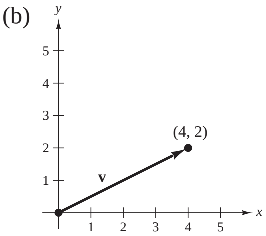
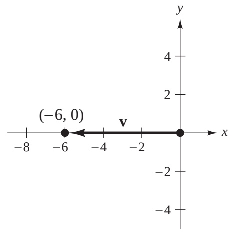
Chapter 11

Section 11.1 *(page 755)*

1. (a) $\langle 4, 2\rangle$
(b)

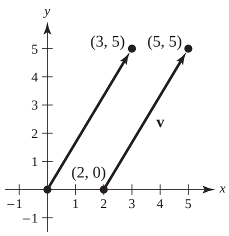


3. (a) $\langle -6, 0\rangle$
(b)



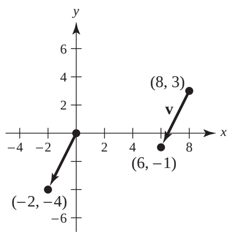
5. $\mathbf{u} = \mathbf{v} = \langle 2, 4\rangle$ **7.** $\mathbf{u} = \mathbf{v} = \langle 6, -5\rangle$

9. (a) and (d)



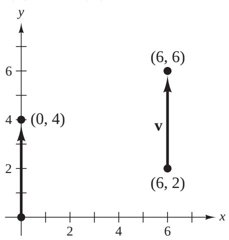
(b) $\langle 3, 5\rangle$
(c) $\mathbf{v} = 3\mathbf{i} + 5\mathbf{j}$

11. (a) and (d)



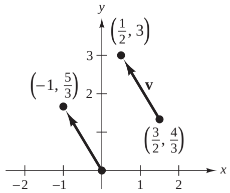
(b) $\langle -2, -4\rangle$
(c) $\mathbf{v} = -2\mathbf{i} - 4\mathbf{j}$

13. (a) and (d)



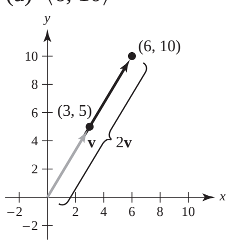
(b) $\langle 0, 4\rangle$ (c) $\mathbf{v} = 4\mathbf{j}$

15. (a) and (d)

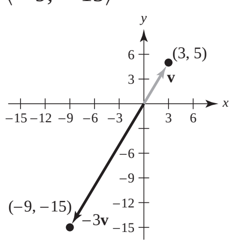


(b) $\left\langle -1, \frac{5}{3}\right\rangle$
(c) $\mathbf{v} = -\mathbf{i} + \frac{5}{3}\mathbf{j}$

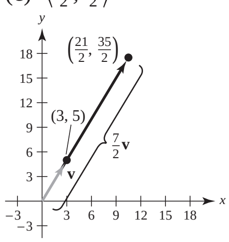
17. (a) $\langle 6, 10\rangle$



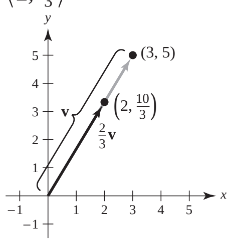
(b) $\langle -9, -15\rangle$



(c) $\left\langle \frac{21}{2}, \frac{35}{2}\right\rangle$

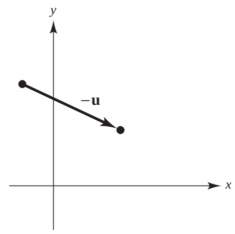


(d) $\left\langle 2, \frac{10}{3}\right\rangle$

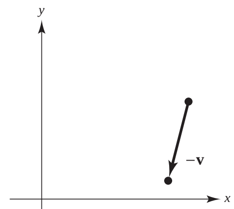


19. (a) $\left\langle \frac{8}{3}, 6\right\rangle$ (b) $\langle 6, -15\rangle$ (c) $\langle -2, -14\rangle$ (d) $\langle 18, -7\rangle$

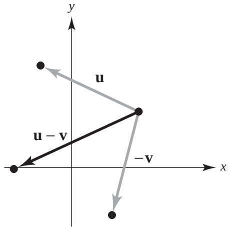
21.



23.



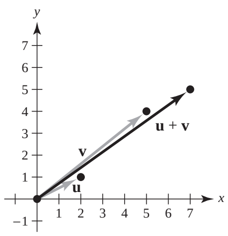
25.



27. $(3, 5)$ **29.** 7
31. 5 **33.** $\sqrt{61}$
35. $\left\langle \sqrt{17}/17, 4\sqrt{17}/17\right\rangle$
37. $\left\langle 3\sqrt{34}/34, 5\sqrt{34}/34\right\rangle$
39. (a) $\sqrt{2}$ (b) $\sqrt{5}$ (c) 1
(d) 1 (e) 1 (f) 1

41. (a) $\sqrt{5}/2$ (b) $\sqrt{13}$ (c) $\sqrt{85}/2$ (d) 1 (e) 1 (f) 1

43.



$\|\mathbf{u}\| + \|\mathbf{v}\| = \sqrt{5} + \sqrt{41}$ and $\|\mathbf{u} + \mathbf{v}\| = \sqrt{74}$
$\sqrt{74} \le \sqrt{5} + \sqrt{41}$

45. $\langle 0, 6\rangle$ **47.** $\left\langle -\sqrt{5}, 2\sqrt{5}\right\rangle$ **49.** $\langle 3, 0\rangle$

51. $\left\langle -\sqrt{3}, 1\right\rangle$ **53.** $\left\langle \dfrac{2 + 3\sqrt{2}}{2}, \dfrac{3\sqrt{2}}{2}\right\rangle$

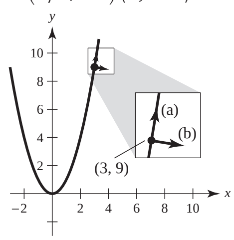
55. $\langle 2\cos 4 + \cos 2, 2\sin 4 + \sin 2\rangle$

57. Answers will vary. Example: A scalar is a single real number, such as 2. A vector is a line segment having both direction and magnitude. The vector $\left\langle \sqrt{3}, 1\right\rangle$, given in component form, has a direction of $\pi/6$ and a magnitude of 2.

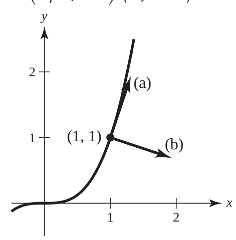
59. $(-4, -1), (6, 5), (10, 3)$ **61.** $a = 1, b = 1$

63. $a = 1, b = 2$ **65.** $a = \frac{2}{3}, b = \frac{1}{3}$

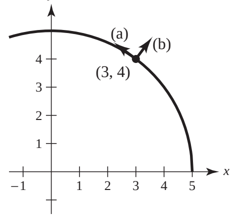
67. (a) $\pm\left(1/\sqrt{37}\right)\langle 1, 6\rangle$
(b) $\pm\left(1/\sqrt{37}\right)\langle 6, -1\rangle$



69. (a) $\pm\left(1/\sqrt{10}\right)\langle 1, 3\rangle$
(b) $\pm\left(1/\sqrt{10}\right)\langle 3, -1\rangle$



71. (a) $\pm\frac{1}{5}\langle -4, 3\rangle$
(b) $\pm\frac{1}{5}\langle 3, 4\rangle$

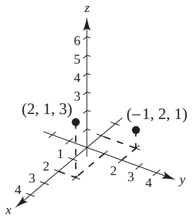


73. $\left\langle -\sqrt{2}/2, \sqrt{2}/2\right\rangle$
75. 10.7°, 584.6 lb
77. 71.3°, 228.5 lb
79. (a) $\theta = 0°$ (b) $\theta = 180°$
(c) No, the resultant can only be less than or equal to the sum.
81. Horizontal: 1193.43 ft/sec
Vertical: 125.43 ft/sec
83. 38.3° north of west
882.9 km/h

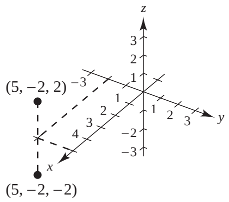
85. True **87.** True **89.** False. $\|a\mathbf{i} + b\mathbf{j}\| = \sqrt{2}|a|$
91–93. Proofs **95.** $x^2 + y^2 = 25$

Section 11.2 *(page 763)*

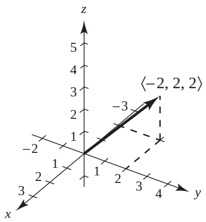
1.



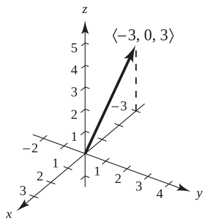
3.



5. $(-3, 4, 5)$ **7.** $(12, 0, 0)$ **9.** 0
11. Six units above the xy-plane
13. Three units behind the yz-plane
15. To the left of the xz-plane
17. Within three units of the xz-plane
19. Three units below the xy-plane, and below either quadrant I or quadrant III
21. Above the xy-plane and above quadrants II or IV, *or* below the xy-plane and below quadrants I or III
23. $\sqrt{69}$ **25.** $\sqrt{61}$ **27.** $7, 7\sqrt{5}, 14$; Right triangle
29. $\sqrt{41}, \sqrt{41}, \sqrt{14}$; Isosceles triangle
31. $(0, 0, 9), (2, 6, 12), (6, 4, -3)$ **33.** $(2, 6, 3)$
35. $\left(\frac{3}{2}, -3, 5\right)$ **37.** $(x-0)^2 + (y-2)^2 + (z-5)^2 = 4$
39. $(x-1)^2 + (y-3)^2 + (z-0)^2 = 10$
41. $(x-1)^2 + (y+3)^2 + (z+4)^2 = 25$
Center: $(1, -3, -4)$ Radius: 5
43. $\left(x - \frac{1}{3}\right)^2 + (y+1)^2 + z^2 = 1$
Center: $\left(\frac{1}{3}, -1, 0\right)$ Radius: 1
45. (a) $\langle -2, 2, 2\rangle$
(b) $\mathbf{v} = -2\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$
(c)

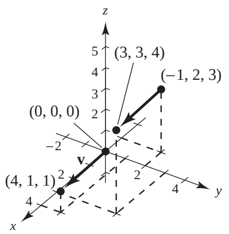


47. (a) $\langle -3, 0, 3\rangle$
(b) $\mathbf{v} = -3\mathbf{i} + 3\mathbf{k}$
(c)



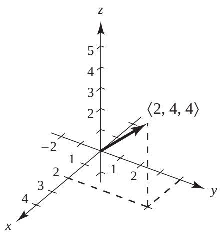
49. $\mathbf{v} = \langle 1, -1, 6\rangle$
$\|\mathbf{v}\| = \sqrt{38}$
$\mathbf{u} = \dfrac{1}{\sqrt{38}}\langle 1, -1, 6\rangle$

51. (a) and (d)

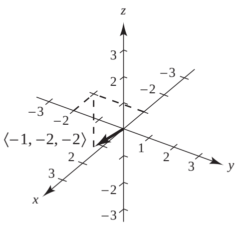


(b) $\langle 4, 1, 1\rangle$
(c) $\mathbf{v} = 4\mathbf{i} + \mathbf{j} + \mathbf{k}$

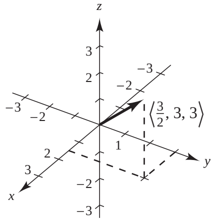
53. $(3, 1, 8)$
55. (a)



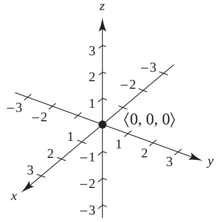
(b)



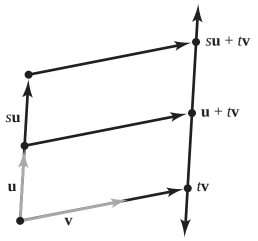
(c)



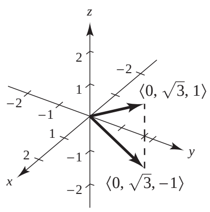
(d)



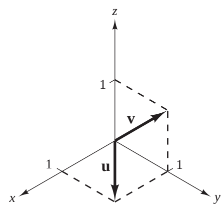
57. $\langle 7, 0, -4\rangle$ **59.** $\left\langle \frac{7}{2}, 3, \frac{5}{2}\right\rangle$ **61.** a and b
63. a **65.** Collinear **67.** Not collinear
69. $\overrightarrow{AB} = \langle 1, 2, 3\rangle, \overrightarrow{CD} = \langle 1, 2, 3\rangle, \overrightarrow{BD} = \langle -2, 1, 1\rangle,$
$\overrightarrow{AC} = \langle -2, 1, 1\rangle$; Because $\overrightarrow{AB} = \overrightarrow{CD}$ and $\overrightarrow{BD} = \overrightarrow{AC}$, the given points form the vertices of a parallelogram.
71. 0 **73.** $\sqrt{34}$ **75.** $\sqrt{14}$
77. (a) $\frac{1}{3}\langle 2, -1, 2\rangle$ (b) $-\frac{1}{3}\langle 2, -1, 2\rangle$
79. (a) $\dfrac{2\sqrt{2}}{5}\mathbf{i} - \dfrac{\sqrt{2}}{2}\mathbf{j} + \dfrac{3\sqrt{2}}{10}\mathbf{k}$ (b) $-\dfrac{2\sqrt{2}}{5}\mathbf{i} + \dfrac{\sqrt{2}}{2}\mathbf{j} - \dfrac{3\sqrt{2}}{10}\mathbf{k}$
81. The terminal points of the vectors $t\mathbf{u}$, $\mathbf{u} + t\mathbf{v}$, and $s\mathbf{u} + t\mathbf{v}$ are collinear.



