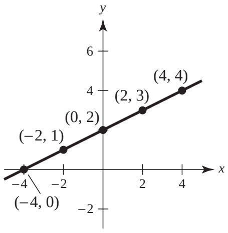
Answers to Odd-Numbered Exercises

Chapter 1

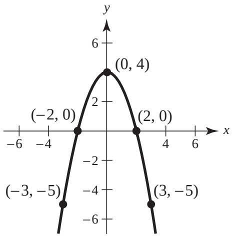
Section 1.1 *(page 8)*

1. b **2.** d **3.** a **4.** c

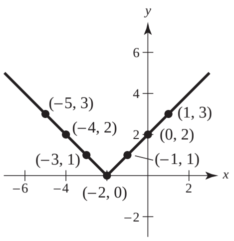
5.



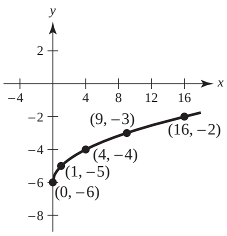
7.



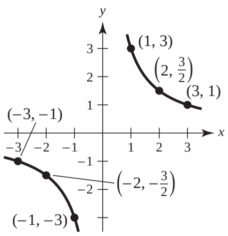
9.



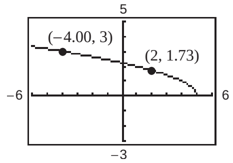
11.



13.



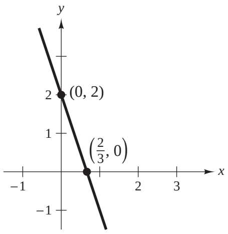
15.



(a) $y \approx 1.73$ (b) $x = -4$

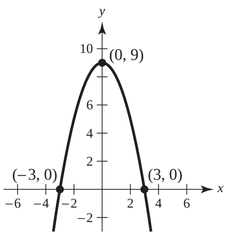
17. $(0, -5), \left(\frac{5}{2}, 0\right)$ **19.** $(0, -2), (-2, 0), (1, 0)$
21. $(0, 0), (4, 0), (-4, 0)$ **23.** $(0, 2), (4, 0)$ **25.** $(0, 0)$
27. Symmetric with respect to the y-axis
29. Symmetric with respect to the x-axis
31. Symmetric with respect to the origin **33.** No symmetry
35. Symmetric with respect to the origin
37. Symmetric with respect to the y-axis

39.



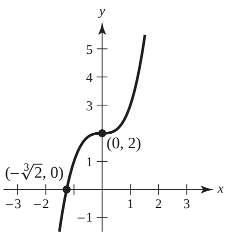
Symmetry: none

41.



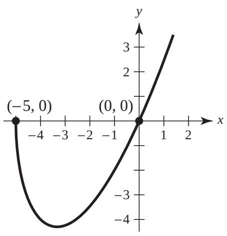
Symmetry: y-axis

43.



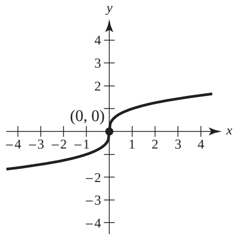
Symmetry: none

45.



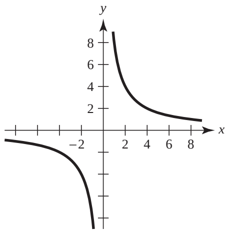
Symmetry: none

47.



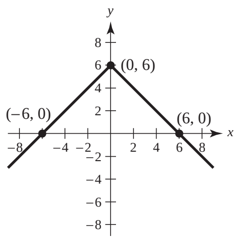
Symmetry: origin

49.



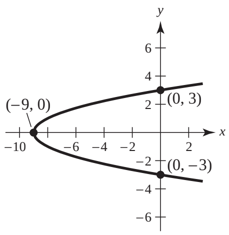
Symmetry: origin

51.



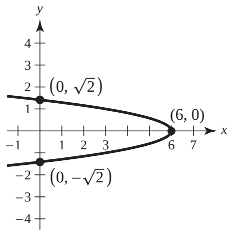
Symmetry: y-axis

53.



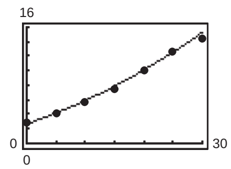
Symmetry: x-axis

55.



Symmetry: x-axis

57. $(3, 5)$
59. $(-1, 5), (2, 2)$
61. $(-1, -2), (2, 1)$
63. $(-1, -5), (0, -1), (2, 1)$
65. $(-2, 2), \left(-3, \sqrt{3}\right)$
67. (a) $y = 0.005t^2 + 0.27t + 2.7$
(b)

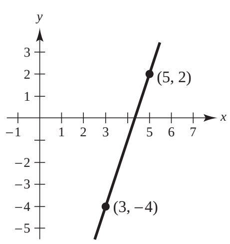


The model is a good fit for the data.
(c) $21.5 trillion
69. 4480 units
71. (a) $k = 4$ (b) $k = -\frac{1}{8}$
(c) All real numbers k (d) $k = 1$
73. Answers will vary. Sample answer: $y = (x + 4)(x - 3)(x - 8)$
75. (a) and (b) Proofs
77. False. $(4, -5)$ is not a point on the graph of $x = y^2 - 29$.
79. True

Section 1.2 *(page 16)*

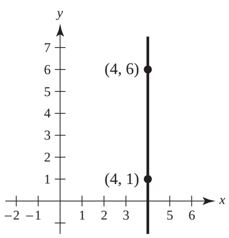
1. $m = 2$ **3.** $m = -1$

5.



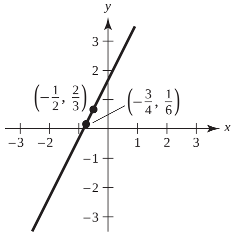
$m = 3$

7.



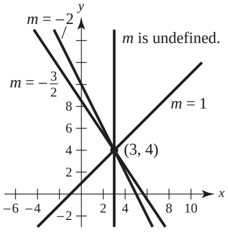
m is undefined.

9.



$m = 2$

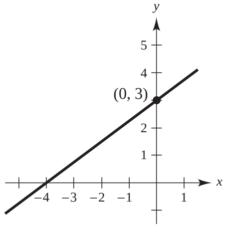
11.



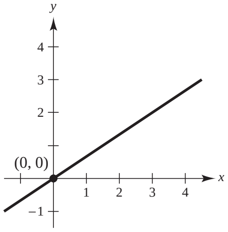
13. Answers will vary. Sample answers: (0, 2), (1, 2), (5, 2)

15. Answers will vary. Sample answers: (0, 10), (2, 4), (3, 1)

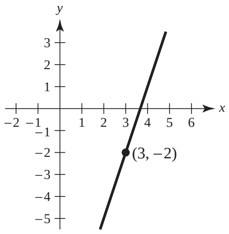
17. $3x - 4y + 12 = 0$



19. $2x - 3y = 0$



21. $3x - y - 11 = 0$

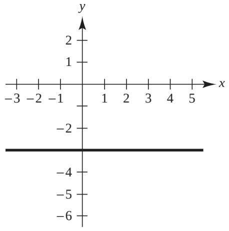


23. (a) $\frac{1}{3}$ (b) $10\sqrt{10}$ ft

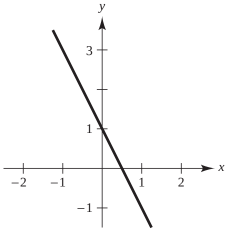
25. $m = 4, (0, -3)$ **27.** $m = -\frac{1}{5}, (0, 4)$

29. m is undefined, no y-intercept

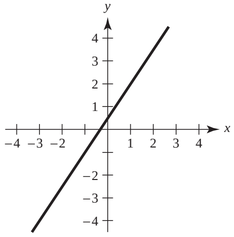
31.



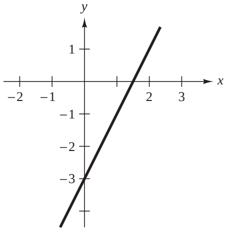
33.



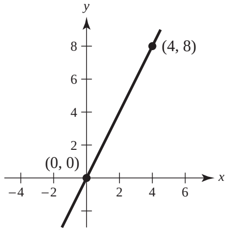
35.



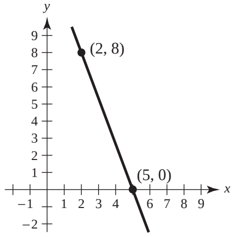
37.



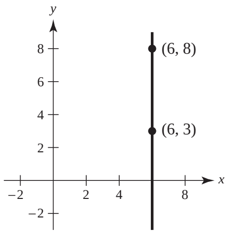
39. $2x - y = 0$



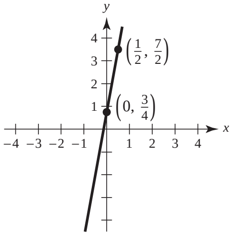
41. $8x + 3y - 40 = 0$



43. $x - 6 = 0$



45. $22x - 4y + 3 = 0$



47. $x - 3 = 0$ **49.** $3x + 2y - 6 = 0$ **51.** $x + y - 3 = 0$

53. $x + 2y - 5 = 0$ **55.** (a) $x + 7 = 0$ (b) $y + 2 = 0$

57. (a) $x - y + 3 = 0$ (b) $x + y - 7 = 0$

59. (a) $2x - y - 3 = 0$ (b) $x + 2y - 4 = 0$

61. (a) $40x - 24y - 9 = 0$ (b) $24x + 40y - 53 = 0$

63. $V = 250t + 1350$ **65.** $V = -1600t + 20{,}400$

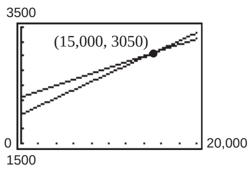
67. Not collinear, because $m_1 \neq m_2$

69. $\left(0, \dfrac{-a^2 + b^2 + c^2}{2c}\right)$ **71.** $\left(b, \dfrac{a^2 - b^2}{c}\right)$

73. (a) The line is parallel to the x-axis when $a = 0$ and $b \neq 0$.
(b) The line is parallel to the y-axis when $b = 0$ and $a \neq 0$.
(c) Answers will vary. Sample answer: $a = -5$ and $b = 8$
(d) Answers will vary. Sample answer: $a = 5$ and $b = 2$
(e) $a = \frac{5}{2}$ and $b = 3$

75. $5F - 9C - 160 = 0$; 72°F ≈ 22.2°C

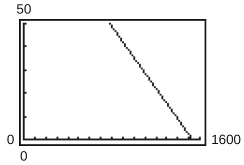
77. (a) Current job: $W = 2000 + 0.07s$
Job offer: $W = 2300 + 0.05s$
(b)



You will make more money at the job offer until you sell $15,000. When your sales exceed $15,000, your current job will pay you more.

(c) No, because you will make more money at your current job.

