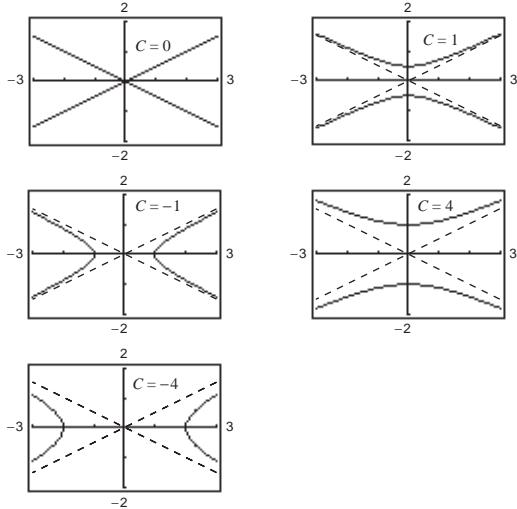


Chapter 6**Section 6.1 (page 385)**

- 1–11.** Proofs **13.** Not a solution **15.** Solution
17. Solution **19.** Solution **21.** Not a solution
23. Solution **25.** Not a solution **27.** Not a solution
29. $y = 3e^{-x/2}$ **31.** $4y^2 = x^3$

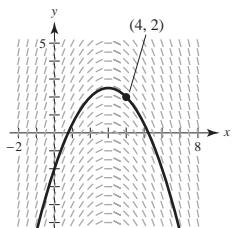
33.

- 35.** $y = 3e^{-2x}$ **37.** $y = 2 \sin 3x - \frac{1}{3} \cos 3x$
39. $y = -2x + \frac{1}{2}x^3$ **41.** $2x^3 + C$
43. $y = \frac{1}{2} \ln(1+x^2) + C$ **45.** $y = x - \ln x^2 + C$
47. $y = -\frac{1}{2} \cos 2x + C$
49. $y = \frac{2}{5}(x-6)^{5/2} + 4(x-6)^{3/2} + C$ **51.** $y = \frac{1}{2}e^{x^2} + C$

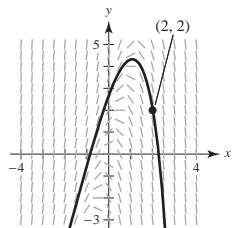
53.	x	-4	-2	0	2	4	8
	y	2	0	4	4	6	8
	dy/dx	-4	Undef.	0	1	$\frac{4}{3}$	2

55.

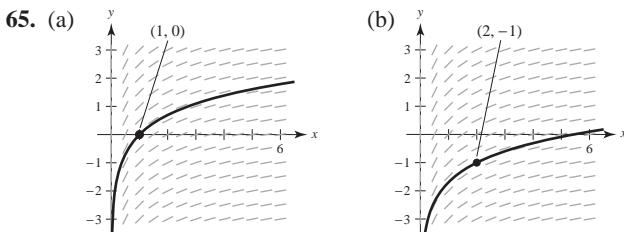
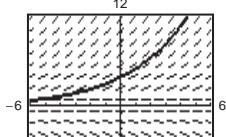
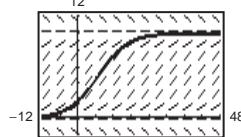
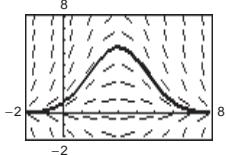
x	-4	-2	0	2	4	8
y	2	0	4	4	6	8
dy/dx	$-2\sqrt{2}$	-2	0	0	$-2\sqrt{2}$	-8

57. b **58. c** **59. d** **60. a****61. (a) and (b)****63. (a) and (b)**

(c) As $x \rightarrow \infty$, $y \rightarrow -\infty$.
As $x \rightarrow -\infty$, $y \rightarrow -\infty$.



(c) As $x \rightarrow \infty$, $y \rightarrow -\infty$.
As $x \rightarrow -\infty$, $y \rightarrow -\infty$.