83. $\left\langle 0, 10/\sqrt{2}, 10/\sqrt{2}\right\rangle$
85. $\left\langle 1, -1, \frac{1}{2}\right\rangle$
87.



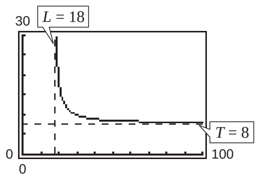
$\langle 0, \sqrt{3}, \pm 1\rangle$ **89.** $(2, -1, 2)$
91. (a)



(b) $a = 0, a + b = 0, b = 0$
(c) $a = 1, a + b = 2, b = 1$
(d) Not possible
93. x_0 is directed distance to yz-plane.
y_0 is directed distance to xz-plane.
z_0 is directed distance to xy-plane.
95. $(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$ **97.** **0**
99. $\left(\sqrt{3}/3\right)\langle 1, 1, 1\rangle$
101. (a) $T = 8L/\sqrt{L^2 - 18^2}, L > 18$
(b)

L	20	25	30	35	40	45	50
T	18.4	11.5	10	9.3	9.0	8.7	8.6

(c)



(d) Proof (e) 30 in.

103. Tension in cable AB: 202.919 N
Tension in cable AC: 157.909 N
Tension in cable AD: 226.521 N

105. $\left(x - \frac{4}{3}\right)^2 + (y - 3)^2 + \left(z + \frac{1}{3}\right)^2 = \frac{44}{9}$
Sphere; center: $\left(\frac{4}{3}, 3, -\frac{1}{3}\right)$, radius: $\frac{2\sqrt{11}}{3}$

Section 11.3 *(page 773)*

1. (a) 17 (b) 25 (c) 25 (d) $\langle -17, 85\rangle$ (e) 34
3. (a) -26 (b) 52 (c) 52 (d) $\langle 78, -52\rangle$ (e) -52
5. (a) 2 (b) 29 (c) 29 (d) $\langle 0, 12, 10\rangle$ (e) 4
7. (a) 1 (b) 6 (c) 6 (d) $\mathbf{i} - \mathbf{k}$ (e) 2
9. (a) $\pi/2$ (b) $90°$ **11.** (a) 1.7127 (b) $98.1°$
13. (a) 1.0799 (b) $61.9°$ **15.** (a) 2.0306 (b) $116.3°$
17. 20 **19.** Orthogonal **21.** Neither **23.** Orthogonal
25. Right triangle; answers will vary.
27. Acute triangle; answers will vary.

29. $\cos\alpha = \frac{1}{3}, \alpha \approx 70.5°$
$\cos\beta = \frac{2}{3}, \beta \approx 48.2°$
$\cos\gamma = \frac{2}{3}, \gamma \approx 48.2°$

31. $\cos\alpha = \frac{3}{\sqrt{17}}, \alpha \approx 43.3°$
$\cos\beta = \frac{2}{\sqrt{17}}, \beta \approx 61.0°$
$\cos\gamma = -\frac{2}{\sqrt{17}}, \gamma \approx 119.0°$

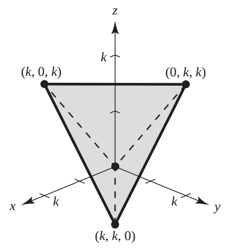
33. $\cos\alpha = 0, \alpha \approx 90°$
$\cos\beta = 3/\sqrt{13}, \beta \approx 33.7°$
$\cos\gamma = -2/\sqrt{13}, \gamma \approx 123.7°$

35. (a) $\langle 2, 8\rangle$ (b) $\langle 4, -1\rangle$ **37.** (a) $\left\langle \frac{5}{2}, \frac{1}{2}\right\rangle$ (b) $\left\langle -\frac{1}{2}, \frac{5}{2}\right\rangle$
39. (a) $\langle -2, 2, 2\rangle$ (b) $\langle 2, 1, 1\rangle$
41. (a) $\left\langle 0, \frac{33}{25}, \frac{44}{25}\right\rangle$ (b) $\left\langle 2, -\frac{8}{25}, \frac{6}{25}\right\rangle$
43. See "Definition of Dot Product" on page 766.
45. (a) and (b) are defined. (c) and (d) are not defined because it is not possible to find the dot product of a scalar and a vector or to add a scalar to a vector.
47. See Figure 11.29 on page 770.
49. Yes.

$$\left\|\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v}\right\| = \left\|\frac{\mathbf{v}\cdot\mathbf{u}}{\|\mathbf{u}\|^2}\mathbf{u}\right\|$$
$$|\mathbf{u}\cdot\mathbf{v}|\frac{\|\mathbf{v}\|}{\|\mathbf{v}\|^2} = |\mathbf{v}\cdot\mathbf{u}|\frac{\|\mathbf{u}\|}{\|\mathbf{u}\|^2}$$
$$\frac{1}{\|\mathbf{v}\|} = \frac{1}{\|\mathbf{u}\|}$$
$$\|\mathbf{u}\| = \|\mathbf{v}\|$$

51. \$17,490.25; Total revenue
53. Answers will vary. Example: $\langle 12, 2\rangle$ and $\langle -12, -2\rangle$
55. Answers will vary. Example: $\langle 2, 0, 3\rangle$ and $\langle -2, 0, -3\rangle$
57. $\arccos\left(1/\sqrt{3}\right) \approx 54.7°$
59. (a) 8335.1 lb (b) 47,270.8 lb
61. 425 ft-lb **63.** 2900.2 km-N
65. False. For example, $\langle 1, 1\rangle \cdot \langle 2, 3\rangle = 5$ and $\langle 1, 1\rangle \cdot \langle 1, 4\rangle = 5$, but $\langle 2, 3\rangle \neq \langle 1, 4\rangle$.
67. (a) $(0, 0), (1, 1)$
(b) To $y = x^2$ at $(1, 1)$: $\left\langle \pm\sqrt{5}/5, \pm 2\sqrt{5}/5\right\rangle$
To $y = x^{1/3}$ at $(1, 1)$: $\left\langle \pm 3\sqrt{10}/10, \pm\sqrt{10}/10\right\rangle$
To $y = x^2$ at $(0, 0)$: $\langle \pm 1, 0\rangle$
To $y = x^{1/3}$ at $(0, 0)$: $\langle 0, \pm 1\rangle$
(c) At $(1, 1)$: $\theta = 45°$
At $(0, 0)$: $\theta = 90°$

69. (a) $(-1, 0), (1, 0)$
(b) To $y = 1 - x^2$ at $(1, 0)$: $\left\langle \pm\sqrt{5}/5, \mp 2\sqrt{5}/5\right\rangle$
To $y = x^2 - 1$ at $(1, 0)$: $\left\langle \pm\sqrt{5}/5, \pm 2\sqrt{5}/5\right\rangle$
To $y = 1 - x^2$ at $(-1, 0)$: $\left\langle \pm\sqrt{5}/5, \pm 2\sqrt{5}/5\right\rangle$
To $y = x^2 - 1$ at $(-1, 0)$: $\left\langle \pm\sqrt{5}/5, \mp 2\sqrt{5}/5\right\rangle$
(c) At $(1, 0)$: $\theta = 53.13°$
At $(-1, 0)$: $\theta = 53.13°$
71. Proof
73. (a)

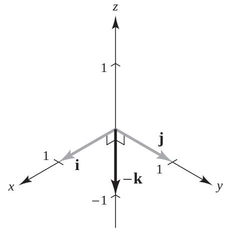


(b) $k\sqrt{2}$ (c) $60°$ (d) $109.5°$

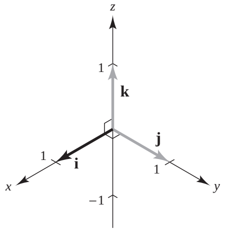
75–77. Proofs

Section 11.4 *(page 781)*

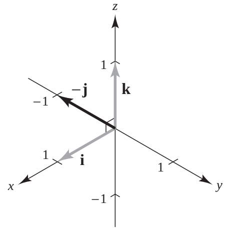
1. $-\mathbf{k}$



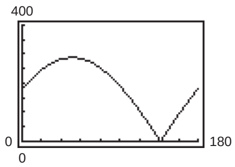
3. $\mathbf{i}$



5. $-\mathbf{j}$



7. (a) $20\mathbf{i} + 10\mathbf{j} - 16\mathbf{k}$ (b) $-20\mathbf{i} - 10\mathbf{j} + 16\mathbf{k}$ (c) $\mathbf{0}$
9. (a) $17\mathbf{i} - 33\mathbf{j} - 10\mathbf{k}$ (b) $-17\mathbf{i} + 33\mathbf{j} + 10\mathbf{k}$ (c) $\mathbf{0}$
11. $\langle 0, 0, 54\rangle$ **13.** $\langle -1, -1, -1\rangle$ **15.** $\langle -2, 3, -1\rangle$
17. $\left\langle -\frac{7}{9\sqrt{3}}, -\frac{5}{9\sqrt{3}}, \frac{13}{9\sqrt{3}}\right\rangle$ or $\left\langle \frac{7}{9\sqrt{3}}, \frac{5}{9\sqrt{3}}, -\frac{13}{9\sqrt{3}}\right\rangle$
19. $\left\langle \frac{3}{\sqrt{59}}, \frac{7}{\sqrt{59}}, \frac{1}{\sqrt{59}}\right\rangle$ or $\left\langle -\frac{3}{\sqrt{59}}, -\frac{7}{\sqrt{59}}, -\frac{1}{\sqrt{59}}\right\rangle$
21. 1 **23.** $6\sqrt{5}$ **25.** $9\sqrt{5}$ **27.** $\frac{11}{2}$
29. $10\cos 40° \approx 7.66$ ft-lb
31. (a) $\mathbf{F} = -180(\cos\theta\mathbf{j} + \sin\theta\mathbf{k})$
(b) $\|\overrightarrow{AB} \times \mathbf{F}\| = |225\sin\theta + 180\cos\theta|$
(c) $\|\overrightarrow{AB} \times \mathbf{F}\| = 225(1/2) + 180(\sqrt{3}/2) \approx 268.38$
(d) $\theta = 141.34°$
$\overrightarrow{AB}$ and $\mathbf{F}$ are perpendicular.
(e)



From part (d), the zero is $\theta \approx 141.34°$, when the vectors are parallel.

33. 1 **35.** 6 **37.** 2 **39.** 75
41. (a) = (b) = (c) = (h) and (e) = (f) = (g)
43. See "Definition of Cross Product of Two Vectors in Space" on page 775.