79. (a) $x = (1530 - p)/15$
(b)



45 units

(c) 49 units

81. $12y + 5x - 169 = 0$ **83.** $(5\sqrt{2})/2$ **85.** $2\sqrt{2}$

87–91. Proofs **93.** True **95.** True

Section 1.3 *(page 27)*

1. (a) -4 (b) -25 (c) $7b - 4$ (d) $7x - 11$

3. (a) 5 (b) 0 (c) 1 (d) $4 + 2t - t^2$

5. (a) 1 (b) 0 (c) $-\frac{1}{2}$ (d) 1

7. $3x^2 + 3x\,\Delta x + (\Delta x)^2, \Delta x \neq 0$

9. $\left(\sqrt{x - 1} - x + 1\right)/[(x - 2)(x - 1)]$

11. Domain: $(-\infty, \infty)$; Range: $[0, \infty)$

13. Domain: $(-\infty, \infty)$; Range: $(-\infty, \infty)$

15. Domain: $[0, \infty)$; Range: $[0, \infty)$

17. Domain: $[-4, 4]$; Range: $[0, 4]$

19. Domain: All real numbers t such that $t \neq 4n + 2$, where n is an integer; Range: $(-\infty, -1] \cup [1, \infty)$

21. Domain: $(-\infty, 0) \cup (0, \infty)$; Range: $(-\infty, 0) \cup (0, \infty)$

23. Domain: $[0, 1]$

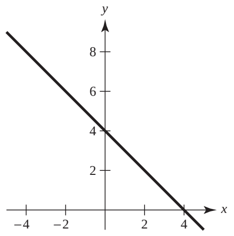
25. Domain: All real numbers x such that $x \neq 2n\pi$, where n is an integer

27. Domain: $(-\infty, -3) \cup (-3, \infty)$

29. (a) -1 (b) 2 (c) 6 (d) $2t^2 + 4$
Domain: $(-\infty, \infty)$; Range: $(-\infty, 1) \cup [2, \infty)$

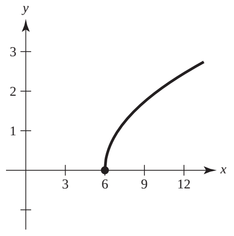
31. (a) 4 (b) 0 (c) -2 (d) $-b^2$
Domain: $(-\infty, \infty)$; Range: $(-\infty, 0] \cup [1, \infty)$

33.



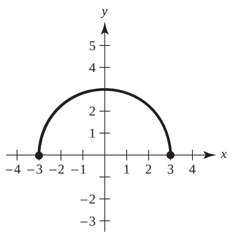
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

35.



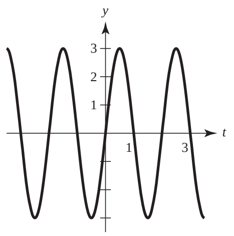
Domain: $[6, \infty)$
Range: $[0, \infty)$

37.



Domain: $[-3, 3]$
Range: $[0, 3]$

39.



Domain: $(-\infty, \infty)$
Range: $[-3, 3]$

41. The student travels $\frac{1}{2}$ mile/minute during the first 4 minutes, is stationary for the next 2 minutes, and travels 1 mile/minute during the final 4 minutes.

43. y is not a function of x. **45.** y is a function of x.

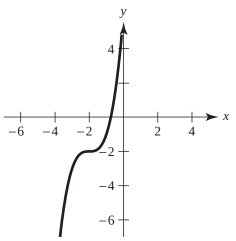
47. y is not a function of x. **49.** y is not a function of x.

51. Horizontal shift to the right two units; $y = \sqrt{x - 2}$

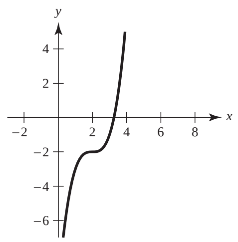
53. Horizontal shift to the right two units and vertical shift down one unit; $y = (x - 2)^2 - 1$

55. *d* **56.** *b* **57.** *c* **58.** *a* **59.** *e* **60.** *g*

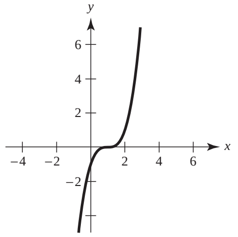
61. (a)



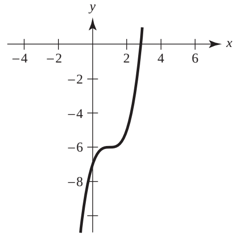
(b)



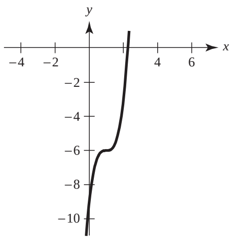
(c)



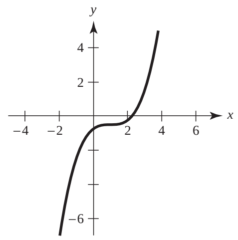
(d)



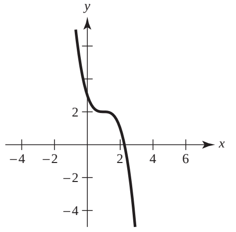
(e)



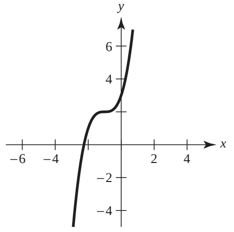
(f)



(g)



(h)



63. (a) $3x$ (b) $3x - 8$ (c) $12x - 16$ (d) $\frac{3}{4}x - 1$

65. (a) 0 (b) 0 (c) -1 (d) $\sqrt{15}$
(e) $\sqrt{x^2 - 1}$ (f) $x - 1$ $(x \geq 0)$

67. $(f \circ g)(x) = x$; Domain: $[0, \infty)$
$(g \circ f)(x) = |x|$; Domain: $(-\infty, \infty)$
No, their domains are different.

69. $(f \circ g)(x) = 3/(x^2 - 1)$;
Domain: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$
$(g \circ f)(x) = (9/x^2) - 1$; Domain: $(-\infty, 0) \cup (0, \infty)$
No

71. (a) 4 (b) -2
(c) Undefined. The graph of g does not exist at $x = -5$.
(d) 3 (e) 2
(f) Undefined. The graph of f does not exist at $x = -4$.

73. Answers will vary.
Sample answer: $f(x) = \sqrt{x}$; $g(x) = x - 2$; $h(x) = 2x$

75. (a) $\left(\frac{3}{2}, 4\right)$ (b) $\left(\frac{3}{2}, -4\right)$

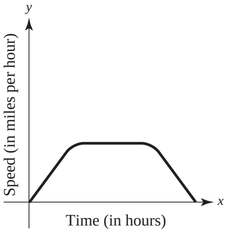
77. f is even. g is neither even nor odd. h is odd.

79. Even; zeros: $x = -2, 0, 2$

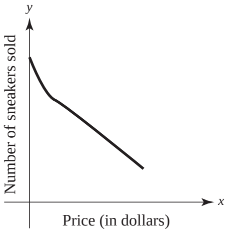
81. Odd; zeros: $x = 0, \dfrac{\pi}{2} + n\pi$, where n is an integer

83. $f(x) = -5x - 6, -2 \leq x \leq 0$ **85.** $y = -\sqrt{-x}$

87. Answers will vary.
Sample answer:



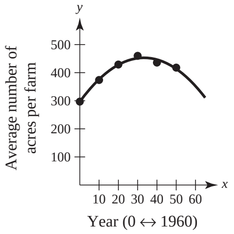
89. Answers will vary.
Sample answer:



91. $c = 25$

93. (a) $T(4) = 16°\text{C}$, $T(15) \approx 23°\text{C}$
(b) The changes in temperature occur 1 hour later.
(c) The temperatures are 1° lower.

95. (a)



(b) $A(25) \approx$ 443 acres/farm

97. $f(x) = |x| + |x - 2| = \begin{cases} 2x - 2, & x \geq 2 \\ 2, & 0 < x < 2 \\ -2x + 2, & x \leq 0 \end{cases}$

99–101. Proofs **103.** $L = \sqrt{x^2 + \left(\dfrac{2x}{x - 3}\right)^2}$

105. False. For example, if $f(x) = x^2$, then $f(-1) = f(1)$.

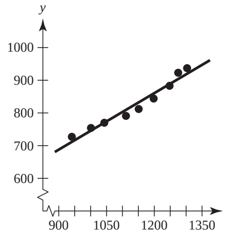
107. True

109. False. $f(x) = 0$ is symmetric with respect to the x-axis.

111. Putnam Problem A1, 1988

Section 1.4 *(page 34)*

1. (a) and (b)

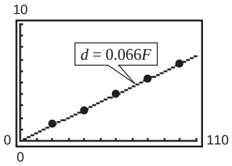


Approximately linear

(c) \$790

3. (a) $d = 0.066F$

(b)

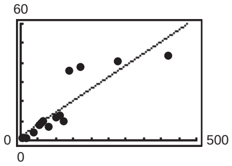


The model fits well.

(c) 3.63 cm

5. (a) $y = 0.122x + 2.07$, $r \approx 0.87$

(b)

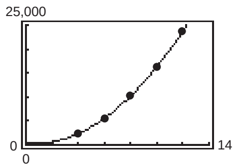


(c) Greater per capita energy consumption by a country tends to correspond to greater per capita gross national product of the country. The three countries that differ most from the linear model are Canada, Italy, and Japan.

(d) $y = 0.142x - 1.66$, $r \approx 0.97$

7. (a) $S = 180.89x^2 - 205.79x + 272$

(b)



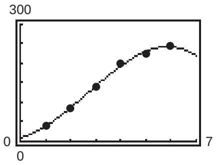
(c) When $x = 2$, $S \approx 583.98$ pounds.

(d) About 4 times greater

(e) About 4.37 times greater; No; Answers will vary.

9. (a) $y = -1.806x^3 + 14.58x^2 + 16.4x + 10$

(b)



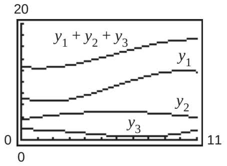
(c) 214 hp

11. (a) $y_1 = -0.0172t^3 + 0.305t^2 - 0.87t + 7.3$

$y_2 = -0.038t^2 + 0.45t + 3.5$

$y_3 = 0.0063t^3 - 0.072t^2 + 0.02t + 1.8$

(b)



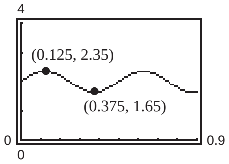
About 15.31 cents/mi

13. (a) Yes. At time t, there is one and only one displacement y.

(b) Amplitude: 0.35; Period: 0.5

(c) $y = 0.35 \sin(4\pi t) + 2$

(d)



The model appears to fit the data well.

