Now we make our last big move. This is the moment in the 6th sense when we get to realize that we’ve been seeing dead people, that Verbal Kent is really Kiser Sorse, that the girl in the crying game is really that guy in the crying game. Today we pull the wool off from over our eyes.



*Theorem* ***The Fundamental Theorem of Calculus, Part I (FTC I) (The Evaluation Theorem)***

If is continuous on and is *any* antiderivative of on , then .

Recall A function is an ***antiderivative*** of on an interval if for all in . Also, the ***indefinite integral*** of with respect to , , is the set of all antiderivatives of . The ***definite integral*** of over is .

Note: Since the FTC I works for *any* antiderivative, we choose .

*Basic Rule of Integration*

(Power Rule)

Ex 2 Evaluate the integral. (For some, do not evaluate completely.)

Ex 4­ Differentiate .

Conclusion:

*Defn* If is integrable on , then its ***average value on*** , also called its ***mean value***, is

Ex: Find the average value of the function

*Theorem* ***The Mean Value Theorem for Definite Integrals***

If is continuous on , then at some point in , .

Ex 1 Find the value(s) of c guaranteed by the Mean Value Theorem (MVT) for the function over the indicated interval. To do this use the Mean Value Theorem for Integrals (MVTI).

1. on .
2. on .

*Theorem* ***The Fundamental Theorem of Calculus, Part II (FTC II)***

If is continous on , then we can define a new function . This new function is continuous on and differentiable on and its derivative is . In symbols,

Note: If ,, then measures area under from to .

The fact that is continuous and differentiable means there’s a small and “smooth” change in area when there is a small change in .

If were not continuous, we can see that there can be a drastic change in area.

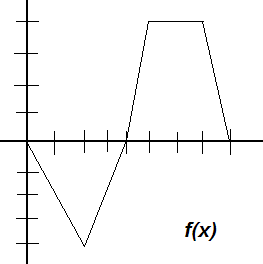
Ex: 5.4.80 Find as a function of x and evaluate it at

Ex: Same directions as above: try on your own:

Ans:

Ex (5.4.103)

If then find

Ex 5 (5.4.86)

Let given in the figure

a) Estimate .

b) Find the largest open interval on which is increasing.

Find the largest open interval on which is decreasing.

c) Identify any extrema of

d) Sketch a rough graph of g

On your own:

The graph of the function shown above consists of a semicircle and three line segments. Let be the function given by .

1. Find and .
2. Find all values of in the open interval at which attains a relative maximum. Justify your answer.
3. Find the absolute minimum value of on the closed interval . Justify your answer.
4. Find all values of in the open interval at which the graph of has a point of inflection.