As $x \rightarrow \infty$, $y \rightarrow \infty$.**67. (a) and (b)**As $x \rightarrow \infty$, $y \rightarrow \infty$.**69. (a) and (b)****71. (a) and (b)****73.**

n	0	1	2	3	4	5	6
x_n	0	0.1	0.2	0.3	0.4	0.5	0.6
y_n	2	2.2	2.43	2.693	2.992	3.332	3.715

n	7	8	9	10
x_n	0.7	0.8	0.9	1.0
y_n	4.146	4.631	5.174	5.781

75.

n	0	1	2	3	4	5	6
x_n	0	0.05	0.1	0.15	0.2	0.25	0.3
y_n	3	2.7	2.438	2.209	2.010	1.839	1.693

n	7	8	9	10
x_n	0.35	0.4	0.45	0.5
y_n	1.569	1.464	1.378	1.308

77.

n	0	1	2	3	4	5	6
x_n	0	0.1	0.2	0.3	0.4	0.5	0.6
y_n	1	1.1	1.212	1.339	1.488	1.670	1.900

n	7	8	9	10
x_n	0.7	0.8	0.9	1.0
y_n	2.213	2.684	3.540	5.958

79.

x	0	0.2	0.4	0.6	0.8	1
$y(x)$ (exact)	3.0000	3.6642	4.4755	5.4664	6.6766	8.1548
$y(x)$ ($h = 0.2$)	3.0000	3.6000	4.3200	5.1840	6.2208	7.4650
$y(x)$ ($h = 0.1$)	3.0000	3.6300	4.3923	5.3147	6.4308	7.7812

81.

x	0	0.2	0.4	0.6	0.8	1
$y(x)$ (exact)	0.0000	0.2200	0.4801	0.7807	1.1231	1.5097
$y(x)$ ($h = 0.2$)	0.0000	0.2000	0.4360	0.7074	1.0140	1.3561
$y(x)$ ($h = 0.1$)	0.0000	0.2095	0.4568	0.7418	1.0649	1.4273

83. (a) $y(1) = 112.7141^\circ$; $y(2) = 96.3770^\circ$; $y(3) = 86.5954^\circ$

(b) $y(1) = 113.2441^\circ$; $y(2) = 97.0158^\circ$; $y(3) = 87.1729^\circ$

(c) Euler's Method: $y(1) = 112.9828^\circ$; $y(2) = 96.6998^\circ$; $y(3) = 86.8863^\circ$

Exact solution: $y(1) = 113.2441^\circ$; $y(2) = 97.0158^\circ$; $y(3) = 87.1729^\circ$

The approximations are better using $h = 0.05$.

85. The general solution is a family of curves that satisfies the differential equation. A particular solution is one member of the family that satisfies given conditions.

87. Begin with a point (x_0, y_0) that satisfies the initial condition $y(x_0) = y_0$. Then, using a small step size h , calculate the point $(x_1, y_1) = (x_0 + h, y_0 + hF(x_0, y_0))$. Continue generating the sequence of points $(x_n + h, y_n + hF(x_n, y_n))$ or (x_{n+1}, y_{n+1}) .89. False. $y = x^3$ is a solution of $xy' - 3y = 0$, but $y = x^3 + 1$ is not a solution.

91. True

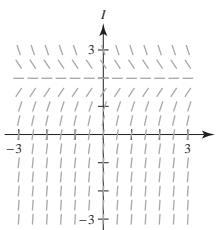
93. (a)

x	0	0.2	0.4	0.6	0.8	1
y	4	2.6813	1.7973	1.2048	0.8076	0.5413
y_1	4	2.56	1.6384	1.0486	0.6711	0.4295
y_2	4	2.4	1.44	0.864	0.5184	0.3110
e_1	0	0.1213	0.1589	0.1562	0.1365	0.1118
e_2	0	0.2813	0.3573	0.3408	0.2892	0.2303
r		0.4312	0.4447	0.4583	0.4720	0.4855

(b) If h is halved, then the error is approximately halved because r is approximately 0.5.

(c) The error will again be halved.