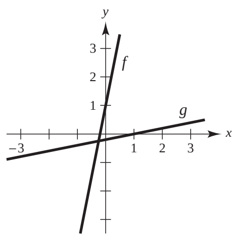
15. Answers will vary. **17.** Putnam Problem A2, 2004

Section 1.5 *(page 44)*

1. (a) $f(g(x)) = 5[(x - 1)/5] + 1 = x$;

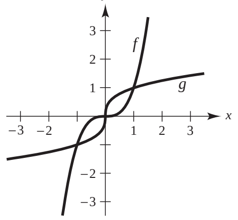
$g(f(x)) = [(5x + 1) - 1]/5 = x$

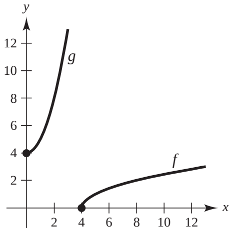
(b)

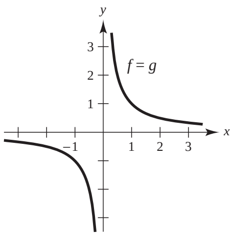


3. (a) $f(g(x)) = \left(\sqrt[3]{x}\right)^3 = x$; $g(f(x)) = \sqrt[3]{x^3} = x$

(b)

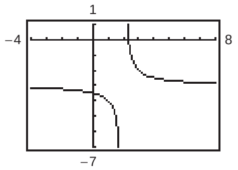


5. (a) $f(g(x)) = \sqrt{x^2 + 4 - 4} = x$;
$g(f(x)) = \left(\sqrt{x - 4}\right)^2 + 4 = x$
(b)


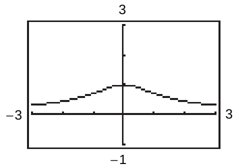
7. (a) $f(g(x)) = \dfrac{1}{1/x} = x$; $g(f(x)) = \dfrac{1}{1/x} = x$
(b)


9. c **10.** b **11.** a **12.** d

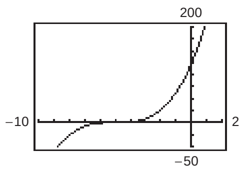
13. One-to-one, inverse exists. **15.** Not one-to-one, inverse does not exist.

17.


One-to-one, inverse exists.

19.


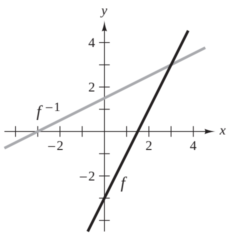
Not one-to-one, inverse does not exist.

21.


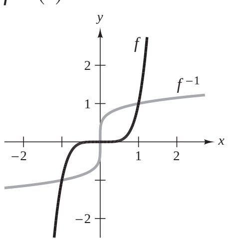
One-to-one, inverse exists.

23. Not one-to-one, inverse does not exist.

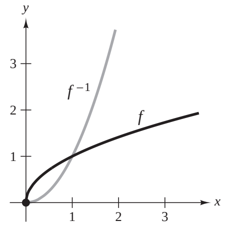
25. One-to-one, inverse exists.

27. (a) $f^{-1}(x) = (x + 3)/2$
(b)


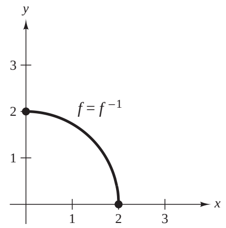
(c) f and f^{-1} are symmetric about $y = x$.
(d) Domain of f and f^{-1}: $(-\infty, \infty)$
Range of f and f^{-1}: $(-\infty, \infty)$

29. (a) $f^{-1}(x) = x^{1/5}$
(b)


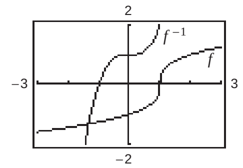
(c) f and f^{-1} are symmetric about $y = x$.
(d) Domain of f and f^{-1}: $(-\infty, \infty)$
Range of f and f^{-1}: $(-\infty, \infty)$

31. (a) $f^{-1}(x) = x^2, \quad x \geq 0$
(b)


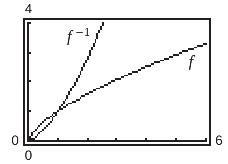
(c) f and f^{-1} are symmetric about $y = x$.
(d) Domain of f and f^{-1}: $[0, \infty)$
Range of f and f^{-1}: $[0, \infty)$

33. (a) $f^{-1}(x) = \sqrt{4 - x^2}, \quad 0 \leq x \leq 2$
(b)


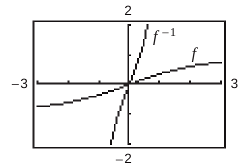
(c) f and f^{-1} are symmetric about $y = x$.
(d) Domain of f and f^{-1}: $[0, 2]$
Range of f and f^{-1}: $[0, 2]$

35. (a) $f^{-1}(x) = x^3 + 1$
(b)


(c) f and f^{-1} are symmetric about $y = x$.
(d) Domain of f and f^{-1}: $(-\infty, \infty)$
Range of f and f^{-1}: $(-\infty, \infty)$

37. (a) $f^{-1}(x) = x^{3/2}, \quad x \geq 0$
(b)


(c) f and f^{-1} are symmetric about $y = x$.
(d) Domain of f and f^{-1}: $[0, \infty)$
Range of f and f^{-1}: $[0, \infty)$

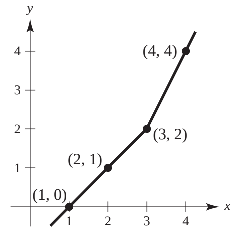
39. (a) $f^{-1}(x) = \sqrt{7}x/\sqrt{1 - x^2}, \quad -1 < x < 1$
(b)


(c) f and f^{-1} are symmetric about $y = x$.
(d) Domain of f: $(-\infty, \infty)$
Range of f: $(-1, 1)$
Domain of f^{-1}: $(-1, 1)$
Range of f^{-1}: $(-\infty, \infty)$

41.

x	0	1	2	4
$f(x)$	1	2	3	4

x	1	2	3	4
$f^{-1}(x)$	0	1	2	4



43. (a) Answers will vary.
(b) $y = \frac{20}{7}(80 - x)$
x: total cost
y: number of pounds of the less expensive commodity
(c) $[62.5, 80]$; The total cost will be between \$62.50 and \$80.00
(d) 20 lb

45. One-to-one; $f^{-1}(x) = x^2 + 2,\ x \ge 0$ **47.** Not one-to-one

49. One-to-one; $f^{-1}(x) = \dfrac{x - b}{a},\ a \ne 0$

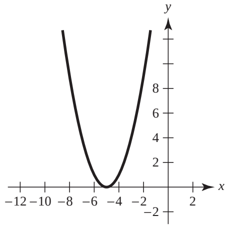
51. The function f passes the Horizontal Line Test on $[4, \infty)$, so it is one-to-one on $[4, \infty)$.

53. The function f passes the Horizontal Line Test on $(0, \infty)$, so it is one-to-one on $(0, \infty)$.

55. The function f passes the Horizontal Line Test on $[0, \pi]$, so it is one-to-one on $[0, \pi]$.

57. Answers will vary. Sample answer: $f^{-1}(x) = \sqrt{x} + 3,\ x \ge 0$

59. (a)

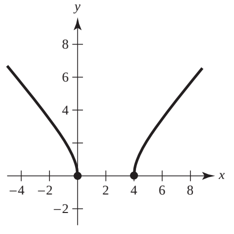


(b) Answers will vary. Sample answer: $[-5, \infty)$

(c) $f^{-1}(x) = \sqrt{x} - 5$

(d) Domain of f^{-1}: $[0, \infty)$

61. (a)

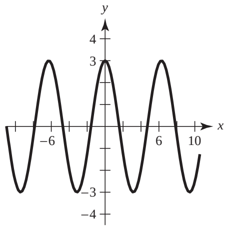


(b) Answers will vary. Sample answer: $[4, \infty)$

(c) $f^{-1}(x) = 2 + \sqrt{x^2 + 4}$

(d) Domain of f^{-1}: $[0, \infty)$

63. (a)



(b) Answers will vary. Sample answer: $[0, \pi]$

(c) $f^{-1}(x) = \arccos\left(\dfrac{x}{3}\right)$

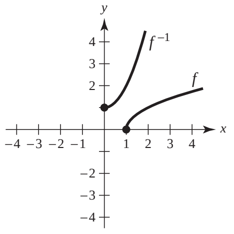
(d) Domain of f^{-1}: $[-3, 3]$

65. 1 **67.** $\dfrac{\pi}{6}$ **69.** 2 **71.** 32 **73.** 600

75. $(g^{-1} \circ f^{-1})(x) = \dfrac{x + 1}{2}$ **77.** $(f \circ g)^{-1}(x) = \dfrac{x + 1}{2}$

79. (a) f is one-to-one because it passes the Horizontal Line Test.
(b) $[-2, 2]$
(c) -4

81.

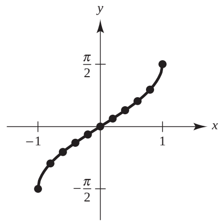


83. (a)

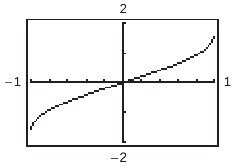
x	-1	-0.8	-0.6	-0.4	-0.2
y	-1.57	-0.93	-0.64	-0.41	-0.20

x	0	0.2	0.4	0.6	0.8	1
y	0	0.20	0.41	0.64	0.93	1.57

(b)



(c)



(d) Intercept: $(0, 0)$; Symmetry: origin

85. $\left(-\sqrt{2}/2, 3\pi/4\right), (1/2, \pi/3), \left(\sqrt{3}/2, \pi/6\right)$ **87.** $\pi/6$

89. $\pi/3$ **91.** $\pi/6$ **93.** $-\pi/4$ **95.** 2.50 **97.** 0.66

99. -0.1 **101.** x **103.** $\dfrac{\sqrt{1 - x^2}}{x}$ **105.** $\dfrac{1}{x}$

107. (a) $\dfrac{3}{5}$ (b) $\dfrac{5}{3}$ **109.** (a) $-\sqrt{3}$ (b) $-\dfrac{13}{5}$

111. $\sqrt{1 - 4x^2}$ **113.** $\dfrac{\sqrt{x^2 - 1}}{|x|}$ **115.** $\dfrac{\sqrt{x^2 - 9}}{3}$

117. $x = \frac{1}{3}\left[\sin\left(\frac{1}{2}\right) + \pi\right] \approx 1.207$ **119.** $x = \frac{1}{3}$

121. $(0.7862, 0.6662)$

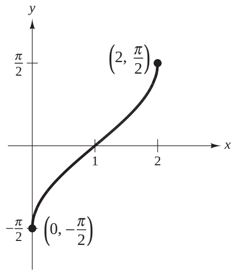
123. Let $y = f(x)$ be one-to-one. Solve for x as a function of y. Interchange x and y to get $y = f^{-1}(x)$. Let the domain of f^{-1} be the range of f. Verify that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.
Sample answer:

$$f(x) = x^3$$
$$y = x^3$$
$$x = \sqrt[3]{y}$$
$$y = \sqrt[3]{x}$$
$$f^{-1}(x) = \sqrt[3]{x}$$

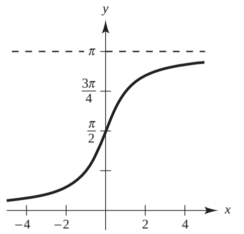
125. The trigonometric functions are not one-to-one. So, their domains must be restricted to define the inverse trigonometric functions.

