\* Must read chapter 1 (except 1.4) prior to doing homework.

Beginning with Rene Descartes in the 1630’s we have begun to not only view relationships and mathematics from one perspective, but from many; namely Algebraically, Geometrically, and Analytically.

 Until this time, no person had tried to combine these different viewpoints for one topic. After Descartes began this line of study, Calculus was developed within the next 80 years.

*Defn* An ***equation*** *from a set D to a set Y is a relation that assigns elements of D to elements of Y.*

*Equations involve equalities, but do not have to be functions.*

*Defn* A ***solution to an equation or solution point*** *of an equation is the value of the unknowns that makes the equation true. Solution points may take the form of an ordered pair* $(x,y)$*. Substituting the* $ x \& y$ *values of the solution point into the equation will lead to some truthful equality statement.*

*Defn* A ***graph***  *of an equation is the set of all solution points for an equation.*

 *I.E.* $\{\left(x,y\right):g\left(x\right)=y where g\left(x\right) is an equation\}$

*Ex: Sketch the curve below by plotting points:*

$$x=y^{2}+1$$

*Defn* An **x-intercept** of a graph is the POINT where the graph intercepts the $x-axis$.

Note: Every x-intercept for any graph always has the same y-component. It is \_\_\_\_\_\_\_\_\_\_\_\_\_\_

*Defn* A **y-intercept** of a graph is the POINT where the graph intercepts the $y-axis$.

Note: Every y-intercept for any graph always has the same x-component. It is \_\_\_\_\_\_\_\_\_\_\_\_\_\_

Ex: Find the x & y intercepts of the following:

$$y^{2}=x^{3}-4x$$

Points of intersection:

If you have two or more equations occurring simultaneously, then the point of intersection will be the solution point that is common to both graphs. To solve for a point of intersection we can treat it as a system of equations and solve by:

1:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 3:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

2:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 4:\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

## Symmetry of a graph

First it is helpful to define odd and even functions (a topic we will return to in a future section)

*Defn* A function $y=f(x)$ is called an

***even function*** if $f\left(-x\right)=f(x)$ (graph of $f$ is symmetric about the $y$-axis)

***odd function*** if $f\left(-x\right)=-f(x)$ (graph of $f$ is symmetric about the origin)

for every $x$ in the function’s domain

Knowing the symmetry of a graph before sketching it can simplify the process of graphing by reducing the number of solution points needed to sketch it.

Types of symmetry:

1. **Symmetry with respect to the y-axis**: the graph will be the reflection of itself across the y-axis. This means if you fold the graph over the y-axis, the graph will perfectly touch itself at all solution points. I.E. if $(x,y)$ is a point of the graph, then so is $(-x,y)$.

The test for a graph being symmetric w.r.t. the y-axis is it is an even function. If it is not a function at all, then replace all x, with (-x), if the equation is exactly the same as before, it has symmetry w.r.t Y-axis.

1. **Symmetry with respect to the x-axis**: the graph will be the reflection of itself across the x-axis. This means if you fold the graph over the x-axis, the graph will perfectly touch itself at all solution points. I.E. if $(x,y)$ is a point of the graph, then so is $(x,-y)$.

This type of equation will not pass the vertical line test and thus is absolutely not a function. The test for a graph being symmetric w.r.t. the y-axis is if you replace all y, with (-y), then if the equation is exactly the same as before, it has symmetry w.r.t x-axis.

1. **Symmetry with respect to the origin**: The graph will be a reflection of itself if you reflect it across the x & y axis’s. This means if you fold the graph over the y-axis then the x-axis, or visa versa, the graph will perfectly touch itself at all solution points. I.E. if $(x,y)$ is a point of the graph, then so is $(-x,-y)$.

The test for a graph being symmetric w.r.t. the origin is if it is an odd function. If it is not a function at all, then replace all x, with (-x) and y with (-y), if the equation is exactly the same as before, then it has symmetry w.r.t origin.

Ex 3 Graph each equation. What symmetries, if any, does each have? Specify the intervals over which the function is increasing and decreasing.

1. $f\left(x\right)=\frac{1}{\left|x\right|}$ - can define this “piecewisely”
2. $g\left(x\right)=\pm \sqrt{x}$

# Modeling equations:

 The “Postal stamp function”

Ex Graph the greatest integer function $f\left(x\right)=[x]$ where $\left[x\right]=$ the greatest integer less than or equal to $x$.

Note: The Least integer function $f\left(x\right)=\left⌈x\right⌉, \left⌈x\right⌉=$ is the smallest integer which is greater than or equal to x.