95. (a)



(b) $\lim_{t \rightarrow \infty} I(t) = 2$

97. $\omega = \pm 4$

99. Putnam Problem 3, Morning Session, 1954

Section 6.2 (page 394)

1. $y = \frac{1}{2}x^2 + 3x + C$

3. $y = Ce^x - 3$

5. $y^2 - 5x^2 = C$

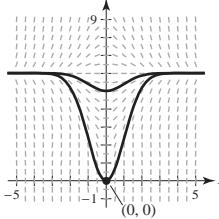
7. $y = Ce^{(2x^{3/2})/3}$

9. $y = C(1 + x^2)$

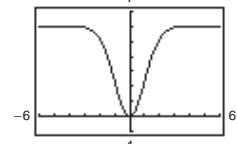
11. $dQ/dt = k/t^2$

$Q = -k/t + C$

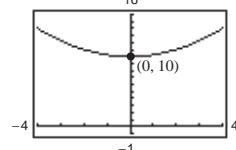
13. (a)



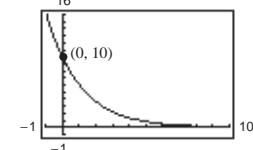
(b) $y = 6 - 6e^{-x^2/2}$



15. $y = \frac{1}{4}t^2 + 10$



17. $y = 10e^{-t/2}$



19. $\frac{8192}{4}$

21. $y = (1/2)e^{[(\ln 10)/5]t} \approx (1/2)e^{0.4605t}$

23. $y = 5(5/2)^{1/4}e^{[(\ln(2/5)/4)t} \approx 6.2872e^{-0.2291t}$

25. C is the initial value of y , and k is the proportionality constant.27. Quadrants I and III; dy/dx is positive when both x and y are positive (Quadrant I) or when both x and y are negative (Quadrant III).

29. Amount after 1000 yr: 12.96 g; Amount after 10,000 yr: 0.26 g

31. Initial quantity: 7.63 g; Amount after 1000 yr: 4.95 g

33. Amount after 1000 yr: 4.43 g; Amount after 10,000 yr: 1.49 g

35. Initial quantity: 2.16 g; Amount after 10,000 yr: 1.62 g

37. 95.76%

39. Time to double: 11.55 yr; Amount after 10 yr: \$7288.48

41. Annual rate: 8.94%; Amount after 10 yr: \$1833.67

43. Annual rate: 9.50%; Time to double: 7.30 yr

45. \$224,174.18 47. \$61,377.75

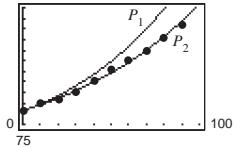
49. (a) 10.24 yr (b) 9.93 yr (c) 9.90 yr (d) 9.90 yr

51. (a) $P = 2.21e^{-0.006t}$ (b) 2.08 million(c) Because $k < 0$, the population is decreasing.53. (a) $P = 33.38e^{0.036t}$ (b) 47.84 million(c) Because $k > 0$, the population is increasing.55. (a) $N = 100.1596(1.2455)^t$ (b) 6.3 h57. (a) $N \approx 30(1 - e^{-0.0502t})$ (b) 36 days

59. (a) Because the population increases by a constant each month, the rate of change from month to month will always be the same. So, the slope is constant, and the model is linear.

- (b) Although the percentage increase is constant each month, the rate of growth is not constant. The rate of change of y is $dy/dt = ry$, which is an exponential model.

61. (a) $P_1 = 106e^{0.01487t} \approx 106(1.01499)^t$
 (b) $P_2 = 107.2727(1.01215)^t$
 (c) 350
 (d) 2029



63. (a) 20 dB (b) 70 dB (c) 95 dB (d) 120 dB

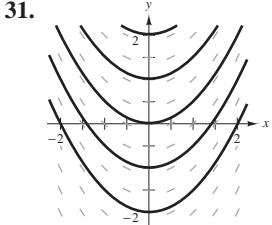
65. 379.2°F

67. False. The rate of growth dy/dx is proportional to y .

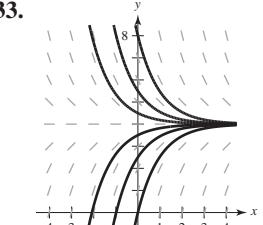
69. False. The prices are rising at a rate of 6.2% per year.