127. $\arcsin\left(\dfrac{9}{\sqrt{x^2 + 81}}\right)$ **129.** Answers will vary.

131.



133.



135. $f^{-1}(8) = -3$ **137–139.** Proofs

141. False. Let $f(x) = x^2$.

143. False. $\arcsin^2 0 + \arccos^2 0 = \left(\dfrac{\pi}{2}\right)^2 \ne 1$

145. True **147.** Answers will vary. **149.** Proof

151. $f^{-1}(x) = \dfrac{-b - \sqrt{b^2 - 4ac + 4ax}}{2a}$

153. $ad - bc \ne 0$; $f^{-1}(x) = \dfrac{b - dx}{cx - a}$

Section 1.6 *(page 53)*

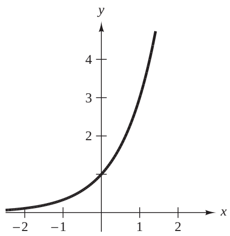
1. (a) 125 (b) 9 (c) $\frac{1}{9}$ (d) $\frac{1}{3}$

3. (a) 5^5 (b) $\frac{1}{5}$ (c) $\frac{1}{5}$ (d) 2^2

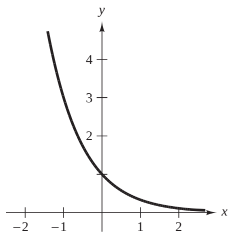
5. (a) e^6 (b) e^{12} (c) $\dfrac{1}{e^6}$ (d) e^2

7. $x = 4$ **9.** $x = 4$ **11.** $x = -5$ **13.** $x = -2$
15. $x = 2$ **17.** $x = 16$ **19.** $x = \ln 5 \approx 1.609$
21. $x = -\frac{5}{2}$ **23.** $2.7182805 < e$

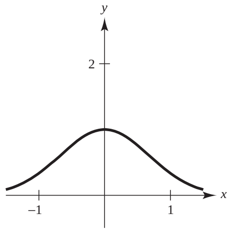
25.



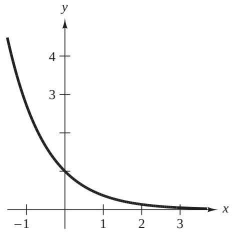
27.



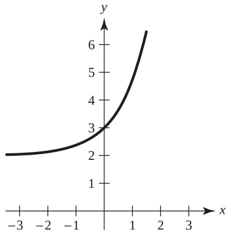
29.



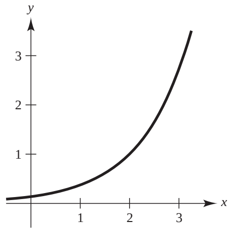
31.



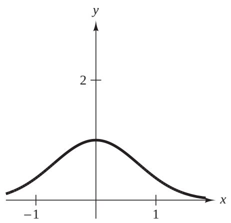
33.



35.

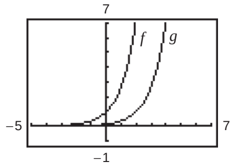


37.



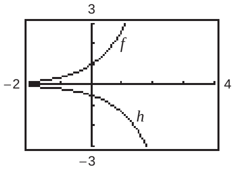
39. Domain: $(-\infty, \infty)$
41. Domain: $(-\infty, 0]$
43. Domain: $(-\infty, \infty)$

45. (a)



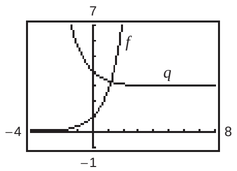
Translation two units to the right

(b)



Reflection in the x-axis and vertical shrink

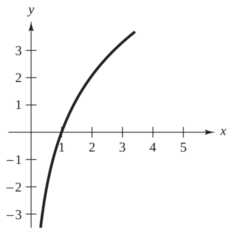
(c)



Reflection in the y-axis and translation three units upward

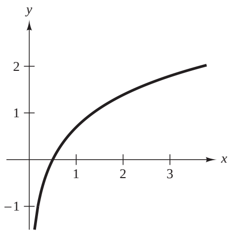
47. c **48.** d **49.** a **50.** b **51.** $y = 2(3^x)$ **53.** b
54. d **55.** a **56.** c **57.** $\ln 1 = 0$ **59.** $e^{0.6931...} = 2$

61.



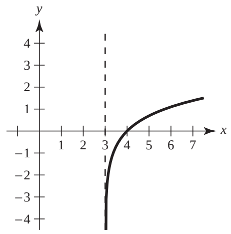
Domain: $x > 0$

63.



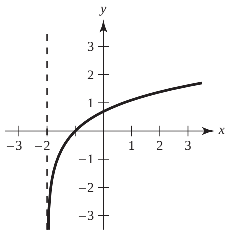
Domain: $x > 0$

65.



Domain: $x > 3$

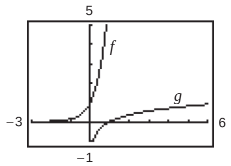
67.



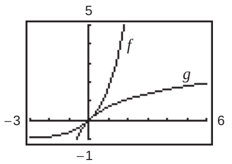
Domain: $x > -2$

69. $g(x) = -e^x - 8$ **71.** $g(x) = \ln(x - 5) - 1$

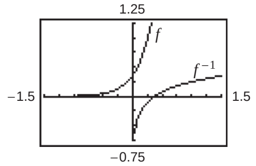
73.



75.

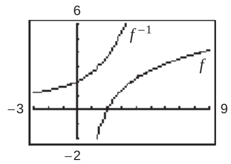


77. (a) $f^{-1}(x) = \dfrac{\ln x + 1}{4}$
(b)



(c) Answers will vary.

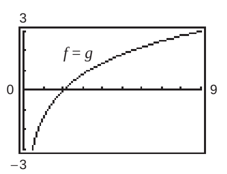
79. (a) $f^{-1}(x) = e^{x/2} + 1$
(b)



(c) Answers will vary.

81. x^2 **83.** $5x + 2$ **85.** $-1 + 2x$
87. (a) 1.7917 (b) −0.4055 (c) 4.3944 (d) 0.5493
89. $\ln x - \ln 4$ **91.** $\ln x + \ln y - \ln z$
93. $\ln x + \frac{1}{2}\ln(x^2 + 5)$ **95.** $\frac{1}{2}[\ln(x - 1) - \ln x]$
97. $2 + \ln 3$ **99.** $\ln(7x)$
101. $\ln \dfrac{x - 2}{x + 2}$ **103.** $\ln \sqrt[3]{\dfrac{x(x + 3)^2}{x^2 - 1}}$ **105.** $\ln \dfrac{9}{\sqrt{x^2 + 1}}$
107. (a) $x = 4$ (b) $x = \frac{3}{2}$
109. (a) $x = e^2 \approx 7.389$ (b) $x = \ln 4 \approx 1.386$
111. $x > \ln 5$ **113.** $e^{-2} < x < 1$

115.



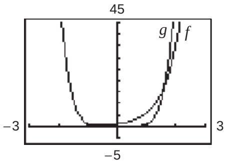
117. Answers will vary. **119.** Answers will vary.

121. (a) False (b) True. $y = \log_2 x$
(c) True. $2^y = x$ (d) False

123. $\beta = 10 \log_{10} I + 160$

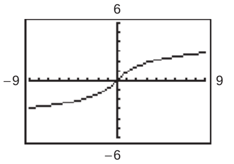
125. False. $\ln x + \ln 25 = \ln 25x$

127.



$(-0.7899, 0.2429)$, $(1.6242, 18.3615)$, and $(6, 46{,}656)$; As x increases, $f(x) = 6^x$ grows more rapidly.

129. (a)



Domain: $(-\infty, \infty)$

(b) Proof

(c) $f^{-1}(x) = \dfrac{e^{2x} - 1}{2e^x}$

131. $12! = 479{,}001{,}600$
Stirling's Formula: $12! \approx 475{,}687{,}487$

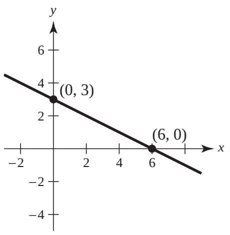
133. Proof

Review Exercises for Chapter 1 *(page 56)*

1. $\left(\frac{8}{5}, 0\right), (0, -8)$ **3.** $(3, 0), \left(0, \frac{3}{4}\right)$ **5.** Not symmetric

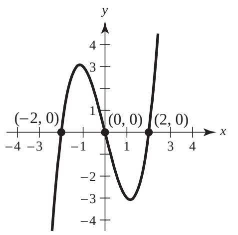
7. Symmetric with respect to the x-axis, the y-axis, and the origin

9.



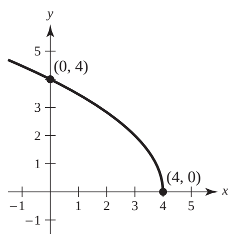
Symmetry: none

11.



Symmetry: origin

13.

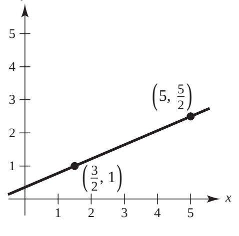


Symmetry: none

15. $(-2, 3)$

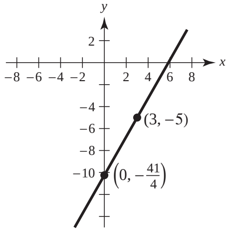
17. $(-2, 3), (3, 8)$

19.

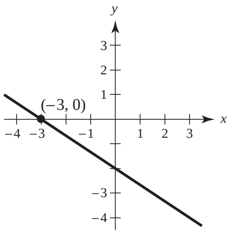


$m = \frac{3}{7}$

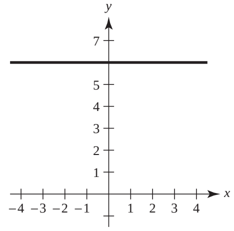
21. $7x - 4y - 41 = 0$



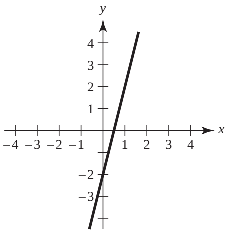
23. $2x + 3y + 6 = 0$



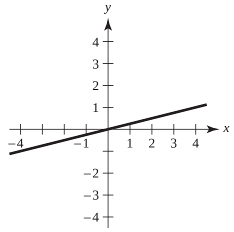
25.