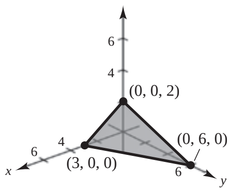
45. The magnitude of the cross product will increase by a factor of 4.
47. False. The cross product of two vectors is not defined in a two-dimensional coordinate system.
49. False. Let $\mathbf{u} = \langle 1, 0, 0 \rangle$, $\mathbf{v} = \langle 1, 0, 0 \rangle$, and $\mathbf{w} = \langle -1, 0, 0 \rangle$. Then $\mathbf{u} \times \mathbf{v} = \mathbf{u} \times \mathbf{w} = \mathbf{0}$, but $\mathbf{v} \neq \mathbf{w}$.
51–59. Proofs

Section 11.5 *(page 790)*

1. (a) Yes (b) No

	Parametric Equations (a)	Symmetric Equations (b)	Direction Numbers
3.	$x = 3t$, $y = t$, $z = 5t$	$\frac{x}{3} = y = \frac{z}{5}$	3, 1, 5
5.	$x = -2 + 2t$, $y = 4t$, $z = 3 - 2t$	$\frac{x+2}{2} = \frac{y}{4} = \frac{z-3}{-2}$	2, 4, -2
7.	$x = 1 + 3t$, $y = -2t$, $z = 1 + t$	$\frac{x-1}{3} = \frac{y}{-2} = \frac{z-1}{1}$	3, -2, 1
9.	$x = 5 + 17t$, $y = -3 - 11t$, $z = -2 - 9t$	$\frac{x-5}{17} = \frac{y+3}{-11} = \frac{z+2}{-9}$	17, -11, -9
11.	$x = 7 - 10t$, $y = -2 + 2t$, $z = 6$	Not possible	-10, 2, 0

13. $x = 2$, $y = 3$, $z = 4 + t$ **15.** $x = 2 + 3t$, $y = 3 + 2t$, $z = 4 - t$ **17.** $x = 5 + 2t$, $y = -3 - t$, $z = -4 + 3t$
19. $x = 2 - t$, $y = 1 + t$, $z = 2 + t$ **21.** $P(3, -1, -2)$; $\mathbf{v} = \langle -1, 2, 0 \rangle$ **23.** $P(7, -6, -2)$; $\mathbf{v} = \langle 4, 2, 1 \rangle$
25. $L_1 = L_2$ and is parallel to L_3. **27.** L_1 and L_3 are identical.
29. $(2, 3, 1)$; $\cos\theta = 7\sqrt{17}/51$ **31.** Not intersecting
33. (a) Yes (b) Yes **35.** $y - 3 = 0$
37. $2x + 3y - z = 10$ **39.** $2x - y - 2z + 6 = 0$
41. $3x - 19y - 2z = 0$ **43.** $4x - 3y + 4z = 10$
45. $z = 3$ **47.** $x + y + z = 5$ **49.** $7x + y - 11z = 5$
51. $y - z = -1$ **53.** $x - z = 0$
55. $9x - 3y + 2z - 21 = 0$ **57.** Orthogonal
59. Neither; 83.5° **61.** Parallel

63.


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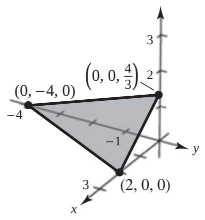
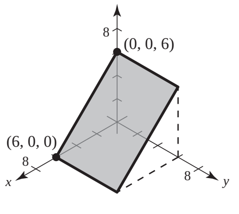
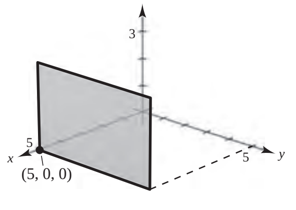
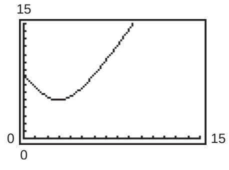


67.


69.


71. P_1 and P_2 are parallel. **73.** $P_1 = P_4$ and is parallel to P_2.
75. (a) $\theta \approx 65.91°$
(b) $x = 2$
$y = 1 + t$
$z = 1 + 2t$
77. $(2, -3, 2)$; The line does not lie in the plane.
79. Not intersecting **81.** $6\sqrt{14}/7$ **83.** $11\sqrt{6}/6$
85. $2\sqrt{26}/13$ **87.** $27\sqrt{94}/188$ **89.** $\sqrt{2533}/17$
91. $7\sqrt{3}/3$ **93.** $\sqrt{66}/3$
95. Parametric equations: $x = x_1 + at$, $y = y_1 + bt$, and $z = z_1 + ct$
Symmetric equations: $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$
You need a vector $\mathbf{v} = \langle a, b, c \rangle$ parallel to the line and a point $P(x_1, y_1, z_1)$ on the line.
97. Simultaneously solve the two linear equations representing the planes and substitute the values back into one of the original equations. Then choose a value for t and form the corresponding parametric equations for the line of intersection.
99. Yes. If $\mathbf{v}_1$ and $\mathbf{v}_2$ are the direction vectors for the lines L_1 and L_2, then $\mathbf{v} = \mathbf{v}_1 \times \mathbf{v}_2$ is perpendicular to both L_1 and L_2.
101. (a)

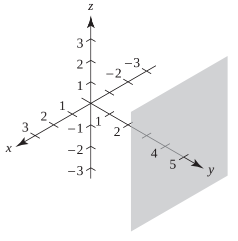
Year	2005	2006	2007	2008	2009	2010
z (approx.)	16.39	17.98	19.78	20.87	19.94	21.04

The approximations are close to the actual values.
(b) An increase
103. (a) $\sqrt{70}$ in.
(b)


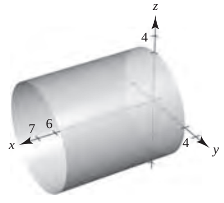
(c) The distance is never zero.
(d) 5 in.
105. $\left(\frac{77}{13}, \frac{48}{13}, -\frac{23}{13}\right)$ **107.** $\left(-\frac{1}{2}, -\frac{9}{4}, \frac{1}{4}\right)$ **109.** True **111.** True
113. False. Plane $7x + y - 11z = 5$ and plane $5x + 2y - 4z = 1$ are both perpendicular to plane $2x - 3y + z = 3$ but are not parallel.

Section 11.6 *(page 802)*

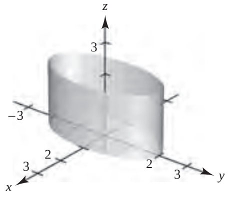
1. c **2.** e **3.** f **4.** b **5.** d **6.** a
7. Plane



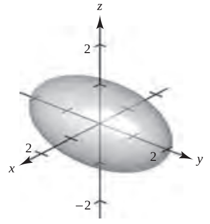
9. Right circular cylinder



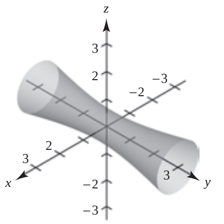
11. Elliptic cylinder



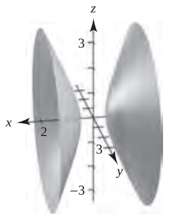
13. Ellipsoid



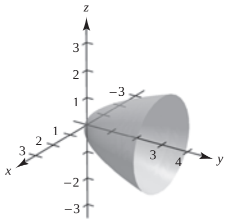
15. Hyperboloid of one sheet



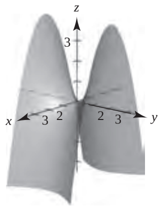
17. Hyperboloid of two sheets



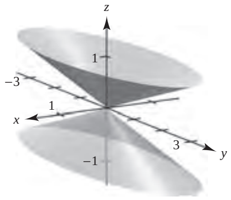
19. Elliptic paraboloid



21. Hyperbolic paraboloid



23. Elliptic cone



25. Let C be a curve in a plane and let L be a line not in a parallel plane. The set of all lines parallel to L and intersecting C is called a cylinder. C is called the generating curve of the cylinder, and the parallel lines are called rulings.

27. See pages 796 and 797.

29. xy-plane: ellipse; three-space: hyperboloid of one sheet

31. $x^2 + z^2 = 4y$ **33.** $4x^2 + 4y^2 = z^2$

35. $y^2 + z^2 = 4/x^2$ **37.** $y = \sqrt{2z}$ $\left(\text{or } x = \sqrt{2z}\right)$ **39.** $128\pi/3$

41. (a) Major axis: $4\sqrt{2}$ (b) Major axis: $8\sqrt{2}$
Minor axis: 4 Minor axis: 8
Foci: $(0, \pm 2, 2)$ Foci: $(0, \pm 4, 8)$

43. $x^2 + z^2 = 8y$; Elliptic paraboloid

45. $x^2/3963^2 + y^2/3963^2 + z^2/3950^2 = 1$

47. $x = at, y = -bt, z = 0$;
$x = at, y = bt + ab^2, z = 2abt + a^2b^2$

49. True **51.** False. A trace of an ellipsoid can be a single point.

53. The Klein bottle does not have both an "inside" and an "outside." It is formed by inserting the small open end through the side of the bottle and making it contiguous with the top of the bottle.

Section 11.7 *(page 809)*

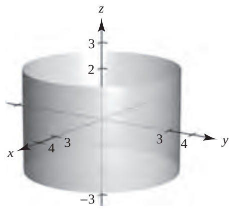
1. $(-7, 0, 5)$ **3.** $\left(3\sqrt{2}/2, 3\sqrt{2}/2, 1\right)$ **5.** $\left(-2\sqrt{3}, -2, 3\right)$

7. $(5, \pi/2, 1)$ **9.** $\left(2\sqrt{2}, -\pi/4, -4\right)$ **11.** $(2, \pi/3, 4)$

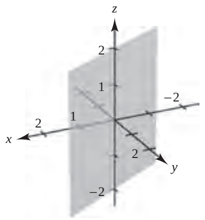
13. $z = 4$ **15.** $r^2 + z^2 = 17$ **17.** $r = \sec\theta\tan\theta$

19. $r^2\sin^2\theta = 10 - z^2$

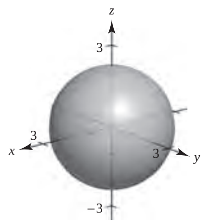
21. $x^2 + y^2 = 9$



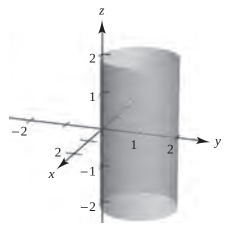
23. $x - \sqrt{3}y = 0$



25. $x^2 + y^2 + z^2 = 5$



27. $x^2 + y^2 - 2y = 0$



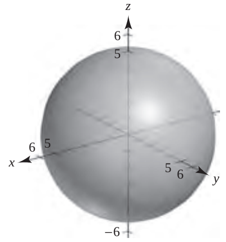
29. $(4, 0, \pi/2)$ **31.** $\left(4\sqrt{2}, 2\pi/3, \pi/4\right)$

33. $(4, \pi/6, \pi/6)$ **35.** $\left(\sqrt{6}, \sqrt{2}, 2\sqrt{2}\right)$ **37.** $(0, 0, 12)$

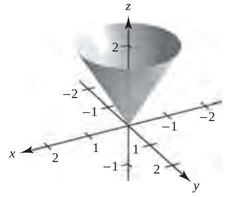
39. $\left(\frac{5}{2}, \frac{5}{2}, -5\sqrt{2}/2\right)$ **41.** $\rho = 2\csc\phi\csc\theta$ **43.** $\rho = 7$

45. $\rho = 4\csc\phi$ **47.** $\tan^2\phi = 2$

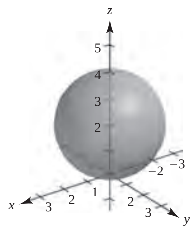
49. $x^2 + y^2 + z^2 = 25$



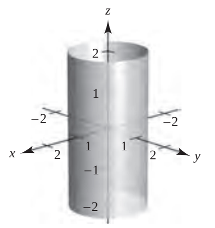
51. $3x^2 + 3y^2 - z^2 = 0$



53. $x^2 + y^2 + (z - 2)^2 = 4$



55. $x^2 + y^2 = 1$



57. d **58.** e **59.** c **60.** a **61.** f **62.** b

63. $(4, \pi/4, \pi/2)$ **65.** $\left(4\sqrt{2}, \pi/2, \pi/4\right)$

67. $\left(2\sqrt{13}, -\pi/6, \arccos[3/\sqrt{13}]\right)$

69. $(13, \pi, \arccos[5/13])$ **71.** $(10, \pi/6, 0)$ **73.** $(36, \pi, 0)$

75. $\left(3\sqrt{3}, -\pi/6, 3\right)$ **77.** $\left(4, 7\pi/6, 4\sqrt{3}\right)$

79. Rectangular to cylindrical:
$r^2 = x^2 + y^2, \tan\theta = y/x, z = z$
Cylindrical to rectangular:
$x = r\cos\theta, y = r\sin\theta, z = z$

81. Rectangular to spherical:
$\rho^2 = x^2 + y^2 + z^2, \tan\theta = y/x, \phi = \arccos\left(z/\sqrt{x^2 + y^2 + z^2}\right)$
Spherical to rectangular:
$x = \rho\sin\phi\cos\theta, y = \rho\sin\phi\sin\theta, z = \rho\cos\phi$

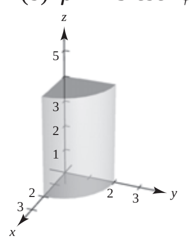
83. (a) $r^2 + z^2 = 25$ (b) $\rho = 5$

85. (a) $r^2 + (z - 1)^2 = 1$ (b) $\rho = 2\cos\phi$

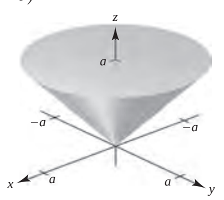
87. (a) $r = 4\sin\theta$ (b) $\rho = 4\sin\theta/\sin\phi = 4\sin\theta\csc\phi$

89. (a) $r^2 = 9/(\cos^2\theta - \sin^2\theta)$
(b) $\rho^2 = 9\csc^2\phi/(\cos^2\theta - \sin^2\theta)$

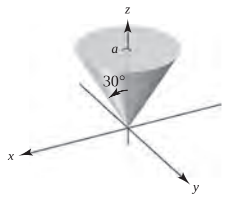
91.