Section 6.3 (page 405)

1. $y^2 - x^2 = C$ 3. $15y^2 + 2x^3 = C$ 5. $r = Ce^{0.75s}$
 7. $y = C(x+2)^3$ 9. $y^2 = C - 8 \cos x$
 11. $y = -\frac{1}{4}\sqrt{1-4x^2} + C$ 13. $y = Ce^{(\ln x)^2/2}$
 15. $y^2 = 4e^x + 5$ 17. $y = e^{-(x^2+2x)/2}$
 19. $y^2 = 4x^2 + 3$ 21. $u = e^{(1-\cos v^2)/2}$ 23. $P = P_0e^{kt}$
 25. $4y^2 - x^2 = 16$ 27. $y = \frac{1}{3}\sqrt{x}$ 29. $f(x) = Ce^{-x/2}$



$$y = \frac{1}{2}x^2 + C$$



$$y = 4 + Ce^{-x}$$

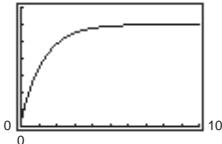
35. (a) $y = 0.1602$ (b) $y = 5e^{-3x^2}$ (c) $y = 0.2489$

37. (a) $y = 3.0318$ (b) $y^3 - 4y = x^2 + 12x - 13$
 (c) $y = 3$

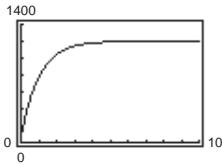
39. 97.9% of the original amount

41. (a) $dy/dx = k(y-4)$ (b) a (c) Proof
 42. (a) $dy/dx = k(x-4)$ (b) b (c) Proof
 43. (a) $dy/dx = ky(y-4)$ (b) c (c) Proof
 44. (a) $dy/dx = ky^2$ (b) d (c) Proof
 45. (a) $w = 1200 - 1140e^{-kt}$

$$(b) w = 1200 - 1140e^{-0.8t}$$

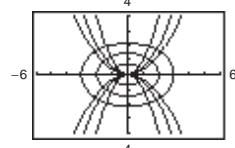
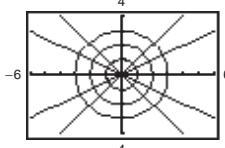


$$w = 1200 - 1140e^{-t}$$

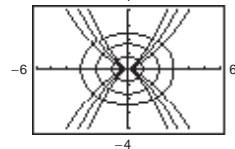


- (c) 1.31 yr; 1.16 yr; 1.05 yr (d) 1200 lb

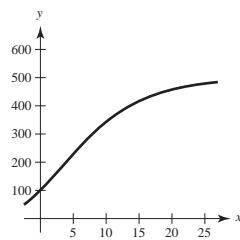
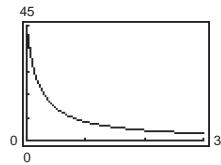
47. Circles: $x^2 + y^2 = C$
 Lines: $y = Kx$
 Graphs will vary.
 49. Parabolas: $x^2 = Cy$
 Ellipses: $x^2 + 2y^2 = K$
 Graphs will vary.



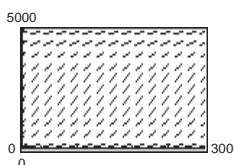
51. Curves: $y^2 = Cx^3$
 Ellipses: $2x^2 + 3y^2 = K$
 Graphs will vary.



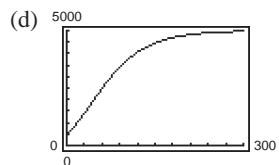
53. $N = 500/(1 + 4e^{-0.2452t})$
 55. $y = 1 - e^{-1.386t}$
 57. $y = 360/(8 + 41t)$
 59. $y = 500e^{-1.6094e^{-0.1451t}}$



61. 34 beavers 63. 92%
 65. (a) $Q = 25e^{-(1/20)t}$ (b) $t \approx 10.2$ min
 67. (a) $y = Ce^{kt}$ (b) About 6.2 h 69. About 3.15 h
 71. $P = Ce^{kt} - N/k$ 73. $A = P/r(e^{rt} - 1)$
 75. \$23,981,015.77



- (b) As $t \rightarrow \infty$, $y \rightarrow L$.
 (c) $y = 5000e^{-2.303e^{-0.02t}}$



The graph is concave upward on $(0, 41.7)$ and concave downward on $(41.7, \infty)$.