27.



29. $x - 4y = 0$



31. (a) $7x - 16y + 101 = 0$ (b) $5x - 3y + 30 = 0$
(c) $4x - 3y + 27 = 0$ (d) $x + 3 = 0$

33. $V = 12{,}500 - 850t$; \$9950

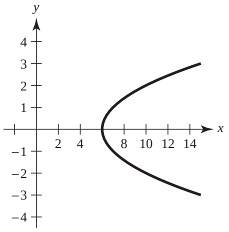
35. (a) 4 (b) 29 (c) -11 (d) $5t + 9$

37. $8x + 4\,\Delta x$, $\Delta x \neq 0$

39. Domain: $(-\infty, \infty)$; Range: $[3, \infty)$

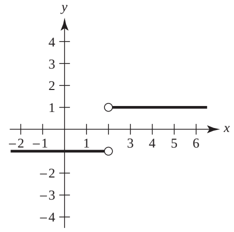
41. Domain: $(-\infty, \infty)$; Range: $(-\infty, 0]$

43.



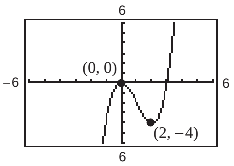
Not a function

45.



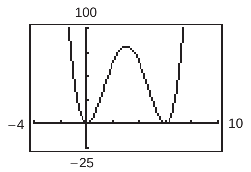
Function

47. $f(x) = x^3 - 3x^2$

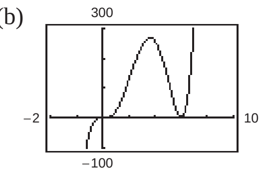


(a) $g(x) = -x^3 + 3x^2 + 1$
(b) $g(x) = (x - 2)^3 - 3(x - 2)^2 + 1$

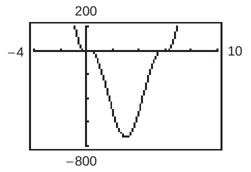
49. (a)



(b)



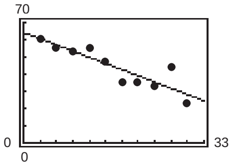
(c)



51. For company (a), the profit rose rapidly for the first year and then leveled off. For company (b), the profit dropped and then rose again later.

53. (a) $y = -1.204x + 64.2667$

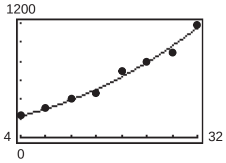
(b)



(c) The data point $(27, 44)$ is probably an error. Without this point, the new model is $y = -1.4344x + 66.4387$.

55. (a) $y = 0.61t^2 + 11.0t + 172$

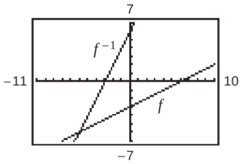
(b)



The model fits the data well.

57. (a) $f^{-1}(x) = 2x + 6$

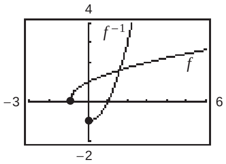
(b)



(c) Answers will vary.

59. (a) $f^{-1}(x) = x^2 - 1,\ x \geq 0$

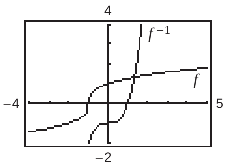
(b)



(c) Answers will vary.

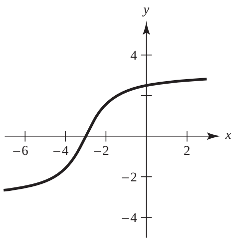
61. (a) $f^{-1}(x) = x^3 - 1$

(b)



(c) Answers will vary.

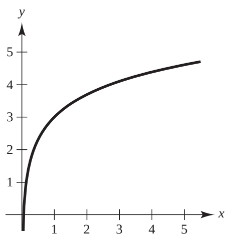
63.



65. $\frac{1}{2}$ **67.** d **68.** a

69. c **70.** b

71.

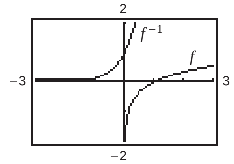


73. $\frac{1}{5}[\ln(2x + 1) + \ln(2x - 1) - \ln(4x^2 + 1)]$

75. $\ln\left(\dfrac{3\sqrt[3]{4 - x^2}}{x}\right)$ **77.** $x = e^4 - 1 \approx 53.598$

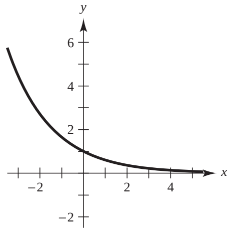
79. (a) $f^{-1}(x) = e^{2x}$

(b)



(c) Answers will vary.

81.

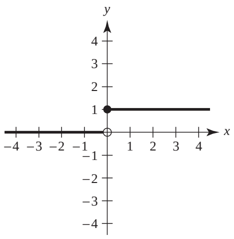


P.S. Problem Solving *(page 59)*

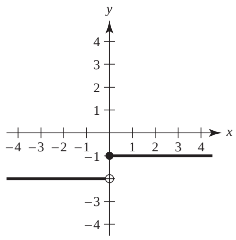
1. (a) Center: $(3, 4)$; Radius: 5

(b) $y = -\frac{3}{4}x$ (c) $y = \frac{3}{4}x - \frac{9}{2}$ (d) $\left(3, -\frac{9}{4}\right)$

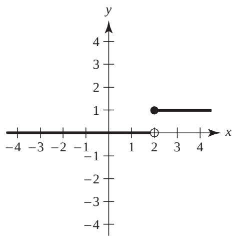
3.



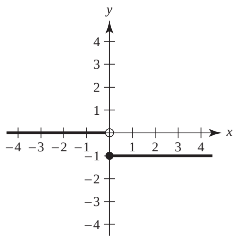
(a)



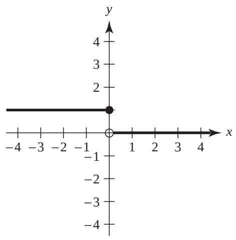
(b)



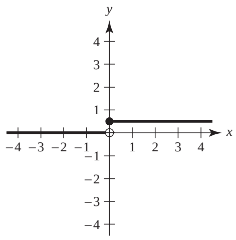
(c)



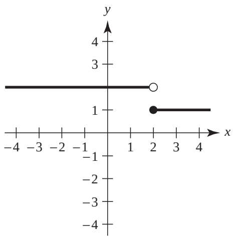
(d)



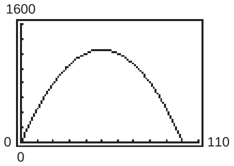
(e)



(f)



5. (a) $A(x) = x[(100 - x)/2]$; Domain: $(0, 100)$
(b)

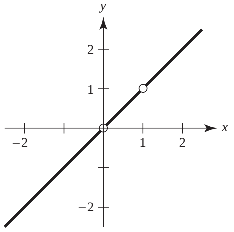


Dimensions 50 m × 25 m yield maximum area of 1250 m^2.
(c) 50 m × 25 m; Area = 1250 m^2

7. $T(x) = \left[2\sqrt{4 + x^2} + \sqrt{(3 - x)^2 + 1}\right]/4$

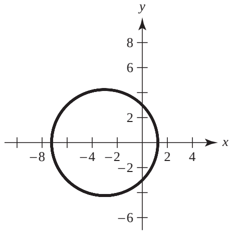
9. (a) 5, less (b) 3, greater (c) 4.1, less
(d) $4 + h$ (e) 4; Answers will vary.

11. (a) Domain: $(-\infty, 1) \cup (1, \infty)$; Range: $(-\infty, 0) \cup (0, \infty)$
(b) $f(f(x)) = \dfrac{x - 1}{x}$; Domain: $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$
(c) $f(f(f(x))) = x$; Domain: $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$
(d)

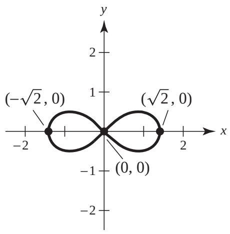


The graph is not a line because there are holes at $x = 0$ and $x = 1$.

13. (a) $x \approx 1.2426, -7.2426$
(b) $(x + 3)^2 + y^2 = 18$



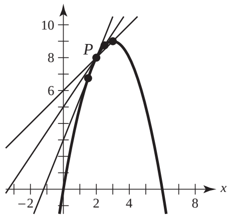
15. Proof



Chapter 2

Section 2.1 *(page 67)*

1. Precalculus: 300 ft
3. Calculus: Slope of the tangent line at $x = 2$ is 0.16.
5. (a) Precalculus: 10 square units
(b) Calculus: 5 square units
7. (a)



(b) 1; $\frac{3}{2}$; $\frac{5}{2}$
(c) 2; Use points closer to P.
9. Area ≈ 10.417; Area ≈ 9.145; Use more rectangles.

Section 2.2 *(page 75)*

1.

x	3.9	3.99	3.999	4
$f(x)$	0.2041	0.2004	0.2000	?

x	4.001	4.01	4.1
$f(x)$	0.2000	0.1996	0.1961

$\lim_{x\to 4} \dfrac{x - 4}{x^2 - 3x - 4} \approx 0.2000$ $\left(\text{Actual limit is } \dfrac{1}{5}.\right)$

3.

x	-0.1	-0.01	-0.001	0
$f(x)$	0.9983	0.99998	1.0000	?

x	0.001	0.01	0.1
$f(x)$	1.0000	0.99998	0.9983

$\lim_{x\to 0} \dfrac{\sin x}{x} \approx 1.0000$ (Actual limit is 1.)

5.

x	-0.1	-0.01	-0.001	0
$f(x)$	0.9516	0.9950	0.9995	?

x	0.001	0.01	0.1
$f(x)$	1.0005	1.0050	1.0517

$\lim_{x\to 0} \dfrac{e^x - 1}{x} \approx 1.0000$ (Actual limit is 1.)

7.

x	0.9	0.99	0.999	1
$f(x)$	0.2564	0.2506	0.2501	?

x	1.001	1.01	1.1
$f(x)$	0.2499	0.2494	0.2439

$\lim_{x\to 1} \dfrac{x - 2}{x^2 + x - 6} \approx 0.2500$ $\left(\text{Actual limit is } \dfrac{1}{4}.\right)$

9.

x	0.9	0.99	0.999	1
$f(x)$	0.7340	0.6733	0.6673	?

x	1.001	1.01	1.1
$f(x)$	0.6660	0.6600	0.6015

$\lim_{x\to 1} \dfrac{x^4 - 1}{x^6 - 1} \approx 0.6666$ $\left(\text{Actual limit is } \dfrac{2}{3}.\right)$

11.

x	-6.1	-6.01	-6.001	-6
$f(x)$	-0.1248	-0.1250	-0.1250	?

x	-5.999	-5.99	-5.9
$f(x)$	-0.1250	-0.1250	-0.1252

$\lim_{x\to -6} \dfrac{\sqrt{10 - x} - 4}{x + 6} \approx -0.1250$ $\left(\text{Actual limit is } -\dfrac{1}{8}.\right)$

13.

x	-0.1	-0.01	-0.001	0
$f(x)$	1.9867	1.9999	2.0000	?

x	0.001	0.01	0.1
$f(x)$	2.0000	1.9999	1.9867

$\lim_{x\to 0} \frac{\sin 2x}{x} \approx 2.0000$ (Actual limit is 2.)

