# An Introduction to Differential Equations (ordinary)

Def: An equation with a differential (derivative) of some function in it is called **a Differential Equation.**

“differential equations” are often affectionately referred to as “diff eqs”.

An example of this may be , where and represents the velocity of some position function, . If the velocity of an object is 2 units/time then

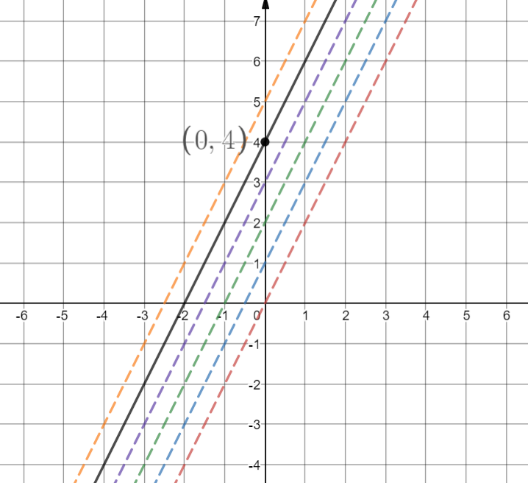
is an example of a diff eq.

To solve this differential equation means to find the function, , which satisfies this differential equation.

So for what function, , does ? The answer to this is called the **solution** to the differential equation. There are different types classes of solutions which will be discussed below.

## What is a General Solution to a differential equation?

By inspection, we could imagine that are all solutions to our diff eq. In truth there are infinite solutions to the diff eq

And all solutions have the general form of

, where c is some constant.

This solution is called the **General Solution** to the differential equation.

The figure to the right shows the general solutions to the differential equation.

Geometrically the general solution looks like a family/group/class of lines with slope of 2 but each line has a potentially different y-intercept. So we imagine a bunch of “twinsies” curves that are all vertical shifts of the others.

## What is a Particular Solution to a differential Equation?

If you are given some initial conditions to your diff eq, like say when the position of the object, is 4 away from a point of reference, i.e. .

With this information we can find which of our “twinsies” lines contains the point , and deduce what the specific value of c must be.

## Analytically we can find the particular solution as follows:

Using the general solution we will attempt to find the particular line that contains the initial condition:

General Solution:

Given the initial condition, .

Using this to plug into the general solution will give us a ***particular value for c.***

Thus

is called the Particular Solution to the differential equation because it is the particular line/equation that satisfies the given diff eq. when the initial condition is .

The types of Differential Equations are varied, there are ordinary and partial diff eqs, with many sub categorizations. We are for the moment only focused on ordinary diff eqs.

The techniques for solving these Differential Equations are also wide and will require lengthy study, but for now we will only focus on the most simple, Separation of Variables, which will look quite familiar to us as a form of integration.

# Example: (If the shoe fits)

(~6.1.2) Verify that is a solution to the differential equation

# Solving Differential Equations using separation of variables.

Separation of variables will be further discussed in the next section, however, the overall strategy is to algebraically separate a diff eq. so that each of the two different variables occur on one side of the equation only.

Example: Separate the diff eqs.

1. b)

## Solution:

Note: don’t for get the chain rule/usub in step 2

# Example: Solve the differential equations.

1. b) c)

## Solution:

L4e 6.2.10

Note: don’t for get the chain rule/usub in step 2

# Exponential Growth and Decay Model

Theorem 6.1: Exponential Growth and Decay Model

If is a differentiable function of t such that and , for some constant , then

is the exponential growth/decay modeling equation where C is the **initial value of**  and is the **proportionality constant**.

A value of will indicate exponential growth and a value of indicates exponential decay.

## Proof:

Proof of Theorem 6.1: Exponential Growth and Decay Model

Assume that is a differentiable function of t such that

And for some constant

Integrating both sides wrt t gives us

Solving for y

Thus

Note: when t=0 & so is the initial value of y.

## Example: Set up and solve the diff eq given by, the rate of change of with respect to is proportional to

## Solution:

L4e: 6.2.12



# Exponential Growth and Decay Modeling Applications

## Example: The rate of change of is proportional to , when & when . What is the value of when ?

## Solution:

L4e: 6.2.24



## Example: Using a new software package generally takes time to become proficient. If we consider the number of “jobs” that a professional could complete in an 8 hour day is 30, then the learning curve for producing N jobs in the same 8 hour day that a layperson can produce after t days (8 hour blocks) of working with the program is given by If after 30 days (8 hour blocks) you find you are able to produce 20 jobs in a day,

## Find the learning curve that describes the persons abilities.

## How many days will be needed to be able to produce 25 jobs in a day?

## Solution:

L4e: 6.2.64



## Example: Find the principal that must be invested at rate , compounded monthly, so that $500,000 will be available for retirement in years.

## Solution:

L4e: 6.2.50



# Newtons Law of Cooling

Newtons Law of Cooling states that the rate of change in the temperature, of an object is proportion to the difference between the objects temperature, and the temperature of the surrounding medium, .

, note this is a diff eq.

Confirm that the solution to this is:

Let be the rate of change of the temerature of an object with a given temperature, , and surrounding temperature, , at a time, then

Thus

## Example: If a cup of hot coffee with initial temp, , is poured in a room with ambient temp, If after 5 min the temp of the coffee is noted to be 100, how long will it take for the coffee to be 80

## Solution:

Given

When