*Derivative of a Constant Function*

If has a constant value of , then .

Proof: Relies on properties of Limits

Ex 2 Find the derivative of at .

Ex 3 Find the derivative of at

Ex4 Find the derivative of at .

Ex 5 Find the derivative of at

Ex 6 Find the derivative of at .

Ex 7 Ex 2 Find the derivative of at

Guess the derivative of at . For Ans: \_\_\_\_\_\_\_\_\_\_

Guess the derivative of Ans: \_\_\_\_\_\_\_\_\_\_\_\_\_

Guess the derivative of Ans: \_\_\_\_\_\_\_\_\_\_\_\_\_\_

*Power Rule for Positive and Negative Integers*

If is a positive integer, then .

If is a negative integer and , then

Note: The rule works for all which we’ll see in Sec 3.7.

Proof (in book, see pg 128)

 Proof: Hint: Relies on Limit Laws

*Constant Multiple Rule*

If is a differentiable function of , and is a constant, then .

*Derivative Sum Rule*

If and are functions of , then their sum is differentiable at every point where and are differentiable. At such points, we have

 Proof: Hint: Relies on Limit Laws

Note: These two rules prove that the operator is a linear operator.

Ex 8 Find the derivative of

1. .

Ans:

Ex 3 Find if . Does (this could imply the derivative of the products is the product of the derivatives!

Ex 2 Find the derivative of . (multiply out)(no product rule yet)

*Derivative of the Natural Exponential Function*

 (Proof requires knowledge of )

Theorem:

 Pic: Note where slope is 0.

 and

Proof: Picture (informal proof)

* Sine Angle Sum Identity:
* From Sec 2.4 example 5a:

 

*g(x) =sin x*

*g'(x) =cos x*

*g''(x) = -sin x*

*g''' (x) = -cosx*



Polynomial functions and trig functions are smooth i.e. infinitely differentiable (where they’re defined).

Ex Let and . Find . Next, find . For what values of will ?

Polynomial functions and trig functions are smooth i.e. infinitely differentiable (where they’re defined).

* To think about: Give an example of a function that is not infinitely differentiable.

Note: We use the word “instantaneous” even if does not represent time.

Ex The volume of a sphere is related to its radius by the equation . How fast does the volume change with respect to its radius when the radius is ?

*Defn*

Suppose that an object is moving along a coordinate axis line, call it the -axis, where its position on that line is a function of time:

The ***displacement*** of an object over the time interval is and the ***average velocity*** of the object over that time interval is

Note: Assume is differentiable on . is increasing on if on . is decreasing on if on . (Converse is true if we change to and to .)

* To find the velocity, we take limit of average velocity as .

*Defn* ***Velocity (instantaneous velocity)*** is the derivative of the position function with respect to time. If a body’s position at time is , then the body’s velocity at time is

 .

Note: Velocity is a vector, meaning it has a magnitude AND a direction.

The ***speed*** is the magnitude or absolute value, of velocity.

The rate at which a body’s velocity changes is the body’s *acceleration*, which is a measure of how quickly the body picks up or loses speed.

*Defn*

***Acceleration*** is also a vector and is the derivative of velocity with respect to time. If a body’s position at time is , then the body’s acceleration at time is .

***Jerk*** is the derivative of acceleration wrt time (:

Ex 2 (# 4) Let be the position function of a body moving on a coordinate line, with in meters and in seconds. . Find **a)** Find the body’s displacement and average velocity for the given time interval **b)** Find the body’s speed and acceleration at the endpoints of the interval **c)** Find when, if ever, during the interval does the body change direction?

a)

c) Change direction when changes signs;

 - + - +

 0 1 2

Ex 3 The position of a particle is given by the equation where is measured in seconds and in meters.

1. Find the velocity at time .
2. What is the velocity after 2 s? After 4 s?
3. When is the particle at rest?
4. When is the particle moving forward (that is, in the positive direction)? Backward?
5. When is the particle’s velocity increasing? Decreasing?
6. Find the total distance traveled by the particle during the first 5 seconds.
7. Draw a diagram to represent the motion of the particle.