15.

x	1.9	1.99	1.999	2
$f(x)$	0.5129	0.5013	0.5001	?

x	2.001	2.01	2.1
$f(x)$	0.4999	0.4988	0.4879

$\lim_{x\to 2} \frac{\ln x - \ln 2}{x - 2} \approx 0.5000$ $\left(\text{Actual limit is } \frac{1}{2}.\right)$

17. 1 **19.** 2

21. Limit does not exist. The function approaches 1 from the right side of 2, but it approaches -1 from the left side of 2.

23. Limit does not exist. The function oscillates between 1 and -1 as x approaches 0.

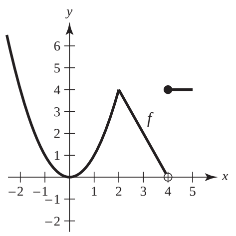
25. (a) 2

(b) Limit does not exist. The function approaches 1 from the right side of 1, but it approaches 3.5 from the left side of 1.

(c) Value does not exist. The function is undefined at $x = 4$.

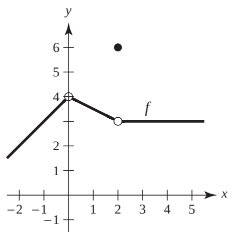
(d) 2

27.



$\lim_{x\to c} f(x)$ exists for all points on the graph except where $c = 4$.

29.



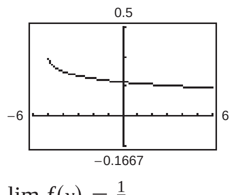
31. $\delta = 0.4$ **33.** $\delta = \frac{1}{11} \approx 0.091$

35. $L = 8$; Let $\delta = 0.01/3 \approx 0.0033$.

37. $L = 1$; Let $\delta = 0.01/5 = 0.002$. **39.** 6 **41.** -3

43. 3 **45.** 0 **47.** 10 **49.** 2 **51.** 4

53.

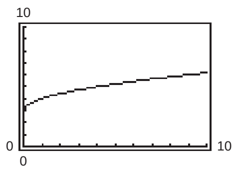


$\lim_{x\to 4} f(x) = \frac{1}{6}$

Domain: $[-5, 4) \cup (4, \infty)$

The graph has a hole at $x = 4$.

55.

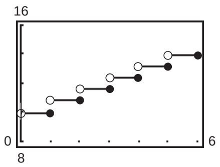


$\lim_{x\to 9} f(x) = 6$

Domain: $[0, 9) \cup (9, \infty)$

The graph has a hole at $x = 9$.

57. (a)



(b)

t	3	3.3	3.4	3.5
C	11.57	12.36	12.36	12.36

t	3.6	3.7	4
C	12.36	12.36	12.36

$\lim_{t\to 3.5} C(t) = 12.36$

(c)

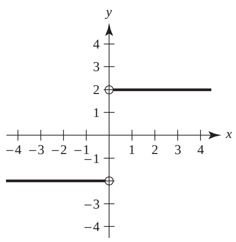
t	2	2.5	2.9	3
C	10.78	11.57	11.57	11.57

t	3.1	3.5	4
C	12.36	12.36	12.36

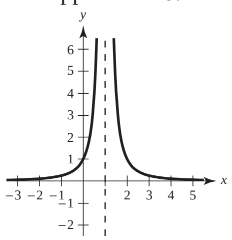
The limit does not exist because the limits from the right and left are not equal.

59. Answers will vary. Sample answer: As x approaches 8 from either side, $f(x)$ becomes arbitrarily close to 25.

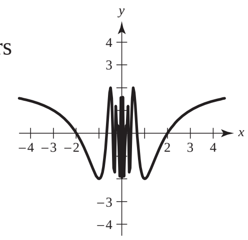
61. (i) The values of f approach different numbers as x approaches c from different sides of c.



(ii) The values of f increase or decrease without bound as x approaches c.



(iii) The values of f oscillate between two fixed numbers as x approaches c.



63. (a) $r = \frac{3}{\pi} \approx 0.9549$ cm

(b) $\frac{5.5}{2\pi} \le r \le \frac{6.5}{2\pi}$, or approximately $0.8754 < r < 1.0345$

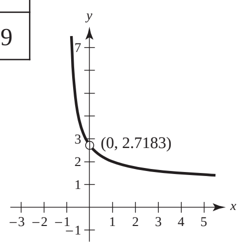
(c) $\lim_{r\to 3/\pi} 2\pi r = 6$; $\varepsilon = 0.5$; $\delta \approx 0.0796$

65.

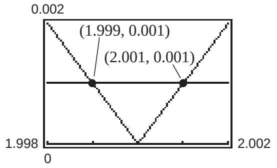
x	-0.001	-0.0001	-0.00001	0
$f(x)$	2.7196	2.7184	2.7183	?

x	0.00001	0.0001	0.001
$f(x)$	2.7183	2.7181	2.7169

$\lim_{x\to 0} f(x) \approx 2.7183$



67.



$\delta = 0.001,\ (1.999, 2.001)$

69. False. The existence or nonexistence of $f(x)$ at $x = c$ has no bearing on the existence of the limit of $f(x)$ as $x \to c$.

71. False. See Exercise 19.

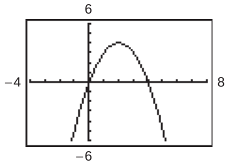
73. Yes. As x approaches 0.25 from either side, $\sqrt{x}$ becomes arbitrarily close to 0.5.

75. $\lim_{x\to 0} \dfrac{\sin nx}{x} = n$ **77–79.** Proofs

81. Putnam Problem B1, 1986

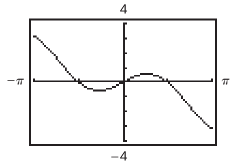
Section 2.3 *(page 87)*

1.



(a) 0 (b) -5

3.



(a) 0 (b) About 0.52 or $\pi/6$

5. 8 **7.** -1 **9.** 0 **11.** 7 **13.** 2 **15.** 1 **17.** $\frac{1}{2}$
19. $\frac{1}{5}$ **21.** 7 **23.** 1 **25.** $\frac{1}{2}$ **27.** 1 **29.** $\frac{1}{2}$ **31.** -1
33. 1 **35.** $\ln 3 + e$ **37.** (a) 4 (b) 64 (c) 64
39. (a) 3 (b) 2 (c) 2 **41.** (a) 10 (b) 5 (c) 6 (d) $\frac{3}{2}$
43. (a) 64 (b) 2 (c) 12 (d) 8

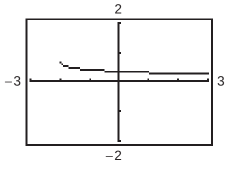
45. $f(x) = \dfrac{x^2 - 1}{x + 1}$ and $g(x) = x - 1$ agree except at $x = -1$.
$\lim_{x\to -1} f(x) = \lim_{x\to -1} g(x) = -2$

47. $f(x) = \dfrac{x^3 - 8}{x - 2}$ and $g(x) = x^2 + 2x + 4$ agree except at $x = 2$.
$\lim_{x\to 2} f(x) = \lim_{x\to 2} g(x) = 12$

49. $-\dfrac{\ln 2}{8} \approx -0.0866$
$f(x) = \dfrac{(x + 4)\ln(x + 6)}{x^2 - 16}$ and $g(x) = \dfrac{\ln(x + 6)}{x - 4}$ agree except at $x = -4$.

51. -1 **53.** $1/8$ **55.** $5/6$ **57.** $1/6$ **59.** $\sqrt{5}/10$
61. $-1/9$ **63.** 2 **65.** $2x - 2$ **67.** $1/5$ **69.** 0
71. 0 **73.** 0 **75.** 1 **77.** 1 **79.** $3/2$

81.



The graph has a hole at $x = 0$.

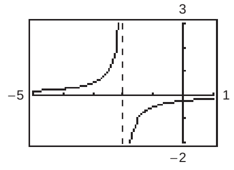
Answers will vary. Sample answer:

x	-0.1	-0.01	-0.001	0
$f(x)$	0.358	0.354	0.354	?

x	0.001	0.01	0.1
$f(x)$	0.354	0.353	0.349

$\lim_{x\to 0} \dfrac{\sqrt{x + 2} - \sqrt{2}}{x} \approx 0.354$; Actual limit is $\dfrac{1}{2\sqrt{2}} = \dfrac{\sqrt{2}}{4}$.

83.



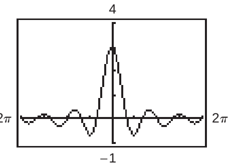
The graph has a hole at $x = 0$.
Answers will vary. Sample answer:

x	-0.1	-0.01	-0.001	0
$f(x)$	-0.263	-0.251	-0.250	?

x	0.001	0.01	0.1
$f(x)$	-0.250	-0.249	-0.238

$\lim_{x\to 0} \dfrac{[1/(2 + x)] - (1/2)}{x} \approx -0.250$; Actual limit is $-\dfrac{1}{4}$.

85.

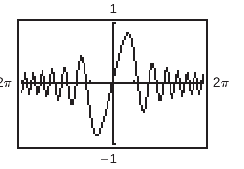


The graph has a hole at $t = 0$.
Answers will vary. Sample answer:

t	-0.1	-0.01	0	0.01	0.1
$f(t)$	2.96	2.9996	?	2.9996	2.96

$\lim_{t\to 0} \dfrac{\sin 3t}{t} \approx 3.0000$; Actual limit is 3.

87.



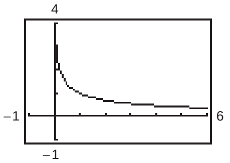
The graph has a hole at $x = 0$.
Answers will vary. Sample answer:

x	-0.1	-0.01	-0.001	0
$f(x)$	-0.1	-0.01	-0.001	?

x	0.001	0.01	0.1
$f(x)$	0.001	0.01	0.1

$\lim_{x\to 0} \dfrac{\sin x^2}{x} = 0$; Actual limit is 0.

89.