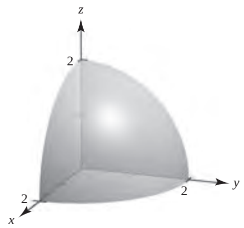
93.



95.



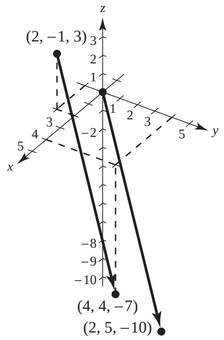
97.



99. Rectangular: $0 \le x \le 10$; $0 \le y \le 10$; $0 \le z \le 10$
101. Spherical: $4 \le \rho \le 6$
103. Cylindrical: $r^2 + z^2 \le 9$, $r \le 3\cos\theta$, $0 \le \theta \le \pi$
105. False. $r = z$ represents a cone.
107. False. See page 805. **109.** Ellipse

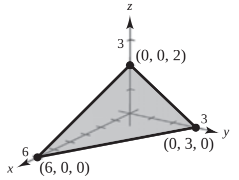
Review Exercises for Chapter 11 *(page 811)*

1. (a) $\mathbf{u} = \langle 3, -1\rangle$, $\mathbf{v} = \langle 4, 2\rangle$ (b) $\mathbf{u} = 3\mathbf{i} - \mathbf{j}$, $\mathbf{v} = 4\mathbf{i} + 2\mathbf{j}$
(c) $\|\mathbf{u}\| = \sqrt{10}$, $\|\mathbf{v}\| = 2\sqrt{5}$ (d) $10\mathbf{i}$
3. $\mathbf{v} = \langle 4, 4\sqrt{3}\rangle$ **5.** $(-5, 4, 0)$ **7.** $\sqrt{22}$
9. $(x-3)^2 + (y+2)^2 + (z-6)^2 = \frac{225}{4}$
11. $(x-2)^2 + (y-3)^2 + z^2 = 9$; Center: $(2, 3, 0)$; Radius: 3
13. (a) and (d)

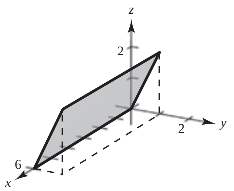


(b) $\mathbf{u} = \langle 2, 5, -10\rangle$
(c) $\mathbf{u} = 2\mathbf{i} + 5\mathbf{j} - 10\mathbf{k}$

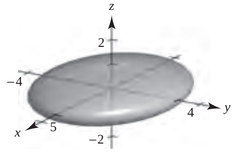
15. Collinear
17. $(1/\sqrt{38})\langle 2, 3, 5\rangle$
19. (a) $\mathbf{u} = \langle -1, 4, 0\rangle$
$\mathbf{v} = \langle -3, 0, 6\rangle$
(b) 3 (c) 45
21. (a) $\dfrac{\pi}{12}$ (b) 15°
23. (a) π (b) 180°
25. Orthogonal
27. $\langle 2, 10\rangle$
29. $\langle 1, 0, 1\rangle$
31. Answers will vary. Example: $\langle -6, 5, 0\rangle$, $\langle 6, -5, 0\rangle$
33. (a) $-9\mathbf{i} + 26\mathbf{j} - 7\mathbf{k}$ (b) $9\mathbf{i} - 26\mathbf{j} + 7\mathbf{k}$ (c) $\mathbf{0}$
35. (a) $-8\mathbf{i} - 10\mathbf{j} + 6\mathbf{k}$ (b) $8\mathbf{i} + 10\mathbf{j} - 6\mathbf{k}$ (c) $\mathbf{0}$
37. $\left\langle \dfrac{8}{\sqrt{377}}, \dfrac{12}{\sqrt{377}}, \dfrac{13}{\sqrt{377}}\right\rangle$ or $\left\langle -\dfrac{8}{\sqrt{377}}, -\dfrac{12}{\sqrt{377}}, -\dfrac{13}{\sqrt{377}}\right\rangle$
39. $100 \sec 20° \approx 106.4$ lb
41. (a) $x = 3 + 6t$, $y = 11t$, $z = 2 + 4t$
(b) $(x-3)/6 = y/11 = (z-2)/4$
43. $x = 1$, $y = 2 + t$, $z = 3$ **45.** $x = t$, $y = -1 + t$, $z = 1$
47. $27x + 4y + 32z + 33 = 0$ **49.** $x + 2y = 1$ **51.** $\frac{8}{7}$
53. $\sqrt{35}/7$
55. Plane



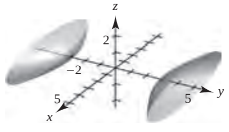
57. Plane



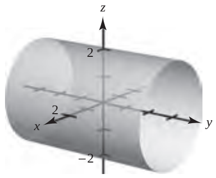
59. Ellipsoid



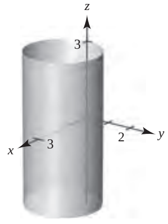
61. Hyperboloid of two sheets



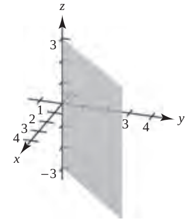
63. Cylinder



65. $x^2 + z^2 = 2y$
67. (a) $(4, 3\pi/4, 2)$ (b) $\left(2\sqrt{5}, 3\pi/4, \arccos\left[\sqrt{5}/5\right]\right)$
69. $\left(50\sqrt{5}, -\pi/6, \arccos\left[1/\sqrt{5}\right]\right)$
71. $\left(25\sqrt{2}/2, -\pi/4, -25\sqrt{2}/2\right)$
73. (a) $r^2 \cos 2\theta = 2z$ (b) $\rho = 2\sec 2\theta \cos\phi \csc^2\phi$
75. $\left(x - \frac{5}{2}\right)^2 + y^2 = \frac{25}{4}$

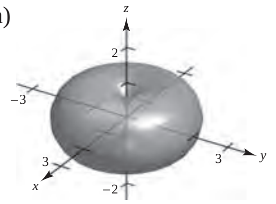


77. $x = y$

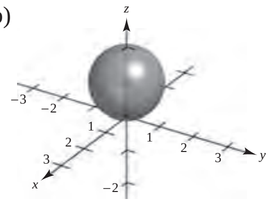


P.S. Problem Solving *(page 813)*

1–3. Proofs **5.** (a) $3\sqrt{2}/2 \approx 2.12$ (b) $\sqrt{5} \approx 2.24$
7. (a) $\pi/2$ (b) $\frac{1}{2}(\pi abk)k$
(c) $V = \frac{1}{2}(\pi ab)k^2$
$V = \frac{1}{2}$(area of base)height
9. Proof
11. (a)



(b)

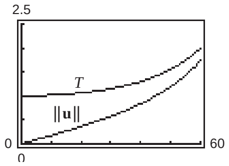


13. (a) Tension: $2\sqrt{3}/3 \approx 1.1547$ lb
Magnitude of $\mathbf{u}$: $\sqrt{3}/3 \approx 0.5774$ lb
(b) $T = \sec\theta$; $\|\mathbf{u}\| = \tan\theta$; Domain: $0° \le \theta \le 90°$
(c)

θ	0°	10°	20°	30°
T	1	1.0154	1.0642	1.1547
$\|\mathbf{u}\|$	0	0.1763	0.3640	0.5774

θ	40°	50°	60°
T	1.3054	1.5557	2
$\|\mathbf{u}\|$	0.8391	1.1918	1.7321

(d)



(e) Both are increasing functions
(f) $\lim_{\theta \to \pi/2^-} T = \infty$ and $\lim_{\theta \to \pi/2^-} \|\mathbf{u}\| = \infty$
Yes. As θ increases, both T and $\|\mathbf{u}\|$ increase.
15. $\langle 0, 0, \cos\alpha \sin\beta - \cos\beta \sin\alpha\rangle$; Proof
17. $D = \dfrac{|\overrightarrow{PQ} \cdot \mathbf{n}|}{\|\mathbf{n}\|} = \dfrac{|\mathbf{w} \cdot (\mathbf{u} \times \mathbf{v})|}{\|\mathbf{u} \times \mathbf{v}\|} = \dfrac{|(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}|}{\|\mathbf{u} \times \mathbf{v}\|} = \dfrac{|\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})|}{\|\mathbf{u} \times \mathbf{v}\|}$
19. Proof

Chapter 12

Section 12.1 *(page 821)*

1. $(-\infty, -1) \cup (-1, \infty)$ **3.** $(0, \infty)$

5. $[0, \infty)$ **7.** $(-\infty, \infty)$

9. (a) $\frac{1}{2}\mathbf{i}$ (b) $\mathbf{j}$ (c) $\frac{1}{2}(s+1)^2\mathbf{i} - s\mathbf{j}$
(d) $\frac{1}{2}\Delta t(\Delta t + 4)\mathbf{i} - \Delta t\mathbf{j}$

11. (a) $\ln 2\mathbf{i} + \dfrac{1}{2}\mathbf{j} + 6\mathbf{k}$ (b) Not possible
(c) $\ln(t-4)\mathbf{i} + \dfrac{1}{t-4}\mathbf{j} + 3(t-4)\mathbf{k}$
(d) $\ln(1+\Delta t)\mathbf{i} - \dfrac{\Delta t}{1+\Delta t}\mathbf{j} + 3\Delta t\mathbf{k}$

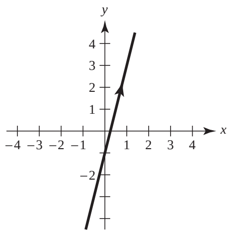
13. $\mathbf{r}(t) = 3t\mathbf{i} + t\mathbf{j} + 2t\mathbf{k}, \quad 0 \le t \le 1$
$x = 3t,\ y = t,\ z = 2t, \quad 0 \le t \le 1$

15. $\mathbf{r}(t) = (-2+t)\mathbf{i} + (5-t)\mathbf{j} + (-3+12t)\mathbf{k}, \quad 0 \le t \le 1$
$x = -2+t,\ y = 5-t,\ z = -3+12t, \quad 0 \le t \le 1$

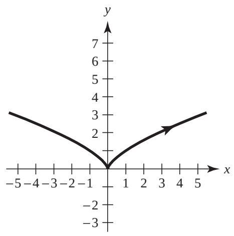
17. $t^2(5t-1)$; No, the dot product is a scalar.

19. b **20.** c **21.** d **22.** a

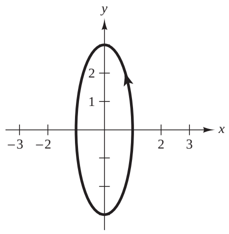
23.



25.



27.



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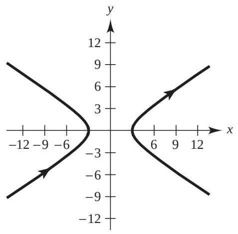

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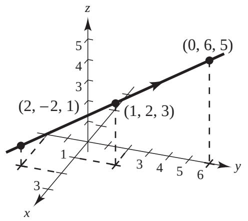

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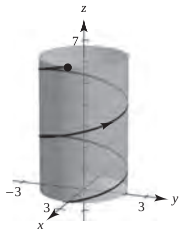

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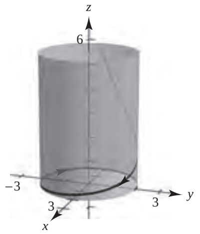

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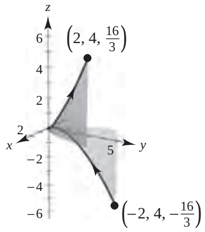
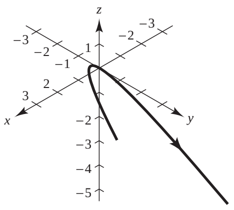
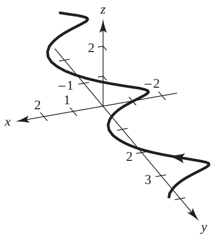


39.



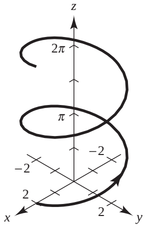
Parabola

41.



Helix

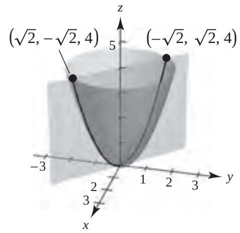
43.



(a) The helix is translated two units back on the x-axis.
(b) The height of the helix increases at a greater rate.
(c) The orientation of the graph is reversed.
(d) The axis of the helix is the x-axis.
(e) The radius of the helix is increased from 2 to 6.

