Math 401 **Preparation for Calculus II**

This is a set of review problems to help get you 'warmed – up' for Calculus II. I recommend putting these problems into a notebook that you will use for lecture notes and practice problems for the semester. I will not collect these problems. An additional handout will be made available online that contains selected solutions.

Here are some key directions to follow that will hold true on all graded work this semester.

Use Pencil.
Use Proper Notation. Do not blend 'scratch' work with 'real' work.
All final solutions should be written without negative numerical exponents or complex fractions.
Show your work whenever multiple steps are involved.
Evaluate functions for exact values whenever possible.

1. Solve for 'x' in each of the following functions.

a) $y = \sqrt[3]{x}$ b) $y = x^2 - 3$ c) $y = 4x$ d) $y = e^x$

e) $y = \cos x$ if $0 \le x \le \pi$ f) $y = \dfrac{e^x - e^{-x}}{2}$ A fun little challenge. ☺

2. Functions related questions.

a) If $f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$, then $f(2x) =$

b) If $f(x) = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \dfrac{x^4}{4!} + \ldots$, then $f(-x) =$

3. Evaluate each limit. If a Limit is undefined, indicate which case it is: $\{\infty, -\infty$, or DNE (Does not exist $\}$. Do not use L'Hopital's Rule if you have been taught this method.

a) $\lim\limits_{x \to 2} \dfrac{x^2 - 4}{x^3 - 8} =$ b) $\lim\limits_{h \to 0} \dfrac{(5+h)^2 - 5^2}{h} =$

c) $\lim\limits_{x \to \infty} \dfrac{x + 5}{x^2 + 2x - 15} =$ d) $\lim\limits_{x \to \infty} \dfrac{7x^3 + 2}{4x^2 - 5x^3 - 1} =$

e) $\lim\limits_{x \to -\infty} e^{-x} =$ f) $\lim\limits_{x \to \infty} e^{-x} =$

g) $\lim\limits_{x \to \infty} (\sin x) =$ h) $\lim\limits_{x \to \frac{\pi}{2}^+} (\tan x) =$

i) $\lim\limits_{x \to \infty} \dfrac{3x^2 + x \sin x}{x^2} =$ j) $\lim\limits_{x \to 0} \dfrac{\sin x}{x}$

4. Use the limit definition of the derivative to evaluate: $\dfrac{d\left(5x^2-3x+2\right)}{dx}$

You must show your work and use the proper notation throughout.

"Here is how it would begin." $\dfrac{d\left(f\left(x\right)\right)}{dx}=\lim\limits_{h\to 0}\left(\dfrac{f\left(x+h\right)-f\left(x\right)}{h}\right)=$

5. Find the derivative of each function using known formulas.

a) $y=x^4$ $\dfrac{dy}{dx}=$

b) $y=\sqrt[3]{u}$ $\dfrac{dy}{du}=$

c) $y=\dfrac{1}{x}$ $y'=$

d) $\dfrac{d\left(\sec\theta\right)}{d\theta}=$

e) $y=e^x$ $\dfrac{dy}{dx}=$

f) $y=\arcsin t$ $\frac{dy}{dt}=$

6. Answer each of the following.

a) Given the function $f\left(x\right)=x^2-\ln x+1$, write an equation of the line tangent to the function at $x=3$.

b) Given the function $g\left(\theta\right)=\tan\theta$, write an equation of the <u>normal line</u> to the function at $\theta=\dfrac{\pi}{6}$.

7. Find the derivative (with respect to x) of each function.

a) $y=\dfrac{x^3}{\sqrt{x}}$ (rewrite this function and use the <u>power rule</u>)

b) $y=\dfrac{x^3}{\sqrt{x}}$ (using the quotient rule)

c) Verify that the results of part (a) and part (b) are the same.

d) $y=x\cot x$

e) $y=\frac{\ln x}{x}$

8. Find the derivative of each function (with respect to x).

a) $y=(4x+3)^9$

b) $f(x)=e^{3x^2+7}$

c) $y=\dfrac{1}{\sqrt{3x^2+7}}$

d) $y=\ln(\sec x+\tan x)$

e) $f(x)=\dfrac{e^x-e^{-x}}{2}$

f) $f(x)=\dfrac{2}{e^{5x}+e^{-5x}}$

9. Solve for $\dfrac{dy}{dx}$ by using implicit differentiation.

a) $x^2+5x^3y-4y^5=7$

b) $x=\sin y$

10. Sketch the region indicated by each definite integral. Use geometry to evaluate. (No Calculus involved.)

a) $\int_1^3 (x+4)\,dx$

b) $\int_0^3 |x-2|\,dx$

c) $\int_{-2}^2 \sqrt{4-x^2}\,dx$

11. Evaluate each indefinite integral.

a) $\int x\,dx$

b) $\int \sin x\,dx$

c) $\int x^4\,dx$

d) $\int e^x\,dx$

e) $\int dx$

f) $\int \dfrac{1}{1+x^2}\,dx$

g) $\int \frac{1}{x}\,dx$

h) $\int \tan x\,dx$

i) $\int \sqrt[5]{x}\,dx$

j) $\int \sec x\,dx$

k) $\int \frac{1}{x^4}\,dx$

l) $\int \dfrac{1}{x\sqrt{x^2-9}}\,dx$

12. Evaluate each definite integral.

a) $\int_{-1}^2 x^2\,dx$

b) $\int_0^{3\pi/4} (4+\sin x)\,dx$

13. Evaluate each indefinite integral. (Show the u-substitution used for each problem.)

a) $\int (6x+5)^{10}\,dx$

b) $\int \frac{\cos x}{\sqrt[3]{4+\sin x}}dx$

c) $\int \frac{t}{3t^2+5}dt$

d) $\int \tan\theta \sec^2\theta d\theta$

e) $\int \frac{\ln y}{y}dy$

f) $\int \frac{-1}{e^x}dx$

14. Expand each sum. (The first 4 terms must be shown. The last term must be shown if it exists.) *Do Not Simplify*.

Example: $\sum_{k=1}^{10} k^2+3 = \left(1^2+3\right)+\left(2^2+3\right)+\left(3^2+3\right)+\left(4^2+3\right)+\cdots+\left(10^2+3\right)$

a) $\sum_{k=1}^{74} k^3+1$

b) $\sum_{k=1}^{n} k^2+3$

c) $\sum_{k=1}^{\infty} \frac{k}{k+4}$

d) $\sum_{k=0}^{\infty} \left(\frac{2}{3}\right)^k$

e) $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$