Answers will vary. Sample answer:

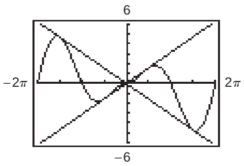
x	0.5	0.9	0.99	1
$f(x)$	1.3863	1.0536	1.0050	?

x	1.01	1.1	1.5
$f(x)$	0.9950	0.9531	0.8109

$\lim_{x\to 1} \frac{\ln x}{x-1} \approx 1$; Actual limit is 1.

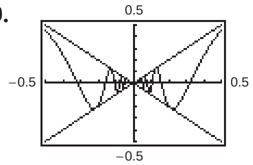
91. 3 **93.** $-1/(x+3)^2$ **95.** 4

97.



0

99.



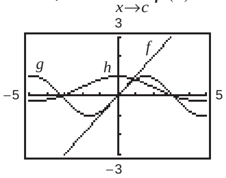
0 (The graph has a hole at $x = 0$.)

101. (a) f and g agree at all but one point if c is a real number such that $f(x) = g(x)$ for all $x \neq c$.

(b) Sample answer: $f(x) = \frac{x^2 - 1}{x - 1}$ and $g(x) = x + 1$ agree at all points except $x = 1$.

103. If a function f is squeezed between two functions h and g, $h(x) \leq f(x) \leq g(x)$, and h and g have the same limit L as $x \to c$, then $\lim_{x\to c} f(x)$ exists and equals L.

105.



The magnitudes of $f(x)$ and $g(x)$ are approximately equal when x is close to 0. Therefore, their ratio is approximately 1.

107. -64 ft/sec (speed $= 64$ ft/sec) **109.** -29.4 m/sec

111. Let $f(x) = 1/x$ and $g(x) = -1/x$. $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ do not exist. However,

$$\lim_{x\to 0} [f(x) + g(x)] = \lim_{x\to 0} \left[\frac{1}{x} + \left(-\frac{1}{x}\right)\right] = \lim_{x\to 0} 0 = 0$$

and therefore does exist.

113–117. Proofs

119. Let $f(x) = \begin{cases} 4, & x \geq 0 \\ -4, & x < 0 \end{cases}$.

$\lim_{x\to 0} |f(x)| = \lim_{x\to 0} 4 = 4$

$\lim_{x\to 0} f(x)$ does not exist because for $x < 0$, $f(x) = -4$ and for $x \geq 0$, $f(x) = 4$.

121. False. The limit does not exist because the function approaches 1 from the right side of 0 and approaches -1 from the left side of 0.

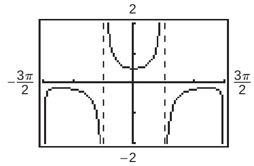
123. True.

125. False. The limit does not exist because $f(x)$ approaches 3 from the left side of 2 and approaches 0 from the right side of 2.

127. Proof

129. (a) All $x \neq 0, \frac{\pi}{2} + n\pi$

(b)



The domain is not obvious. The hole at $x = 0$ is not apparent from the graph.

(c) $\frac{1}{2}$ (d) $\frac{1}{2}$

Section 2.4 *(page 99)*

1. (a) 3 (b) 3 (c) 3; $f(x)$ is continuous on $(-\infty, \infty)$.

3. (a) 0 (b) 0 (c) 0; Discontinuity at $x = 3$

5. (a) -3 (b) 3 (c) Limit does not exist. Discontinuity at $x = 2$

7. $\frac{1}{16}$ **9.** $\frac{1}{10}$

11. Limit does not exist. The function decreases without bound as x approaches -3 from the left.

13. -1 **15.** $-1/x^2$ **17.** $5/2$

19. Limit does not exist. The function decreases without bound as x approaches π from the left and increases without bound as x approaches π from the right.

21. 8

23. Limit does not exist. The function approaches 5 from the left side of 3 but approaches 6 from the right side of 3.

25. Limit does not exist. The function decreases without bound as x approaches 3 from the right.

27. ln 4 **29.** Discontinuities at $x = -2$ and $x = 2$

31. Discontinuities at every integer

33. Continuous on $[-7, 7]$ **35.** Continuous on $[-1, 4]$

37. Nonremovable discontinuity at $x = 0$

39. Continuous for all real x

41. Nonremovable discontinuities at $x = -2$ and $x = 2$

43. Nonremovable discontinuity at $x = 1$
Removable discontinuity at $x = 0$

45. Continuous for all real x

47. Removable discontinuity at $x = -2$
Nonremovable discontinuity at $x = 5$

49. Nonremovable discontinuity at $x = -7$

51. Continuous for all real x

53. Nonremovable discontinuity at $x = 2$

55. Continuous for all real x

57. Nonremovable discontinuity at $x = 0$

59. Nonremovable discontinuities at integer multiples of $\pi/2$

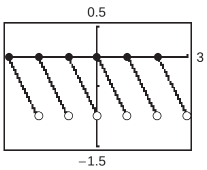
61. Nonremovable discontinuities at each integer

63. $a = 7$ **65.** $a = -1,\ b = 1$ **67.** $a = -1$

69. Continuous for all real x

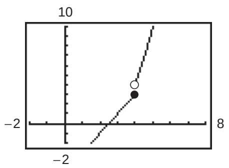
71. Nonremovable discontinuities at $x = 1$ and $x = -1$

73.



Nonremovable discontinuity at each integer

75.



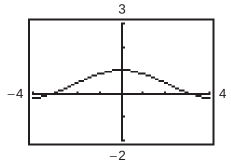
Nonremovable discontinuity at $x = 4$

77. Continuous on $(-\infty, \infty)$ **79.** Continuous on $[0, \infty)$

81. Continuous on the open intervals . . . , $(-6, -2)$, $(-2, 2)$, $(2, 6)$, . . .

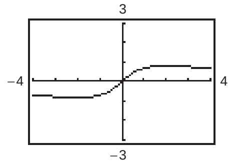
83. Continuous on $(-\infty, \infty)$

85.



The graph has a hole at $x = 0$. The graph appears to be continuous, but the function is not continuous on $[-4, 4]$. It is not obvious from the graph that the function has a discontinuity at $x = 0$.

87.



The graph has a hole at $x = 0$. The graph appears to be continuous, but the function is not continuous on $[-4, 4]$. It is not obvious from the graph that the function has a discontinuity at $x = 0$.

89. Because $f(x)$ is continuous on the interval $[1, 2]$ and $f(1) = 37/12$ and $f(2) = -8/3$, by the Intermediate Value Theorem there exists a real number c in $[1, 2]$ such that $f(c) = 0$.

91. Because $h(x)$ is continuous on the interval $[0, \pi/2]$, and $h(0) = -2$ and $h(\pi/2) \approx 0.9119$, by the Intermediate Value Theorem there exists a real number c in $[0, \pi/2]$ such that $f(c) = 0$.

93. 0.68, 0.6823 **95.** 0.56, 0.5636 **97.** 0.79, 0.7921

99. $f(3) = 11$ **101.** $f(2) = 4$

103. (a) The limit does not exist at $x = c$.
(b) The function is not defined at $x = c$.
(c) The limit exists, but it is not equal to the value of the function at $x = c$.
(d) The limit does not exist at $x = c$.

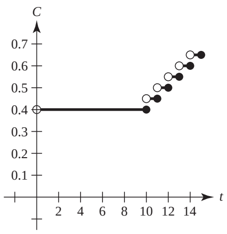
105. If f and g are continuous for all real x, then so is $f + g$ (Theorem 1.11, part 2). However, f/g might not be continuous if $g(x) = 0$. For example, let $f(x) = x$ and $g(x) = x^2 - 1$. Then f and g are continuous for all real x, but f/g is not continuous at $x = \pm 1$.

107. True

109. False. A rational function can be written as $P(x)/Q(x)$, where P and Q are polynomials of degree m and n, respectively. It can have, at most, n discontinuities.

111. The functions differ by 1 for non-integer values of x.

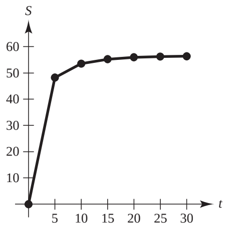
113. $$C = \begin{cases} 0.40, & 0 < t \le 10 \\ 0.40 + 0.05[\![t - 9]\!], & t > 10,\ t \text{ is not an integer.} \\ 0.40 + 0.05(t - 10), & t > 10,\ t \text{ is an integer.} \end{cases}$$



There is a nonremovable discontinuity at each integer greater than or equal to 10.

115–117. Proofs **119.** Answers will vary.

121. (a)

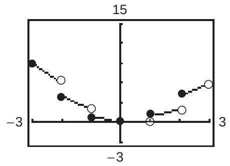


(b) There appears to be a limiting speed, and a possible cause is air resistance.

123. $c = \left(-1 \pm \sqrt{5}\right)/2$

125. Domain: $[-c^2, 0) \cup (0, \infty)$; Let $f(0) = 1/(2c)$.

127. $h(x)$ has a nonremovable discontinuity at every integer except 0.



129. Putnam Problem B2, 1988

Section 2.5 *(page 108)*

1. $\displaystyle\lim_{x\to -2^+} 2\left|\frac{x}{x^2-4}\right| = \infty, \quad \lim_{x\to -2^-} 2\left|\frac{x}{x^2-4}\right| = \infty$

3. $\displaystyle\lim_{x\to -2^+} \tan(\pi x/4) = -\infty, \quad \lim_{x\to -2^-} \tan(\pi x/4) = \infty$

5. $\displaystyle\lim_{x\to 4^+} \frac{1}{x-4} = \infty, \quad \lim_{x\to 4^-} \frac{1}{x-4} = -\infty$

7. $\displaystyle\lim_{x\to 4^+} \frac{1}{(x-4)^2} = \infty, \quad \lim_{x\to 4^-} \frac{1}{(x-4)^2} = \infty$

9.

x	−3.5	−3.1	−3.01	−3.001	−3
$f(x)$	0.31	1.64	16.6	167	?

x	−2.999	−2.99	−2.9	−2.5
$f(x)$	−167	−16.7	−1.69	−0.36

$\displaystyle\lim_{x\to -3^+} f(x) = -\infty, \quad \lim_{x\to -3^-} f(x) = \infty$

11.

x	−3.5	−3.1	−3.01	−3.001	−3
$f(x)$	3.8	16	151	1501	?

x	−2.999	−2.99	−2.9	−2.5
$f(x)$	−1499	−149	−14	−2.3

$\displaystyle\lim_{x\to -3^+} f(x) = -\infty, \quad \lim_{x\to -3^-} f(x) = \infty$

13. $x = 0$ **15.** $x = \pm 2$ **17.** No vertical asymptote

19. $x = -2, x = 1$ **21.** No vertical asymptote

23. $x = 1$ **25.** $t = -2$ **27.** $x = 0$

29. $x = n$, n is an integer. **31.** $t = n\pi$, n is a nonzero integer.