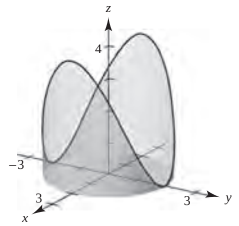
45–51. Answers will vary.

53.



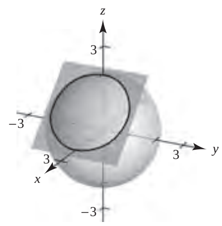
$\mathbf{r}(t) = t\mathbf{i} - t\mathbf{j} + 2t^2\mathbf{k}$

55.



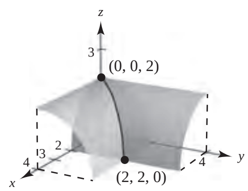
$\mathbf{r}(t) = 2\sin t\mathbf{i} + 2\cos t\mathbf{j} + 4\sin^2 t\mathbf{k}$

57.



$\mathbf{r}(t) = (1+\sin t)\mathbf{i} + \sqrt{2}\cos t\mathbf{j} + (1-\sin t)\mathbf{k}$ and
$\mathbf{r}(t) = (1+\sin t)\mathbf{i} - \sqrt{2}\cos t\mathbf{j} + (1-\sin t)\mathbf{k}$

59.

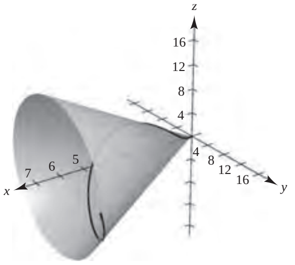


$\mathbf{r}(t) = t\mathbf{i} + t\mathbf{j} + \sqrt{4-t^2}\,\mathbf{k}$

61. Let $x = t$, $y = 2t\cos t$, and $z = 2t\sin t$. Then

$$\begin{aligned} y^2 + z^2 &= (2t\cos t)^2 + (2t\sin t)^2 \\ &= 4t^2\cos^2 t + 4t^2\sin^2 t \\ &= 4t^2(\cos^2 t + \sin^2 t) \\ &= 4t^2. \end{aligned}$$

Because $x = t$, $y^2 + z^2 = 4x^2$.



63. $\pi\mathbf{i} - \mathbf{j}$ **65.** $\mathbf{0}$ **67.** $\mathbf{i} + \mathbf{j} + \mathbf{k}$

69. $(-\infty, 0), (0, \infty)$ **71.** $[-1, 1]$

73. $(-\pi/2 + n\pi, \pi/2 + n\pi)$, n is an integer.

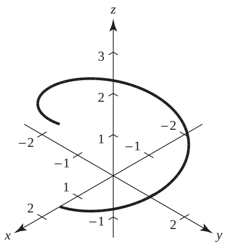
75. $\mathbf{s}(t) = t^2\mathbf{i} + (t-3)\mathbf{j} + (t+3)\mathbf{k}$

77. $\mathbf{s}(t) = (t^2 - 2)\mathbf{i} + (t-3)\mathbf{j} + t\mathbf{k}$

79. A vector-valued function $\mathbf{r}$ is continuous at $t = a$ if the limit of $\mathbf{r}(t)$ exists as $t \to a$ and $\lim_{t\to a} \mathbf{r}(t) = \mathbf{r}(u)$. The function

$$\mathbf{r}(t) = \begin{cases} \mathbf{i} + \mathbf{j}, & t \ge 2 \\ -\mathbf{i} + \mathbf{j}, & t < 2 \end{cases}$$ is not continuous at $t = 0$.

81. Answers will vary. Sample answer:



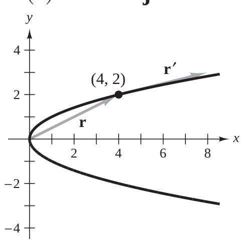
$$\mathbf{r}(t) = 1.5\cos t\mathbf{i} + 1.5\sin t\mathbf{j} + \frac{1}{\pi}t\mathbf{k}, \quad 0 \le t \le 2\pi$$

83–85. Proofs **87.** Yes; Yes **89.** Not necessarily

91. True **93.** True

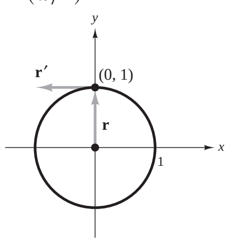
Section 12.2 *(page 830)*

1. $\mathbf{r}'(t) = 2t\mathbf{i} + \mathbf{j}$
$\mathbf{r}(2) = 4\mathbf{i} + 2\mathbf{j}$
$\mathbf{r}'(2) = 4\mathbf{i} + \mathbf{j}$



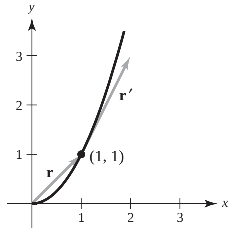
$\mathbf{r}'(t_0)$ is tangent to the curve at t_0.

3. $\mathbf{r}'(t) = -\sin t\mathbf{i} + \cos t\mathbf{j}$
$\mathbf{r}(\pi/2) = \mathbf{j}$
$\mathbf{r}'(\pi/2) = -\mathbf{i}$



$\mathbf{r}'(t_0)$ is tangent to the curve at t_0.

5. $\mathbf{r}'(t) = \langle e^t, 2e^{2t}\rangle$
$\mathbf{r}(0) = \mathbf{i} + \mathbf{j}$
$\mathbf{r}'(0) = \mathbf{i} + 2\mathbf{j}$

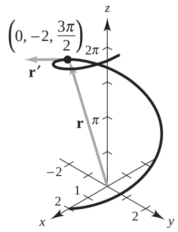


$\mathbf{r}'(t_0)$ is tangent to the curve at t_0.

7. $\mathbf{r}'(t) = -2\sin t\mathbf{i} + 2\cos t\mathbf{j} + \mathbf{k}$

$$\mathbf{r}\left(\frac{3\pi}{2}\right) = -2\mathbf{j} + \left(\frac{3\pi}{2}\right)\mathbf{k}$$

$$\mathbf{r}'\left(\frac{3\pi}{2}\right) = 2\mathbf{i} + \mathbf{k}$$



9. $3t^2\mathbf{i} - 3\mathbf{j}$ **11.** $-2\sin t\mathbf{i} + 5\cos t\mathbf{j}$

13. $6\mathbf{i} - 14t\mathbf{j} + 3t^2\mathbf{k}$ **15.** $-3a\sin t\cos^2 t\mathbf{i} + 3a\sin^2 t\cos t\mathbf{j}$

17. $-e^{-t}\mathbf{i} + (5te^t + 5e^t)\mathbf{k}$

19. $\langle \sin t + t\cos t, \cos t - t\sin t, 1\rangle$

21. (a) $3t^2\mathbf{i} + t\mathbf{j}$ (b) $6t\mathbf{i} + \mathbf{j}$ (c) $18t^3 + t$

23. (a) $-4\sin t\mathbf{i} + 4\cos t\mathbf{j}$ (b) $-4\cos t\mathbf{i} - 4\sin t\mathbf{j}$ (c) 0

25. (a) $t\mathbf{i} - \mathbf{j} + \frac{1}{2}t^2\mathbf{k}$ (b) $\mathbf{i} + t\mathbf{k}$ (c) $t^3/2 + t$
(d) $-t\mathbf{i} - \frac{1}{2}t^2\mathbf{j} + \mathbf{k}$

27. (a) $\langle t\cos t, t\sin t, 1\rangle$
(b) $\langle \cos t - t\sin t, \sin t + t\cos t, 0\rangle$ (c) t
(d) $\langle -\sin t - t\cos t, \cos t - t\sin t, t^2\rangle$

29. $(-\infty, 0), (0, \infty)$ **31.** $(n\pi/2, (n+1)\pi/2)$

33. $(-\infty, \infty)$ **35.** $(-\infty, 0), (0, \infty)$

37. $(-\pi/2 + n\pi, \pi/2 + n\pi)$, n is an integer.

39. (a) $\mathbf{i} + 3\mathbf{j} + 2t\mathbf{k}$ (b) $-\mathbf{i} + (9 - 2t)\mathbf{j} + (6t - 3t^2)\mathbf{k}$
(c) $40t\mathbf{i} + 15t^2\mathbf{j} + 20t^3\mathbf{k}$ (d) $8t + 9t^2 + 5t^4$
(e) $8t^3\mathbf{i} + (12t^2 - 4t^3)\mathbf{j} + (3t^2 - 24t)\mathbf{k}$
(f) $2\mathbf{i} + 6\mathbf{j} + 8t\mathbf{k}$

41. (a) $7t^6$ (b) $12t^5\mathbf{i} - 5t^4\mathbf{j}$ **43.** $t^2\mathbf{i} + t\mathbf{j} + t\mathbf{k} + \mathbf{C}$

45. $\ln|t|\mathbf{i} + t\mathbf{j} - \frac{2}{5}t^{5/2}\mathbf{k} + \mathbf{C}$

47. $(t^2 - t)\mathbf{i} + t^4\mathbf{j} + 2t^{3/2}\mathbf{k} + \mathbf{C}$ **49.** $\tan t\mathbf{i} + \arctan t\mathbf{j} + \mathbf{C}$

51. $4\mathbf{i} + \frac{1}{2}\mathbf{j} - \mathbf{k}$ **53.** $a\mathbf{i} + a\mathbf{j} + (\pi/2)\mathbf{k}$

55. $2\mathbf{i} + (e^2 - 1)\mathbf{j} - (e^2 + 1)\mathbf{k}$

57. $2e^{2t}\mathbf{i} + 3(e^t - 1)\mathbf{j}$ **59.** $600\sqrt{3}t\mathbf{i} + (-16t^2 + 600t)\mathbf{j}$

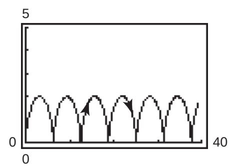
61. $((2 - e^{-t^2})/2)\mathbf{i} + (e^{-t} - 2)\mathbf{j} + (t+1)\mathbf{k}$

63. See "Definition of the Derivative of a Vector-Valued Function" and Figure 12.8 on page 824.

65. The three components of $\mathbf{u}$ are increasing functions of t at $t = t_0$.

67–73. Proofs

75. (a)



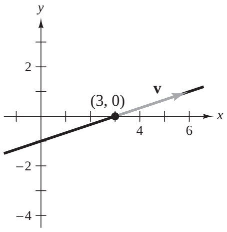
The curve is a cycloid.

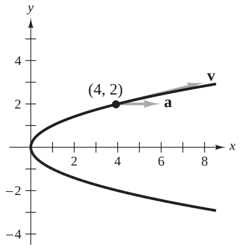
(b) The maximum of $\|\mathbf{r}'\|$ is 2; the minimum of $\|\mathbf{r}'\|$ is 0. The maximum and the minimum of $\|\mathbf{r}''\|$ are 1.

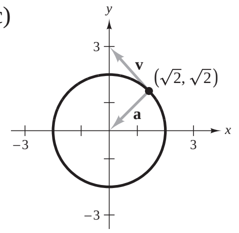
77. Proof **79.** True

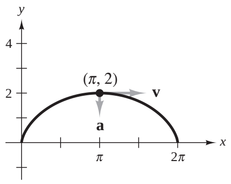
81. False. Let $\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + \mathbf{k}$, then $d/dt[\|\mathbf{r}(t)\|] = 0$, but $\|\mathbf{r}'(t)\| = 1$.