79. Answers will vary. 81. Separable: $\frac{1}{y} dy = -\frac{(1+x)}{x} dx$

83. Not separable

85. (a) $v = 20(1 - e^{-1.386t})$
 (b) $s \approx 20t + 14.43(e^{-1.386t} - 1)$

87. Homogeneous of degree 3

89. Homogeneous of degree 3 91. Not homogeneous

93. Homogeneous of degree 0 95. $|x| = C(x - y)^2$

97. $|y^2 + 2xy - x^2| = C$ 99. $y = Ce^{-x^2/(2y^2)}$

101. False. $y' = x/y$ is separable, but $y = 0$ is not a solution.

103. True

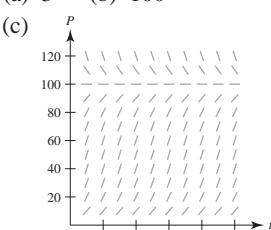
Section 6.4 (page 414)

1. d 2. a 3. b 4. c
 5. $y(0) = 4$ 7. $y(0) = \frac{12}{7}$

9. (a) 0.75 (b) 2100 (c) 70 (d) 4.49 yr
 (e) $dP/dt = 0.75P[1 - (P/2100)]$

11. (a) 0.8 (b) 6000 (c) 1.2 (d) 10.65 yr
 (e) $dP/dt = 0.8P[1 - (P/6000)]$

13. (a) 3 (b) 100



(d) 50

17. $y = 36/(1 + 8e^{-t})$; 34.16; 36

19. $y = 120/(1 + 14e^{-0.8t})$; 95.51; 120

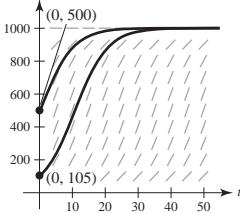
21. c

22. d

23. b

24. a

25. (a)



27. L represents the value that y approaches as t approaches infinity.
 L is the carrying capacity.

29. Yes. It can be written as $\frac{dy}{dt} = ky\left(1 - \frac{y}{L}\right)$.

31. (a) $P = \frac{200}{1 + 7e^{-0.2640t}}$ (b) 70 panthers (c) 7.37 yr

(d) $dP/dt = 0.2640P(1 - P/200)$; 69.25 panthers

(e) 100 yr

33. False. $dy/dt < 0$ and the population decreases to approach L .

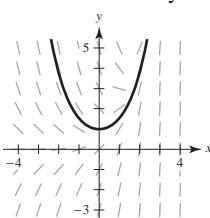
35. Answers will vary. Sample answer: There might be limits on available food or space.

37. Proof

Section 6.5 (page 420)

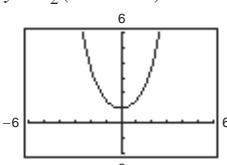
1. Linear; can be written in the form $dy/dx + P(x)y = Q(x)$
 3. Not linear; cannot be written in the form $dy/dx + P(x)y = Q(x)$
 5. $y = 2x^2 + x + C/x$ 7. $y = -16 + Ce^x$
 9. $y = -1 + Ce^{\sin x}$ 11. $y = (x^3 - 3x + C)/[3(x - 1)]$
 13. $y = e^{x^3}(x + C)$

15. (a) Answers will vary.



