# Continuing our look at solids of revolution

Consider the solid of revolution formed by rotating the region bounded by

Rotated about the y-axis.

This shape will look like a solid cylinder with a parabola shaped cup taken out of the inside, but this cup is only half the depth of the cylinder. Consider the images below.

Create the differential element in each picture that depicts the cross sections of the disk method from 7.2, then set up the integral to find the volume.



Region Front view of solid 3-D view of solid

Note that for this shape, the differential element must be a dy, and doing so requires two integrals, which is less than ideal.

Now, create the differential element dx in the region below, then draw a picture of what this element will look like when it alone is revolved around the y-axis. What does this shape look like, describe it?!



# The Shell Method (Hollow Pipe)



where distance to the shell from the axis of rotation, in our case is a function of x,

 height of the sides of the shell, which is the height of the curve and is also a function of x,

 thickness of the shell, in our picture, this will be

In more generalized terms

So then,

Meaning the volume is the limit of the sum of all the shells volumes, and each shell volume is

Where

# The Shell Method (hollow pipe)

To find the volume of a solid of revolution with the shell Method, use one of the following formulas, which formula you use will depend on the shape of the 2-D rotated region and the vertical or horizontal orientation of the axis of rotation.

For horizontal Axis of revolution For vertical axis of revolution

# Example: Use the shell method to find the volume of the solid of revolution formed by rotating the region bounded by 1 about the y-axis.

## Solution:

Lets draw the differential element dx and rotate it about the y-axis, since this shape creates a shell, it was the correct choice, if we used a dy element and rotated it about the y-axis we would get a disk, then the disk method would need to be used.



# Example: A torus is formed by revolving the region bounded by the circle centered at the origin with radius 1 given by about the line . Find the volume of the torus.

## Solution:

L4e: 7.3.43 cont.



L4e: 7.3.43

 x=2

# Example: Determine if shell or disk method is preferred when finding the volume of the region bounded by when rotated about the y-axis. Then find the volume.

## Solution:

L4e: 7.3.7 cont.

 

L4e: 7.3.7

If a dy element is used then the equation will need to be written explicitly in terms of y, which will be difficult to do, since using dx is ok the shell method is preferred.

So x is on [0,2]

# Example: Set up but do not integrate the problem to find the volume of the solid generated by rotating the region bounded by about the line a) b) c) the line (use any method)

## Solution:

L4e: 7.3.28

  