Section 12.3 *(page 838)*

1. (a) $\mathbf{v}(t) = 3\mathbf{i} + \mathbf{j}$
$\|\mathbf{v}(t)\| = \sqrt{10}$
$\mathbf{a}(t) = \mathbf{0}$
(b) $\mathbf{v}(1) = 3\mathbf{i} + \mathbf{j}$
$\mathbf{a}(1) = \mathbf{0}$
(c)


3. (a) $\mathbf{v}(t) = 2t\mathbf{i} + \mathbf{j}$
$\|\mathbf{v}(t)\| = \sqrt{4t^2 + 1}$
$\mathbf{a}(t) = 2\mathbf{i}$
(b) $\mathbf{v}(2) = 4\mathbf{i} + \mathbf{j}$
$\mathbf{a}(2) = 2\mathbf{i}$
(c)


5. (a) $\mathbf{v}(t) = -2\sin t\mathbf{i} + 2\cos t\mathbf{j}$
$\|\mathbf{v}(t)\| = 2$
$\mathbf{a}(t) = -2\cos t\mathbf{i} - 2\sin t\mathbf{j}$
(b) $\mathbf{v}(\pi/4) = -\sqrt{2}\,\mathbf{i} + \sqrt{2}\,\mathbf{j}$
$\mathbf{a}(\pi/4) = -\sqrt{2}\,\mathbf{i} - \sqrt{2}\,\mathbf{j}$
(c)


7. (a) $\mathbf{v}(t) = \langle 1 - \cos t, \sin t\rangle$
$\|\mathbf{v}(t)\| = \sqrt{2 - 2\cos t}$
$\mathbf{a}(t) = \langle \sin t, \cos t\rangle$
(b) $\mathbf{v}(\pi) = \langle 2, 0\rangle$
$\mathbf{a}(\pi) = \langle 0, -1\rangle$
(c)


9. (a) $\mathbf{v}(t) = \mathbf{i} + 5\mathbf{j} + 3\mathbf{k}$
$\|\mathbf{v}(t)\| = \sqrt{35}$
$\mathbf{a}(t) = \mathbf{0}$
(b) $\mathbf{v}(1) = \mathbf{i} + 5\mathbf{j} + 3\mathbf{k}$
$\mathbf{a}(1) = \mathbf{0}$

11. (a) $\mathbf{v}(t) = \mathbf{i} + 2t\mathbf{j} + t\mathbf{k}$
$\|\mathbf{v}(t)\| = \sqrt{1 + 5t^2}$
$\mathbf{a}(t) = 2\mathbf{j} + \mathbf{k}$
(b) $\mathbf{v}(4) = \mathbf{i} + 8\mathbf{j} + 4\mathbf{k}$
$\mathbf{a}(4) = 2\mathbf{j} + \mathbf{k}$

13. (a) $\mathbf{v}(t) = \mathbf{i} + \mathbf{j} - \left(t/\sqrt{9 - t^2}\right)\mathbf{k}$
$\|\mathbf{v}(t)\| = \sqrt{(18 - t^2)/(9 - t^2)}$
$\mathbf{a}(t) = (-9/(9 - t^2)^{3/2})\mathbf{k}$
(b) $\mathbf{v}(0) = \mathbf{i} + \mathbf{j}$
$\mathbf{a}(0) = -\frac{1}{3}\mathbf{k}$

15. (a) $\mathbf{v}(t) = 4\mathbf{i} - 3\sin t\mathbf{j} + 3\cos t\mathbf{k}$
$\|\mathbf{v}(t)\| = 5$
$\mathbf{a}(t) = -3\cos t\mathbf{j} - 3\sin t\mathbf{k}$
(b) $\mathbf{v}(\pi) = \langle 4, 0, -3\rangle$
$\mathbf{a}(\pi) = \langle 0, 3, 0\rangle$

17. (a) $\mathbf{v}(t) = (e^t\cos t - e^t\sin t)\mathbf{i} + (e^t\sin t + e^t\cos t)\mathbf{j} + e^t\mathbf{k}$
$\|\mathbf{v}(t)\| = e^t\sqrt{3}$
$\mathbf{a}(t) = -2e^t\sin t\mathbf{i} + 2e^t\cos t\mathbf{j} + e^t\mathbf{k}$
(b) $\mathbf{v}(0) = \langle 1, 1, 1\rangle$
$\mathbf{a}(0) = \langle 0, 2, 1\rangle$

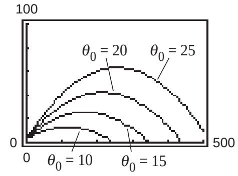
19. $\mathbf{v}(t) = t(\mathbf{i} + \mathbf{j} + \mathbf{k})$
$\mathbf{r}(t) = (t^2/2)(\mathbf{i} + \mathbf{j} + \mathbf{k})$
$\mathbf{r}(2) = 2(\mathbf{i} + \mathbf{j} + \mathbf{k})$

21. $\mathbf{v}(t) = \left(t^2/2 + \frac{9}{2}\right)\mathbf{j} + \left(t^2/2 - \frac{1}{2}\right)\mathbf{k}$
$\mathbf{r}(t) = \left(t^3/6 + \frac{9}{2}t - \frac{14}{3}\right)\mathbf{j} + \left(t^3/6 - \frac{1}{2}t + \frac{1}{3}\right)\mathbf{k}$
$\mathbf{r}(2) = \frac{17}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$

23. $\mathbf{v}(t) = -\sin t\mathbf{i} + \cos t\mathbf{j} + \mathbf{k}$
$\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k}$
$\mathbf{r}(2) = (\cos 2)\mathbf{i} + (\sin 2)\mathbf{j} + 2\mathbf{k}$

25. Maximum height: 45.5 ft; The ball will clear the fence.

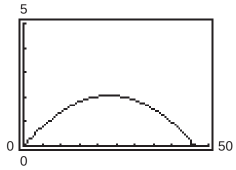
27. $v_0 = 40\sqrt{6}$ ft/sec; 78 ft **29.** Proof

31. (a) $\mathbf{r}(t) = \left(\frac{440}{3}\cos\theta_0\right)t\mathbf{i} + \left[3 + \left(\frac{440}{3}\sin\theta_0\right)t - 16t^2\right]\mathbf{j}$
(b)


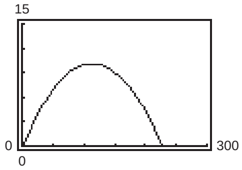
The minimum angle appears to be $\theta_0 = 20°$.
(c) $\theta_0 \approx 19.38°$

33. (a) $v_0 = 28.78$ ft/sec; $\theta = 58.28°$ (b) $v_0 \approx 32$ ft/sec

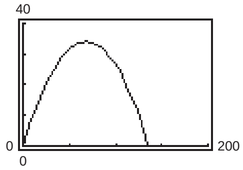
35. 1.91°

37. (a)


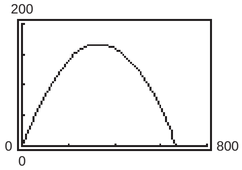
Maximum height: 2.1 ft
Range: 46.6 ft

(b)


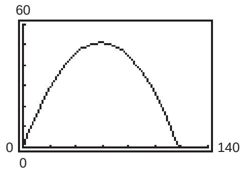
Maximum height: 10.0 ft
Range: 227.8 ft

(c)


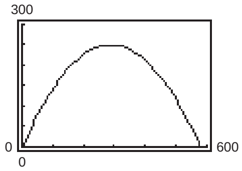
Maximum height: 34.0 ft
Range: 136.1 ft

(d)


Maximum height: 166.5 ft
Range: 666.1 ft

(e)


Maximum height: 51.0 ft
Range: 117.9 ft

(f)


Maximum height: 249.8 ft
Range: 576.9 ft

39. Maximum height: 129.1 m; Range: 886.3 m

41. Proof

43. $\mathbf{v}(t) = b\omega[(1 - \cos\omega t)\mathbf{i} + \sin\omega t\mathbf{j}]$
$\mathbf{a}(t) = b\omega^2(\sin\omega t\mathbf{i} + \cos\omega t\mathbf{j})$
(a) $\|\mathbf{v}(t)\| = 0$ when $\omega t = 0, 2\pi, 4\pi, \ldots$.
(b) $\|\mathbf{v}(t)\|$ is maximum when $\omega t = \pi, 3\pi, \ldots$.

45. $\mathbf{v}(t) = -b\omega\sin\omega t\mathbf{i} + b\omega\cos\omega t\mathbf{j}$
$\mathbf{v}(t) \cdot \mathbf{r}(t) = 0$

47. $\mathbf{a}(t) = -b\omega^2(\cos\omega t\mathbf{i} + \sin\omega t\mathbf{j}) = -\omega^2\mathbf{r}(t)$; $\mathbf{a}(t)$ is a negative multiple of a unit vector from (0, 0) to $(\cos\omega t, \sin\omega t)$, so $\mathbf{a}(t)$ is directed toward the origin.

49. $8\sqrt{10}$ ft/sec

51. The velocity of an object involves both magnitude and direction of motion, whereas speed involves only magnitude.

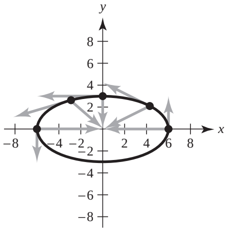
53. Proof

55. (a) $\mathbf{v}(t) = -6 \sin t\,\mathbf{i} + 3 \cos t\,\mathbf{j}$
$\|\mathbf{v}(t)\| = 3\sqrt{3 \sin^2 t + 1}$
$\mathbf{a}(t) = -6 \cos t\,\mathbf{i} - 3 \sin t\,\mathbf{j}$

(b)

t	0	$\pi/4$	$\pi/2$	$2\pi/3$	π
Speed	3	$3\sqrt{10}/2$	6	$3\sqrt{13}/2$	3

(c)



(d) The speed is increasing when the angle between **v** and **a** is in the interval $[0, \pi/2)$, and decreasing when the angle is in the interval $(\pi/2, \pi]$.

57. Proof

59. False. Acceleration is the derivative of the velocity.

61. True

Section 12.4 *(page 848)*

1. $\mathbf{T}(1) = (\sqrt{2}/2)(\mathbf{i} + \mathbf{j})$ **3.** $\mathbf{T}(\pi/4) = (\sqrt{2}/2)(-\mathbf{i} + \mathbf{j})$

5. $\mathbf{T}(e) = (3e\mathbf{i} - \mathbf{j})/\sqrt{9e^2 + 1} \approx 0.9926\mathbf{i} - 0.1217\mathbf{j}$

7. $\mathbf{T}(0) = (\sqrt{2}/2)(\mathbf{i} + \mathbf{k})$
$x = t$
$y = 0$
$z = t$

9. $\mathbf{T}(0) = (\sqrt{10}/10)(3\mathbf{j} + \mathbf{k})$
$x = 3$
$y = 3t$
$z = t$

11. $\mathbf{T}(\pi/4) = \frac{1}{2}\langle -\sqrt{2}, \sqrt{2}, 0\rangle$
$x = \sqrt{2} - \sqrt{2}t$
$y = \sqrt{2} + \sqrt{2}t$
$z = 4$

13. $\mathbf{N}(2) = (\sqrt{5}/5)(-2\mathbf{i} + \mathbf{j})$

15. $\mathbf{N}(2) = (-\sqrt{5}/5)(2\mathbf{i} - \mathbf{j})$

17. $\mathbf{N}(1) = (-\sqrt{14}/14)(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$

19. $\mathbf{N}(3\pi/4) = (\sqrt{2}/2)(\mathbf{i} - \mathbf{j})$

21. $\mathbf{T}(1) = (\sqrt{2}/2)(\mathbf{i} - \mathbf{j})$
$\mathbf{N}(1) = (\sqrt{2}/2)(\mathbf{i} + \mathbf{j})$
$a_{\mathbf{T}} = -\sqrt{2}$
$a_{\mathbf{N}} = \sqrt{2}$

23. $\mathbf{T}(1) = (-\sqrt{5}/5)(\mathbf{i} - 2\mathbf{j})$
$\mathbf{N}(1) = (-\sqrt{5}/5)(2\mathbf{i} + \mathbf{j})$
$a_{\mathbf{T}} = 14\sqrt{5}/5$
$a_{\mathbf{N}} = 8\sqrt{5}/5$

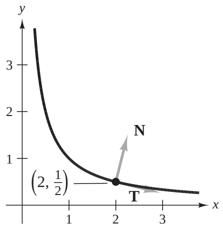
25. $\mathbf{T}(0) = (\sqrt{5}/5)(\mathbf{i} - 2\mathbf{j})$
$\mathbf{N}(0) = (\sqrt{5}/5)(2\mathbf{i} + \mathbf{j})$
$a_{\mathbf{T}} = -7\sqrt{5}/5$
$a_{\mathbf{N}} = 6\sqrt{5}/5$

27. $\mathbf{T}(\pi/2) = (\sqrt{2}/2)(-\mathbf{i} + \mathbf{j})$
$\mathbf{N}(\pi/2) = (-\sqrt{2}/2)(\mathbf{i} + \mathbf{j})$
$a_{\mathbf{T}} = \sqrt{2}e^{\pi/2}$
$a_{\mathbf{N}} = \sqrt{2}e^{\pi/2}$

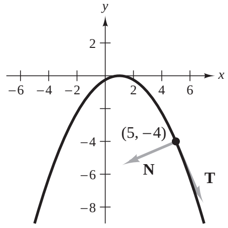
29. $\mathbf{T}(t) = -\sin(\omega t)\mathbf{i} + \cos(\omega t)\mathbf{j}$
$\mathbf{N}(t) = -\cos(\omega t)\mathbf{i} - \sin(\omega t)\mathbf{j}$
$a_{\mathbf{T}} = 0$
$a_{\mathbf{N}} = a\omega^2$

31. $\|\mathbf{v}(t)\| = a\omega$; The speed is constant because $a_{\mathbf{T}} = 0$.

