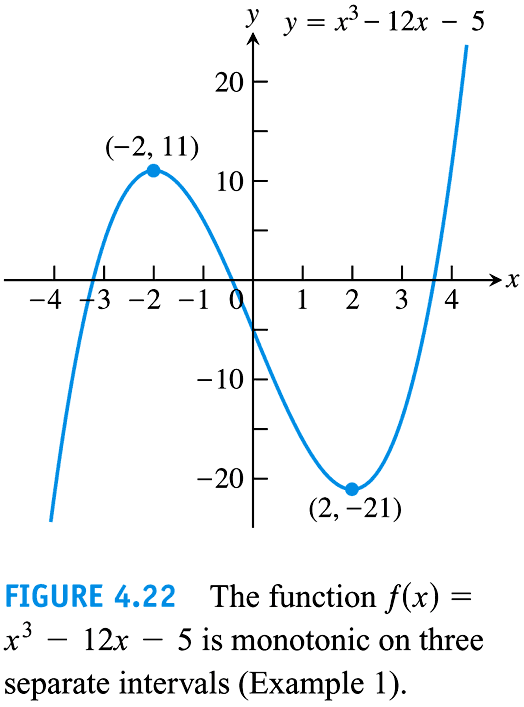
* We have discussed how to find relative extrema with derivatives, but have yet to use derivatives to determine if these points are local max or local min. In this section we introduce the 1st Derivative Test which will help us do that and learn more about the behavior of a (differentiable) function.

Consider the function below.

What can we say about the derivative of the function where the function is increasing?

Decreasing?

Try to sketch the derivative over the top of this graph.

*Defn* Let be a function defined on an interval and let and be **ANY** two points in with .

1. If then is said to be ***strictly increasing*** or just **increasing** on .
2. If then is said to be ***strictly decreasing*** just **decreasing** on .

A function that is only increasing or decreasing (but not both) on is said to be ***monotonic*** on .

Def: A function is said to be **monotonic** if it is strictly increasing or strictly decreasing on an interval I.

*(1st Derivative Test for Monotonic Functions)*

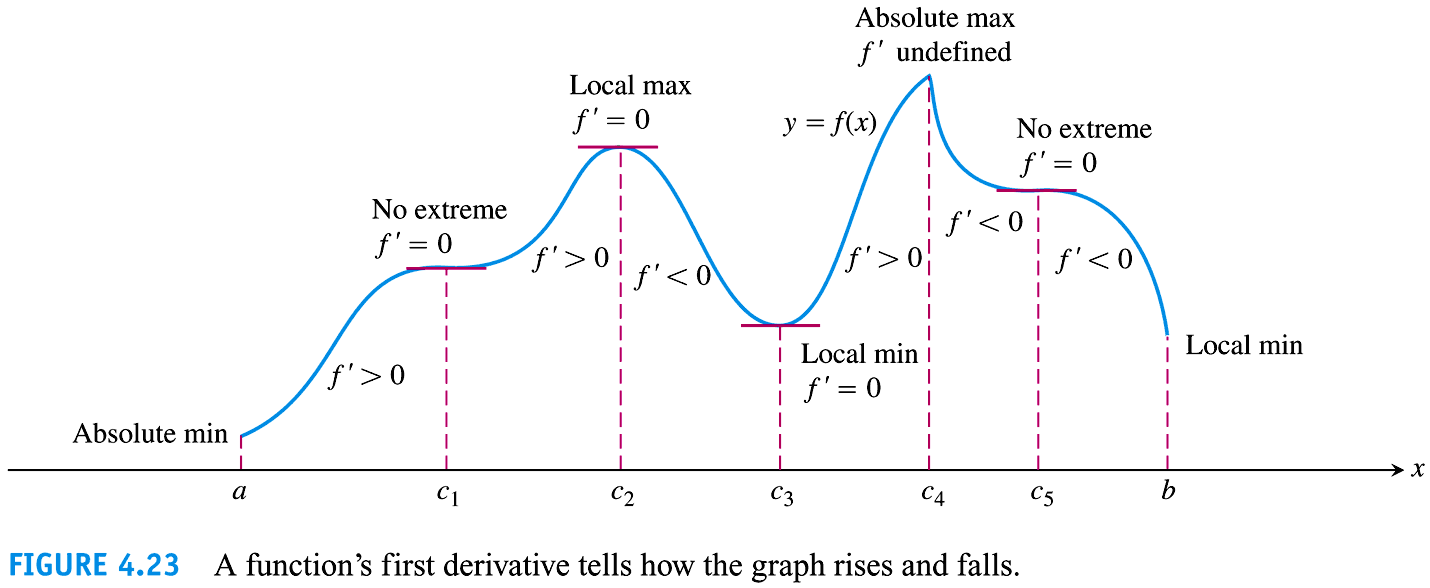
Suppose that is continuous on and differentiable on .