(b) $y = \frac{1}{2}(e^x + e^{-x})$

(c)



17. $y = 1 + 4/e^{\tan x}$ 19. $y = \sin x + (x + 1)\cos x$

21. $xy = 4$ 23. $y = -2 + x \ln|x| + 12x$

25. $P = -N/k + (N/k + P_0)e^{kt}$

27. (a) \$4,212,796.94 (b) \$31,424,909.75

29. (a) $\frac{dN}{dt} = k(75 - N)$ (b) $N = 75 + Ce^{-kt}$

(c) $N = 75 - 55.9296e^{-0.0168t}$

31. $v(t) = -159.47(1 - e^{-0.2007t})$; -159.47 ft/sec

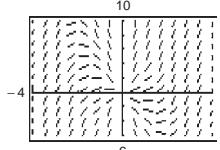
33. $I = \frac{E_0}{R} + Ce^{-Rt/L}$ 35. Proof

37. (a) $Q = 25e^{-t/20}$ (b) $-20 \ln(\frac{3}{5}) \approx 10.2$ min (c) 0

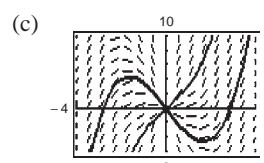
39. a 41. $\frac{dy}{dx} + P(x)y = Q(x)$; $u(x) = e^{\int P(x) dx}$

43. c 44. d 45. a 46. b

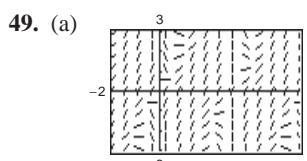
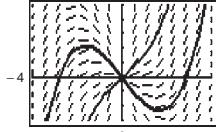
47. (a)



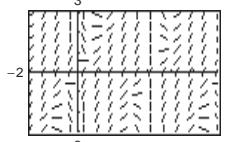
(b) $(-2, 4)$: $y = \frac{1}{2}x(x^2 - 8)$
 $(2, 8)$: $y = \frac{1}{2}x(x^2 + 4)$



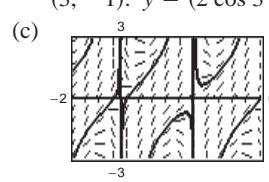
- (c)



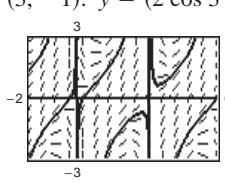
49. (a)



(b) $(1, 1)$: $y = (2 \cos 1 + \sin 1) \csc x - 2 \cot x$
 $(3, -1)$: $y = (2 \cos 3 - \sin 3) \csc x - 2 \cot x$



- (c)



51. $2e^x + e^{-2y} = C$ 53. $y = Ce^{-\sin x} + 1$

55. $y = [e^x(x - 1) + C]/x^2$ 57. $y = \frac{12}{5}x^2 + C/x^3$

59. $1/y^2 = Ce^{2x^3} + \frac{1}{3}$ 61. $y = 1/(Cx - x^2)$

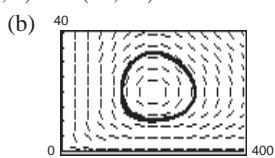
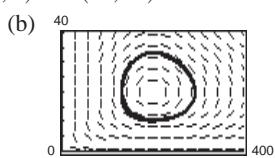
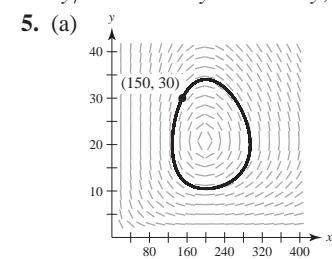
63. $1/y^2 = 2x + Cx^2$ 65. $y^{2/3} = 2e^x + Ce^{2x/3}$

67. False. $y' + xy = x^2$ is linear.

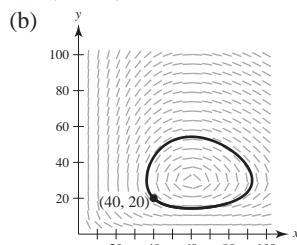
Section 6.6 (page 428)

1. $dx/dt = 0.9x - 0.05xy$,
 $dy/dt = -0.6y + 0.008xy$; (0, 0) and (75, 18)