33. $\mathbf{r}(2) = 2\mathbf{i} + \frac{1}{2}\mathbf{j}$
$\mathbf{T}(2) = (\sqrt{17}/17)(4\mathbf{i} - \mathbf{j})$
$\mathbf{N}(2) = (\sqrt{17}/17)(\mathbf{i} + 4\mathbf{j})$



35. $\mathbf{r}(2) = 5\mathbf{i} - 4\mathbf{j}$
$\mathbf{T}(2) = \dfrac{\mathbf{i} - 2\mathbf{j}}{\sqrt{5}}$
$\mathbf{N}(2) = \dfrac{-2\mathbf{i} - \mathbf{j}}{\sqrt{5}}$, perpendicular to $\mathbf{T}(2)$



37. $\mathbf{T}(1) = (\sqrt{14}/14)(\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$
$\mathbf{N}(1)$ is undefined.
$a_{\mathbf{T}}$ is undefined.
$a_{\mathbf{N}}$ is undefined.

39. $\mathbf{T}(1) = (\sqrt{6}/6)(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$
$\mathbf{N}(1) = (\sqrt{30}/30)(-5\mathbf{i} + 2\mathbf{j} + \mathbf{k})$
$a_{\mathbf{T}} = 5\sqrt{6}/6$
$a_{\mathbf{N}} = \sqrt{30}/6$

41. $\mathbf{T}(0) = (\sqrt{3}/3)(\mathbf{i} + \mathbf{j} + \mathbf{k})$
$\mathbf{N}(0) = (\sqrt{2}/2)(\mathbf{i} - \mathbf{j})$
$a_{\mathbf{T}} = \sqrt{3}$
$a_{\mathbf{N}} = \sqrt{2}$

43. Let C be a smooth curve represented by **r** on an open interval I. The unit tangent vector $\mathbf{T}(t)$ at t is defined as

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|},\ \mathbf{r}'(t) \neq 0.$$

The principal unit normal vector $\mathbf{N}(t)$ at t is defined as

$$\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|},\ \mathbf{T}'(t) \neq 0.$$

The tangential and normal components of acceleration are defined as $\mathbf{a}(t) = a_{\mathbf{T}}\mathbf{T}(t) + a_{\mathbf{N}}\mathbf{N}(t)$.

45. (a) The particle's motion is in a straight line.
(b) The particle's speed is constant.

47. $\mathbf{v}(t) = \mathbf{r}'(t) = 3\mathbf{i} + 4\mathbf{j}$
$\|\mathbf{v}(t)\| = \sqrt{9 + 16} = 5$
$\mathbf{a}(t) = \mathbf{v}'(t) = \mathbf{0}$
$\mathbf{T}(t) = \dfrac{\mathbf{v}(t)}{\|\mathbf{v}(t)\|} = \dfrac{3}{5}\mathbf{i} + \dfrac{4}{5}\mathbf{j}$
$\mathbf{T}'(t) = 0 \implies \mathbf{N}(t)$ does not exist.
The path is a line. The speed is constant (5).

49. (a) $t = \frac{1}{2}$: $a_{\mathbf{T}} = \sqrt{2}\pi^2/2$, $a_{\mathbf{N}} = \sqrt{2}\pi^2/2$
$t = 1$: $a_{\mathbf{T}} = 0$, $a_{\mathbf{N}} = \pi^2$
$t = \frac{3}{2}$: $a_{\mathbf{T}} = -\sqrt{2}\pi^2/2$, $a_{\mathbf{N}} = \sqrt{2}\pi^2/2$
(b) $t = \frac{1}{2}$: Increasing because $a_{\mathbf{T}} > 0$.
$t = 1$: Maximum because $a_{\mathbf{T}} = 0$.
$t = \frac{3}{2}$: Decreasing because $a_{\mathbf{T}} < 0$.

51. $\mathbf{T}(\pi/2) = (\sqrt{17}/17)(-4\mathbf{i} + \mathbf{k})$
$\mathbf{N}(\pi/2) = -\mathbf{j}$
$\mathbf{B}(\pi/2) = (\sqrt{17}/17)(\mathbf{i} + 4\mathbf{k})$

53. $\mathbf{T}(\pi/4) = (\sqrt{2}/2)(\mathbf{j} - \mathbf{k})$
$\mathbf{N}(\pi/4) = -(\sqrt{2}/2)(\mathbf{j} + \mathbf{k})$
$\mathbf{B}(\pi/4) = -\mathbf{i}$

55. $\mathbf{T}(\pi/3) = (\sqrt{5}/5)(\mathbf{i} - \sqrt{3}\mathbf{j} + \mathbf{k})$
$\mathbf{N}(\pi/3) = -\frac{1}{2}(\sqrt{3}\mathbf{i} + \mathbf{j})$
$\mathbf{B}(\pi/3) = (\sqrt{5}/10)(\mathbf{i} - \sqrt{3}\mathbf{j} - 4\mathbf{k})$

57. $\mathbf{N}(t) = \dfrac{1}{\sqrt{16t^2 + 9}}(-4t\mathbf{i} + 3\mathbf{j})$

59. $\mathbf{N}(t) = \dfrac{1}{\sqrt{5t^2 + 25}}(-t\mathbf{i} - 2t\mathbf{j} + 5\mathbf{k})$

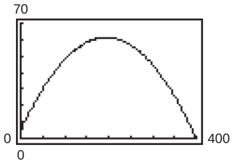
61. $a_{\mathbf{T}} = \dfrac{-32(v_0 \sin\theta - 32t)}{\sqrt{v_0^2\cos^2\theta + (v_0\sin\theta - 32t)^2}}$

$a_{\mathbf{N}} = \dfrac{32v_0\cos\theta}{\sqrt{v_0^2\cos^2\theta + (v_0\sin\theta - 32t)^2}}$

At maximum height, $a_{\mathbf{T}} = 0$ and $a_{\mathbf{N}} = 32$.

63. (a) $\mathbf{r}(t) = 60\sqrt{3}t\mathbf{i} + (5 + 60t - 16t^2)\mathbf{j}$

(b)



Maximum height $\approx$ 61.245 ft
Range $\approx$ 398.186 ft

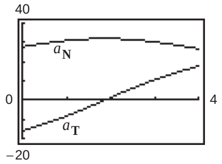
(c) $\mathbf{v}(t) = 60\sqrt{3}\mathbf{i} + (60 - 32t)\mathbf{j}$
$\|\mathbf{v}(t)\| = 8\sqrt{16t^2 - 60t + 225}$
$\mathbf{a}(t) = -32\mathbf{j}$

(d)

t	0.5	1.0	1.5
Speed	112.85	107.63	104.61

t	2.0	2.5	3.0
Speed	104	105.83	109.98

(e)



The speed is decreasing when $a_{\mathbf{T}}$ and $a_{\mathbf{N}}$ have opposite signs.

65. (a) $4\sqrt{625\pi^2 + 1} \approx 314$ mi/h
(b) $a_{\mathbf{T}} = 0$, $a_{\mathbf{N}} = 1000\pi^2$
$a_{\mathbf{T}} = 0$ because the speed is constant.

67. (a) The centripetal component is quadrupled.
(b) The centripetal component is halved.

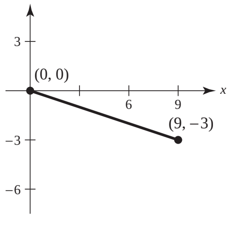
69. 4.74 mi/sec **71.** 4.67 mi/sec

73. False; centripetal acceleration may occur with constant speed.

75. (a) and (b) Proofs **77–79.** Proofs

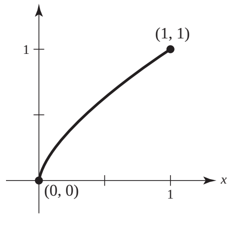
Section 12.5 *(page 860)*

1.



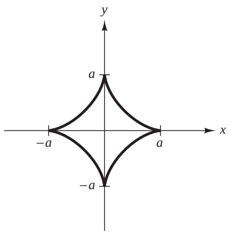
$3\sqrt{10}$

3.



$(13\sqrt{13} - 8)/27$

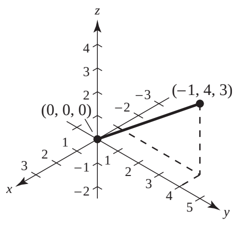
5.



$6a$

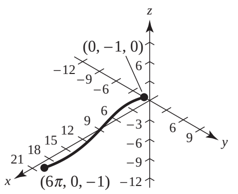
7. (a) $\mathbf{r}(t) = (50t\sqrt{2})\mathbf{i} + (3 + 50t\sqrt{2} - 16t^2)\mathbf{j}$
(b) $\frac{649}{8} \approx 81$ ft (c) 315.5 ft (d) 362.9 ft

9.



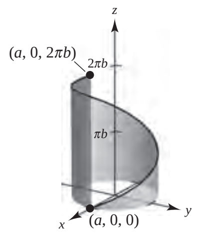
$\sqrt{26}$

11.



$3\sqrt{17}\pi/2$

13.



$2\pi\sqrt{a^2 + b^2}$

15. (a) $2\sqrt{21} \approx 9.165$ (b) 9.529
(c) Increase the number of line segments. (d) 9.571

17. (a) $s = \sqrt{5}t$ (b) $\mathbf{r}(s) = 2\cos\dfrac{s}{\sqrt{5}}\mathbf{i} + 2\sin\dfrac{s}{\sqrt{5}}\mathbf{j} + \dfrac{s}{\sqrt{5}}\mathbf{k}$
(c) $s = \sqrt{5}$: $(1.081, 1.683, 1.000)$
$s = 4$: $(-0.433, 1.953, 1.789)$
(d) Proof

19. 0 **21.** $\frac{2}{5}$ **23.** 0 **25.** $\sqrt{2}/2$ **27.** 1 **29.** $\frac{1}{4}$

31. $1/a$ **33.** $\sqrt{5}/(1 + 5t^2)^{3/2}$ **35.** $\frac{3}{25}$ **37.** $\frac{12}{125}$

39. $7\sqrt{26}/676$ **41.** $K = 0$, $1/K$ is undefined.

43. $K = 4/17^{3/2}$, $1/K = 17^{3/2}/4$ **45.** $K = 4$, $1/K = 1/4$

47. $K = 12/145^{3/2}$, $1/K = 145^{3/2}/12$ **49.** (a) $(1, 3)$ (b) 0

51. (a) $K \to \infty$ as $x \to 0$ (No maximum) (b) 0

53. (a) $(1/\sqrt{2}, -\ln 2/2)$ (b) 0

55. $(0, 1)$ **57.** $(\pi/2 + K\pi, 0)$

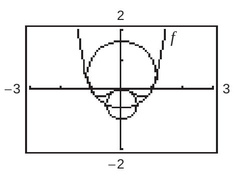
59. $s = \displaystyle\int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}\,dt = \int_a^b \|\mathbf{r}'(t)\|\,dt$

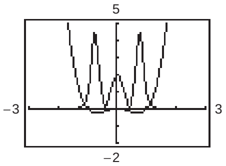
61. The curve is a line.

63. (a) $K = \dfrac{2|6x^2 - 1|}{(16x^6 - 16x^4 + 4x^2 + 1)^{3/2}}$

(b) $x = 0$: $x^2 + \left(y + \dfrac{1}{2}\right)^2 = \dfrac{1}{4}$

$x = 1$: $x^2 + \left(y - \dfrac{1}{2}\right)^2 = \dfrac{5}{4}$



(c) 

The curvature tends to be greatest near the extrema of the function and decreases as $x \to \pm\infty$. However, f and K do not have the same critical numbers.

Critical numbers of f: $x = 0, \pm\sqrt{2}/2 \approx \pm 0.7071$

Critical numbers of K: $x = 0, \pm 0.7647, \pm 0.4082$

65. Proof **67.** (a) 12.25 units (b) $\frac{1}{2}$ **69–71.** Proofs

73. (a) 0 (b) 0 **75.** $\frac{1}{4}$ **77.** Proof

79. $K = [1/(4a)]|\csc(\theta/2)|$ **81.** 3327.5 lb

Minimum: $K = 1/(4a)$

There is no maximum.

83. Proof **85.** False. See Exploration on page 851.

87. True **89–95.** Proofs

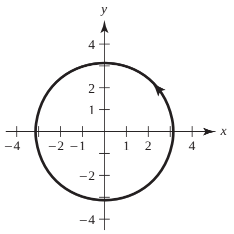
Review Exercises for Chapter 12 *(page 863)*

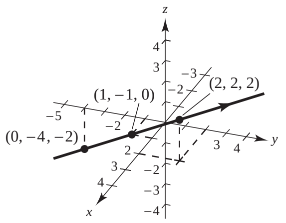
1. (a) All reals except $(\pi/2) + n\pi$, n is an integer.
(b) Continuous except at $t = (\pi/2) + n\pi$, n is an integer.