1. If at every point , then is **strictly increasing** or **increasing** on .
2. If at every point , then is **strictly decreasing** or **decreasing** on .

Is the converse of this statement true, that if is **increasing** on , then If at every point ?

Note: To say a function is increasing on and decreasing on leaves to be a place of some ambiguity. To say a function is increasing on implies that it is increasing at the place where , and it is decreasing where in the interval making the function both increasing and decreasing there. This will contradict our definition of increasing or decreasing which is defined only on **open** intervals because It is impossible for a function to increase at a single place, b. it is important to note that we will only be asking for and looking for OPEN intervals of increasing and decreasing.

Ex: 4.3.22 Find the critical numbers of the function (if any) and find the open intervals on which the function is increasing or decreasing.



*First Derivative Test for Local Extrema*

Suppose that is a critical point of a continuous function and that is differentiable at every point in some interval containing except possibly at itself. Moving across from left to right,

1. if changes from (-) to (+) at , then has a local/relative min at
2. if changes from (+) to (-) at , then has a local/relative max at
3. if does not change sign at , then has no local/relative extremum at

* Remind Students they can use graphing calculator to verify answers in hw. Remind of Virtual TI-83 on webpage.

Ex 1 Answer the questions about each function whose derivative is given

1. What are the critical points of ?
2. On what open intervals is increasing or decreasing?
3. At what points, if any, does assume a local max and min values?
4. 4.3.40

Ex 2 For each function, find:

1. the open intervals on which the function is increasing or decreasing
2. any local extrema, saying where they occur
3. which, if any, of the extreme values absolute
4. 4.3.24
5. 4.3.36

Ex 3 4.3.54 Consider the function on Find:

a) the open intervals on which the function is Inc. or Dec.

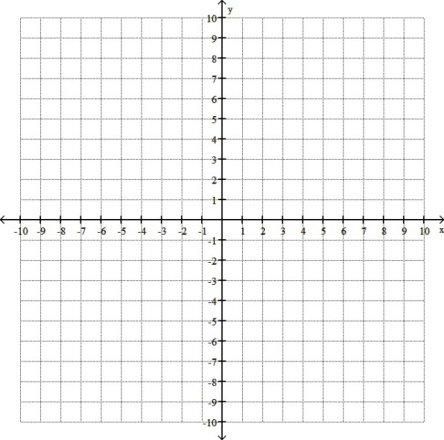
b) all relative extrema

c) (optional) Use graphing utility to confirm your results

Ex: 4.3.64 & 66 The graph of is given. Sketch the graph of over the top of each graph.



Ex: 4.3.70 Use the graph of (shown) to sketch the graph of .



*How to Find Local and Absolute Extrema When an Interval is not Given*

1. Find all critical points of . Note the domain of (whether or not it’s all reals) and any vertical asymptotes. (If the domain is not all reals, then we want to evaluate the endpoints where the function is defined to check for absolute extrema.)
2. Find where is increasing or decreasing.
3. Find where has a local min/max by looking at changes in sign of .
4. Use the above to get a general idea of the graph to determine if any of the local extrema are absolute extrema.

**Summary:**

We can tell a lot about a function based upon what its derivative tells us.

* In 4.1 we learned it identifies CN’s (potential local extrema) and local extrema only occur at CN’s
* In 4.2 we learned Rolles and MVT, which can help us identify intervals containing local extrema
* In 4.3 we learned that it can identify intervals of “increasingness/decreasingness” as well as identify if a particular CN is a relative max, min, or neither.

**Fun Conceptual Questions:** Consider these problems on your own or with friends, we will not discuss them in class.

* Given the following information about a function , sketch a graph of the function. Label the coordinates for all key points that have been indicated.

1. is defined for all real numbers
2. for
3. is undefined at
4. for
5. is undefined at
6. is on and on
7. is on and on

* For what values of and does the function satisfy the hypotheses of the Mean Value Theorem on the interval ?

Def: A function is said to be One-to-one if for any where then . This means that every value a function has corresponds to (came from) one and only one value.

One to one functions are part of the requirements to ensure that a functions inverse will itself be a function.

* Show that strictly increasing functions and strictly decreasing functions are one-to-one. To Show this, show that for any in then this implies that .
* (Thomas # 58) Prove that if Hint: let if you can show that this is an increasing function on , a decreasing function on , that , and that there is a local min at x=0 then you would know that on .