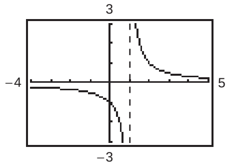
33. Removable discontinuity at $x = -1$

35. Vertical asymptote at $x = -1$ **37.** ∞ **39.** ∞

41. $-\frac{1}{5}$ **43.** $-\infty$ **45.** $-\infty$ **47.** ∞ **49.** $-\infty$

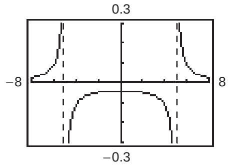
51. $-\infty$ **53.** ∞

55.



$\lim_{x\to 1^+} f(x) = \infty$

57.

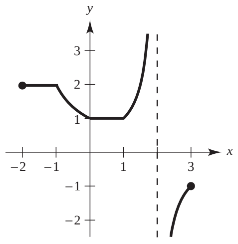


$\lim_{x\to 5^-} f(x) = -\infty$

59. Answers will vary; No

61. Answers will vary. Sample answer: $f(x) = \dfrac{x-3}{x^2-4x-12}$

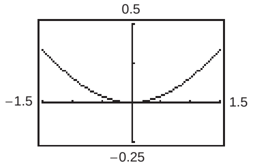
63.



65. (a)

x	1	0.5	0.2	0.1
$f(x)$	0.1585	0.0411	0.0067	0.0017

x	0.01	0.001	0.0001
$f(x)$	≈ 0	≈ 0	≈ 0

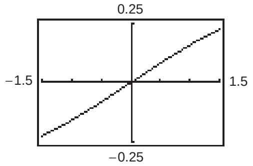


$\lim_{x\to 0^+} \dfrac{x - \sin x}{x} = 0$

(b)

x	1	0.5	0.2	0.1
$f(x)$	0.1585	0.0823	0.0333	0.0167

x	0.01	0.001	0.0001
$f(x)$	0.0017	≈ 0	≈ 0

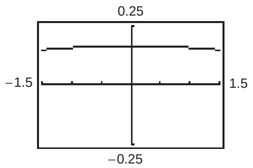


$\lim_{x\to 0^+} \dfrac{x - \sin x}{x^2} = 0$

(c)

x	1	0.5	0.2	0.1
$f(x)$	0.1585	0.1646	0.1663	0.1666

x	0.01	0.001	0.0001
$f(x)$	0.1667	0.1667	0.1667

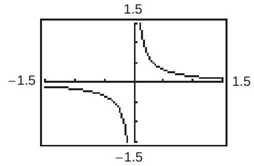


$\lim_{x\to 0^+} \dfrac{x - \sin x}{x^3} = 0.1667 \ (1/6)$

(d)

x	1	0.5	0.2	0.1
$f(x)$	0.1585	0.3292	0.8317	1.6658

x	0.01	0.001	0.0001
$f(x)$	16.67	166.7	1667.0



$\lim_{x\to 0^+} \dfrac{x - \sin x}{x^4} = \infty$

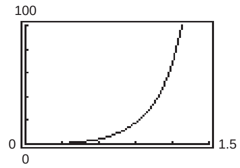
For $n > 3$, $\lim_{x\to 0^+} \dfrac{x - \sin x}{x^n} = \infty$.

67. (a) $\dfrac{7}{12}$ ft/sec (b) $\dfrac{3}{2}$ ft/sec (c) $\lim_{x\to 25^-} \dfrac{2x}{\sqrt{625-x^2}} = \infty$

69. (a) $A = 50 \tan\theta - 50\theta$; Domain: $(0, \pi/2)$

(b)

θ	0.3	0.6	0.9	1.2	1.5
$f(\theta)$	0.47	4.21	18.0	68.6	630.1



(c) $\lim_{\theta\to\pi/2} A = \infty$

71. False. Let $f(x) = (x^2-1)/(x-1)$.

73. False. Let $f(x) = \tan x$.

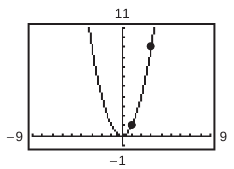
75. Let $f(x) = \dfrac{1}{x^2}$ and $g(x) = \dfrac{1}{x^4}$, and let $c = 0$. $\lim_{x\to 0} \dfrac{1}{x^2} = \infty$ and $\lim_{x\to 0} \dfrac{1}{x^4} = \infty$, but $\lim_{x\to 0}\left(\dfrac{1}{x^2} - \dfrac{1}{x^4}\right) = \lim_{x\to 0}\left(\dfrac{x^2-1}{x^4}\right) = -\infty \neq 0$.

77. Given $\lim_{x\to c} f(x) = \infty$, let $g(x) = 1$. Then $\lim_{x\to c} \dfrac{g(x)}{f(x)} = 0$ by Theorem 2.15.

79. Answers will vary.

Review Exercises for Chapter 2 *(page 111)*

1. Calculus Estimate: 8.3



3.

x	2.9	2.99	2.999	3
$f(x)$	−0.9091	−0.9901	−0.9990	?

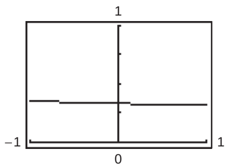
x	3.001	3.01	3.1
$f(x)$	−1.0010	−1.0101	−1.1111

$\lim_{x\to 0} \dfrac{x-3}{x^2-7x+12} \approx -1.0000$

5. (a) 4 (b) 5 **7.** 5; Proof **9.** −3; Proof **11.** 36

13. 16 **15.** $\frac{4}{3}$ **17.** $-\frac{1}{4}$ **19.** $\frac{1}{2}$ **21.** -1 **23.** 0
25. 1 **27.** $\sqrt{3}/2$ **29.** -3 **31.** -5

33.



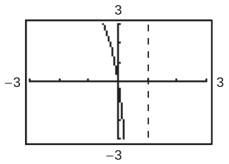
The graph has a hole at $x = 0$.

x	-0.1	-0.01	-0.001	0
$f(x)$	0.3352	0.3335	0.3334	?

x	0.001	0.01	0.1
$f(x)$	0.3333	0.3331	0.3315

$\lim_{x\to 0} \dfrac{\sqrt{2x+9}-3}{x} \approx 0.3333$; Actual limit is $\dfrac{1}{3}$.

35.



x	-0.1	-0.01	-0.001	0
$f(x)$	0.8867	0.0988	0.0100	?

x	0.001	0.01	0.1
$f(x)$	-0.0100	-0.1013	-1.1394

$\lim_{x\to 0} f(x) = 0$; Actual limit is 0.

37. -39.2 m/sec **39.** $\frac{1}{6}$ **41.** $\frac{1}{4}$ **43.** 3 **45.** 0

47. Limit does not exist. The limit as t approaches 1 from the left is 2, whereas the limit as t approaches 1 from the right is 1.

49. Continuous for all real x

51. Nonremovable discontinuity at $x = 5$

53. Nonremovable discontinuities at $x = -1$ and $x = 1$
Removable discontinuity at $x = 0$

55. $c = -\frac{1}{2}$ **57.** Continuous for all real x

59. Continuous on $[4, \infty)$

61. Continuous on $(k, k+1)$ for all integers k

63. Removable discontinuity at $x = 1$
Continuous on $(-\infty, 1) \cup (1, \infty)$

65. Proof **67.** (a) -4 (b) 4 (c) Limit does not exist.

69. $x = 0$ **71.** $x = \pm 3$ **73.** $x = \pm 8$

75. $x = \pm 5$ **77.** $-\infty$ **79.** $\frac{1}{3}$ **81.** $-\infty$

83. $\frac{4}{5}$ **85.** ∞ **87.** $-\infty$

89. (a) \$14,117.65 (b) \$80,000.00 (c) \$720,000.00
(d) ∞; No matter how much the company spends, the company will never be able to remove 100% of the pollutants.

P.S. Problem Solving *(page 113)*

1. (a) Perimeter $\triangle PAO = 1 + \sqrt{(x^2-1)^2 + x^2} + \sqrt{x^4 + x^2}$
Perimeter $\triangle PBO = 1 + \sqrt{x^4 + (x-1)^2} + \sqrt{x^4 + x^2}$

(b)

x	4	2	1
Perimeter $\triangle PAO$	33.0166	9.0777	3.4142
Perimeter $\triangle PBO$	33.7712	9.5952	3.4142
$r(x)$	0.9777	0.9461	1.0000

x	0.1	0.01
Perimeter $\triangle PAO$	2.0955	2.0100
Perimeter $\triangle PBO$	2.0006	2.0000
$r(x)$	1.0475	1.0050

1

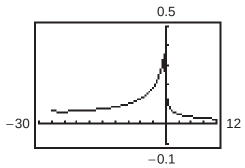
3. (a) Area (hexagon) $= (3\sqrt{3})/2 \approx 2.5981$
Area (circle) $= \pi \approx 3.1416$
Area (circle) $-$ Area (hexagon) ≈ 0.5435
(b) $A_n = (n/2)\sin(2\pi/n)$
(c)

n	6	12	24	48	96
A_n	2.5981	3.0000	3.1058	3.1326	3.1394

3.1416 or π

5. (a) $m = -\frac{12}{5}$ (b) $y = \frac{5}{12}x - \frac{169}{12}$
(c) $m_x = \dfrac{-\sqrt{169 - x^2} + 12}{x - 5}$
(d) $\frac{5}{12}$; It is the same as the slope of the tangent line found in (b).

7. (a) Domain: $[-27, 1) \cup (1, \infty)$
(b)

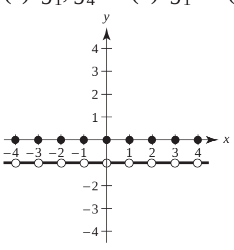


(c) $\frac{1}{14}$ (d) $\frac{1}{12}$

The graph has a hole at $x = 1$.

9. (a) g_1, g_4 (b) g_1 (c) g_1, g_3, g_4

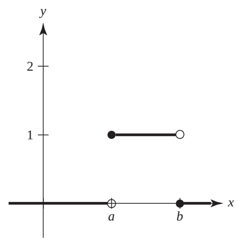
11.



The graph jumps at every integer.

(a) $f(1) = 0,\quad f(0) = 0,\quad f\left(\frac{1}{2}\right) = -1,\quad f(-2.7) = -1$
(b) $\lim_{x\to 1^-} f(x) = -1,\quad \lim_{x\to 1^+} f(x) = -1,\quad \lim_{x\to 1/2} f(x) = -1$
(c) There is a discontinuity at each integer.

13. (a)



(b) (i) $\lim_{x\to a^+} P_{a,b}(x) = 1$
(ii) $\lim_{x\to a^-} P_{a,b}(x) = 0$
(iii) $\lim_{x\to b^+} P_{a,b}(x) = 0$
(iv) $\lim_{x\to b^-} P_{a,b}(x) = 1$

(c) Continuous for all positive real numbers except a and b
(d) The area under the graph of U and above the x-axis is 1.