3. (a) $(0, \infty)$ (b) Continuous for all $t > 0$

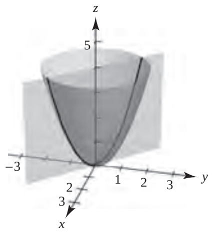
5. (a) $\mathbf{i} - \sqrt{2}\mathbf{k}$ (b) $-3\mathbf{i} + 4\mathbf{j}$
(c) $(2c - 1)\mathbf{i} + (c - 1)^2\mathbf{j} - \sqrt{c + 1}\,\mathbf{k}$
(d) $2\Delta t\mathbf{i} + \Delta t(\Delta t + 2)\mathbf{j} - \left(\sqrt{\Delta t + 3} - \sqrt{3}\right)\mathbf{k}$

7. $\mathbf{r}(t) = (3 - t)\mathbf{i} - 2t\mathbf{j} + (5 - 2t)\mathbf{k}, \quad 0 \le t \le 1$
$x = 3 - t, y = -2t, z = 5 - 2t, \quad 0 \le t \le 1$

9. 

11. 

13. $\mathbf{r}(t) = t\mathbf{i} + \left(-\frac{3}{4}t + 3\right)\mathbf{j}$

15. 

$x = t, y = -t, z = 2t^2$

17. $4\mathbf{i} + \mathbf{k}$

19. (a) $(2t + 4)\mathbf{i} - 6t\mathbf{j}$
(b) $2\mathbf{i} - 6\mathbf{j}$
(c) $40t + 8$

21. (a) $6t^2\mathbf{i} + 4\mathbf{j} - 2t\mathbf{k}$
(b) $12t\mathbf{i} - 2\mathbf{k}$
(c) $72t^3 + 4t$
(d) $-8\mathbf{i} - 12t^2\mathbf{j} - 48t\mathbf{k}$

23. (a) $3\mathbf{i} + \mathbf{j}$ (b) $-5\mathbf{i} + (2t - 2)\mathbf{j} + 2t^2\mathbf{k}$
(c) $18t\mathbf{i} + (6t - 3)\mathbf{j}$ (d) $4t + 3t^2$
(e) $\left(\frac{8}{3}t^3 - 2t^2\right)\mathbf{i} - 8t^3\mathbf{j} + (9t^2 - 2t + 1)\mathbf{k}$
(f) $2\mathbf{i} + 8t\mathbf{j} + 16t^2\mathbf{k}$

25. $t\mathbf{i} + 3t\mathbf{j} + 2t^2\mathbf{k} + \mathbf{C}$ **27.** $2t^{3/2}\mathbf{i} + 2\ln|t|\,\mathbf{j} + t\mathbf{k} + \mathbf{C}$

29. $\frac{32}{3}\mathbf{j}$ **31.** $2(e - 1)\mathbf{i} - 8\mathbf{j} - 2\mathbf{k}$

33. $\mathbf{r}(t) = (t^2 + 1)\mathbf{i} + (e^t + 2)\mathbf{j} - (e^{-t} + 4)\mathbf{k}$

35. (a) $\mathbf{v}(t) = 4\mathbf{i} + 3t^2\mathbf{j} - \mathbf{k}$
$\|\mathbf{v}(t)\| = \sqrt{17 + 9t^4}$
$\mathbf{a}(t) = 6t\mathbf{j}$
(b) $\mathbf{v}(1) = 4\mathbf{i} + 3\mathbf{j} - \mathbf{k}$
$\mathbf{a}(1) = 6\mathbf{j}$

37. (a) $\mathbf{v}(t) = \langle -3\cos^2 t \sin t, 3\sin^2 t \cos t, 3\rangle$
$\|\mathbf{v}(t)\| = 3\sqrt{\sin^2 t \cos^2 t + 1}$
$\mathbf{a}(t) = \langle 3\cos t(2\sin^2 t - \cos^2 t), 3\sin t(2\cos^2 t - \sin^2 t), 0\rangle$
(b) $\mathbf{v}(\pi) = \langle 0, 0, 3\rangle$
$\mathbf{a}(\pi) = \langle 3, 0, 0\rangle$

39. About 191.0 ft **41.** About 38.1 m/sec

43. $\mathbf{T}(1) = \dfrac{\sqrt{10}}{10}\mathbf{i} + \dfrac{3\sqrt{10}}{10}\mathbf{j}$

45. $\mathbf{T}\left(\dfrac{\pi}{3}\right) = -\dfrac{\sqrt{15}}{5}\mathbf{i} + \dfrac{\sqrt{5}}{5}\mathbf{j} + \dfrac{\sqrt{5}}{5}\mathbf{k}$;

$x = -\sqrt{3}t + 1, y = t + \sqrt{3}, z = t + \dfrac{\pi}{3}$

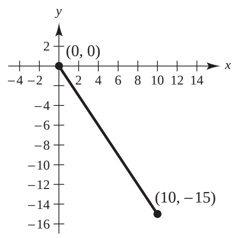
47. $\mathbf{N}(1) = -\dfrac{3\sqrt{10}}{10}\mathbf{i} + \dfrac{\sqrt{10}}{10}\mathbf{j}$ **49.** $\mathbf{N}\left(\dfrac{\pi}{4}\right) = -\mathbf{j}$

51. $\mathbf{T}(3) = -\dfrac{\sqrt{13}}{65}\mathbf{i} - \dfrac{18\sqrt{13}}{65}\mathbf{j}$

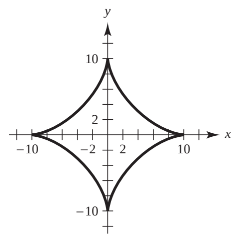
$\mathbf{N}(3) = \dfrac{18\sqrt{13}}{65}\mathbf{i} - \dfrac{\sqrt{13}}{65}\mathbf{j}$

$a_{\mathbf{T}} = -\dfrac{2\sqrt{13}}{585}$

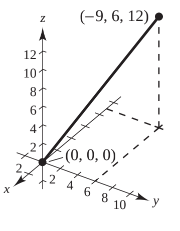
$a_{\mathbf{N}} = \dfrac{4\sqrt{13}}{65}$

53. 

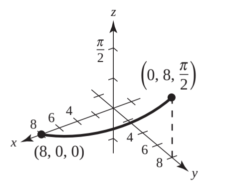
$5\sqrt{13}$

55. 

60

57. 

$3\sqrt{29}$

59. 

$\sqrt{65}\pi/2$

61. 0 **63.** $\left(2\sqrt{5}\right)/(4 + 5t^2)^{3/2}$ **65.** $\sqrt{2}/3$

67. $K = \sqrt{17}/289$; $r = 17\sqrt{17}$ **69.** $K = \sqrt{2}/4$; $r = 2\sqrt{2}$

71. 2016.7 lb

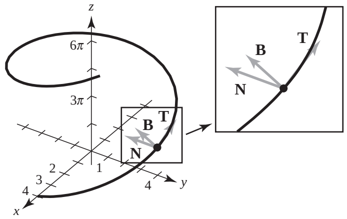
P.S. Problem Solving *(page 865)*

1. (a) a (b) πa (c) $K = \pi a$

3. Initial speed: 447.21 ft/sec; $\theta \approx 63.43°$

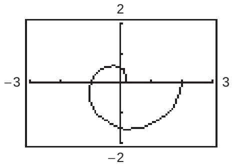
5–7. Proofs

9. Unit tangent: $\left\langle -\frac{4}{5}, 0, \frac{3}{5} \right\rangle$
Unit normal: $\langle 0, -1, 0 \rangle$
Binormal: $\left\langle \frac{3}{5}, 0, \frac{4}{5} \right\rangle$



11. (a) and (b) Proofs

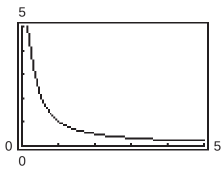
13. (a)



(b) 6.766

(c) $K = [\pi(\pi^2 t^2 + 2)]/(\pi^2 t^2 + 1)^{3/2}$
$K(0) = 2\pi$
$K(1) = [\pi(\pi^2 + 2)]/(\pi^2 + 1)^{3/2} \approx 1.04$
$K(2) \approx 0.51$

(d)



(e) $\lim_{t \to \infty} K = 0$

(f) As $t \to \infty$, the graph spirals outward and the curvature decreases.

Chapter 13

Section 13.1 *(page 876)*

1. Not a function because for some values of x and y (for example $x = y = 0$), there are two z-values.

3. z is a function of x and y. **5.** z is not a function of x and y.

7. (a) 6 (b) -4 (c) 150 (d) $5y$ (e) $2x$ (f) $5t$

9. (a) 5 (b) $3e^2$ (c) $2/e$ (d) $5e^y$ (e) xe^2 (f) te^t

11. (a) $\frac{2}{3}$ (b) 0 (c) $-\frac{3}{2}$ (d) $-\frac{10}{3}$

13. (a) $\sqrt{2}$ (b) $3 \sin 1$ (c) $-3\sqrt{3}/2$ (d) 4

15. (a) -4 (b) -6 (c) $-\frac{25}{4}$ (d) $\frac{9}{4}$

17. (a) $2, \Delta x \neq 0$ (b) $2y + \Delta y, \Delta y \neq 0$

19. Domain: $\{(x, y)$: x is any real number, y is any real number$\}$
Range: $z \geq 0$

21. Domain: $\{(x, y)\colon y \geq 0\}$
Range: all real numbers

23. Domain: $\{(x, y)\colon x \neq 0, y \neq 0\}$
Range: all real numbers

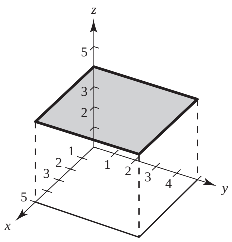
25. Domain: $\{(x, y)\colon x^2 + y^2 \leq 4\}$
Range: $0 \leq z \leq 2$

27. Domain: $\{(x, y)\colon -1 \leq x + y \leq 1\}$
Range: $0 \leq z \leq \pi$

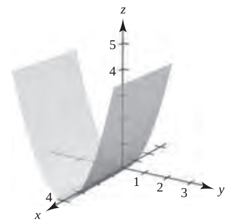
29. Domain: $\{(x, y)\colon y < -x + 4\}$
Range: all real numbers

31. (a) $(20, 0, 0)$ (b) $(-15, 10, 20)$
(c) $(20, 15, 25)$ (d) $(20, 20, 0)$

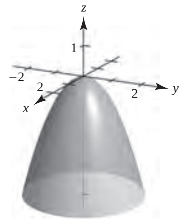
33.



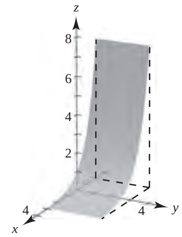
35.



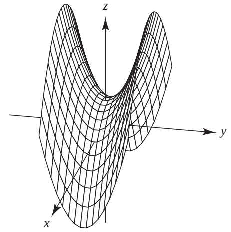
37.



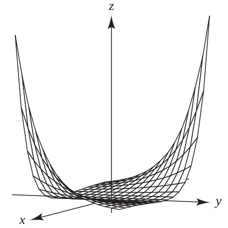
39.



41.

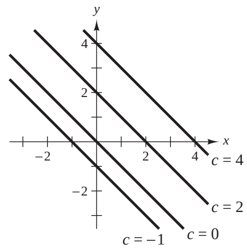


43.

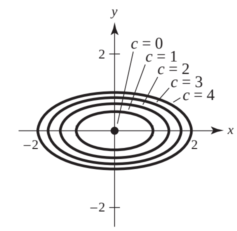


45. c **46.** d **47.** b **48.** a

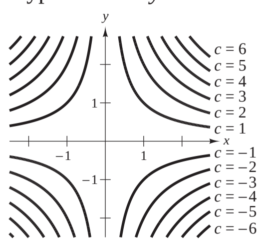
49. Lines: $x + y = c$



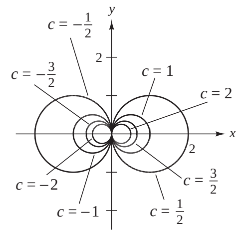
51. Ellipses: $x^2 + 4y^2 = c$
(except $x^2 + 4y^2 = 0$ is the point $(0, 0)$.)



53. Hyperbolas: $xy = c$



55. Circles passing through $(0, 0)$
Centered at $(1/(2c), 0)$



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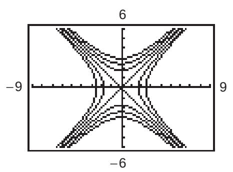
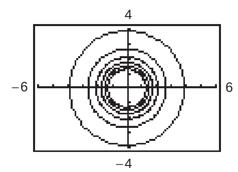


59.



61. The graph of a function of two variables is the set of all points (x, y, z) for which $z = f(x, y)$ and (x, y) is in the domain of f. The graph can be interpreted as a surface in space. Level curves are the scalar fields $f(x, y) = c$, where c is a constant.

63. $f(x, y) = x/y$; the level curves are the lines $y = (1/c)x$.