

$\Rightarrow F_y = 450\left(\frac{37}{4}\right)\left(\frac{2}{31}\right) = 1074 \text{ lb}$ ✓

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* Note Joint F &

$\Sigma F_x = 0 = -E_x - F_x - 450$

$\Rightarrow E_x = -F_x - 450$

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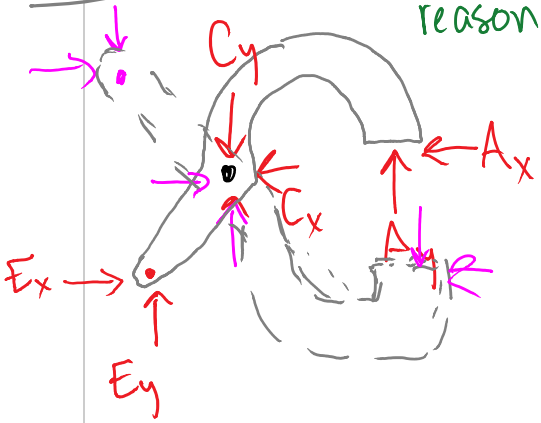
Note: for same reason $C_x = 0$



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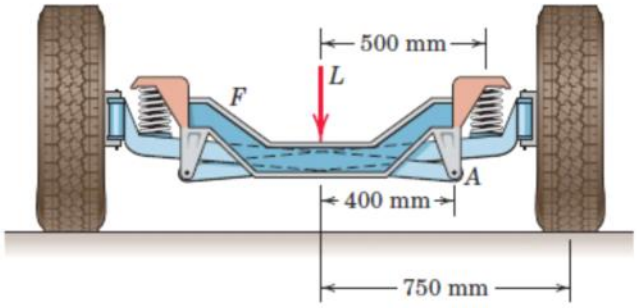
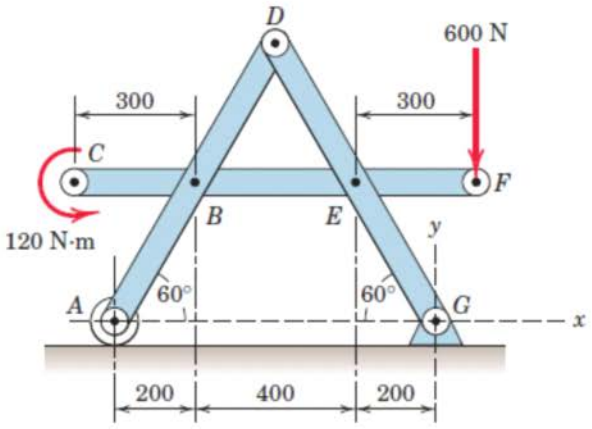


4/115 Calculate the x- and y-components of all forces acting on each member of the loaded frame.



Dimensions in millimeters