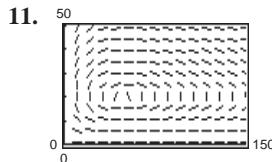
3. $dx/dt = 0.5x - 0.01xy$,
 $dy/dt = -0.49y + 0.007xy$; (0, 0) and (70, 50)



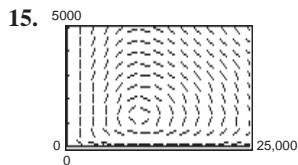
7. (a) $(40, 20)$



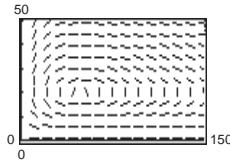
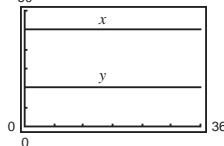
9. $(0, 0), (50, 20)$



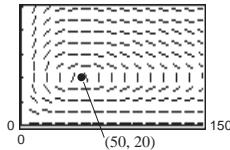
13. $(0, 0), (10,000, 1250)$



17. As t increases, both x and y are constant.



The solution curve reduces to a single point at $(50, 20)$.



19. $dx/dt = 2x - 3x^2 - 2xy$, $dy/dt = 2y - 3y^2 - 2xy$;

$(0, 0), (2/5, 2/5), (0, 2/3)$, and $(2/3, 0)$

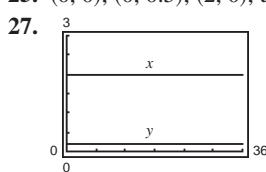
21. $dx/dt = 0.15x - 0.6x^2 - 0.75xy$,

$dy/dt = 0.15y - 1.2y^2 - 0.45xy$;

$(0, 0), (0, 1/8), (1/4, 0)$, and $(3/17, 1/17)$

23. $(0, 0), (0, 0.5), (2, 0)$, and $(45/23, 4/23)$

25. $(0, 0), (0, 0.5), (2, 0)$, and $(-9/38, 17/19)$



As t increases, both x and y are constant.

29. Yes. See bottom of page 425.

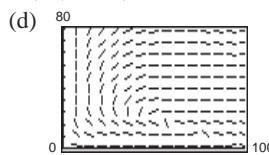
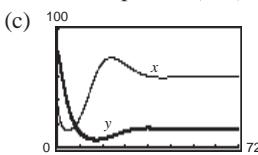
31. Use a critical point as the initial condition.

33. (a) $dx/dt = ax(1 - x/L)$. The equation is logistic.

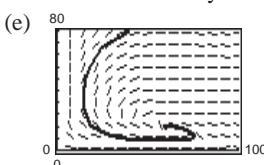
(b) $dx/dt = 0.4x[1 - (x/100)] - 0.01xy$,

$$dy/dt = -0.3y + 0.005xy$$

Critical points: $(0, 0), (60, 16), (100, 0)$



Answers will vary.



Answers will vary.

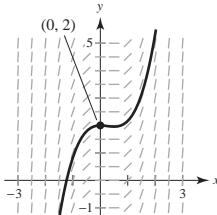
Review Exercises for Chapter 6 (page 430)

1. Solution 3. $y = \frac{4}{3}x^3 + 7x + C$ 5. $y = \frac{1}{2} \sin 2x + C$

7. $y = -e^{2-x} + C$

x	-4	-2	0	2	4	8
y	2	0	4	4	6	8
dy/dx	-10	-4	-4	0	2	8

11. (a) and (b)



13.

n	0	1	2	3	4	5	6
x_n	0	0.05	0.1	0.15	0.2	0.25	0.3
y_n	4	3.8	3.6125	3.4369	3.2726	3.1190	2.9756

n	7	8	9	10
x_n	0.35	0.4	0.45	0.5
y_n	2.8418	2.7172	2.6038	2.4986

15. $y = -\frac{5}{3}x^3 + x^2 + C$ 17. $y = -3 - 1/(x + C)$

19. $y = \frac{Ce^x}{(2+x)^2}$ 21. $\frac{dy}{dt} = \frac{k}{t^3}$, $y = -\frac{k}{2t^2} + C$

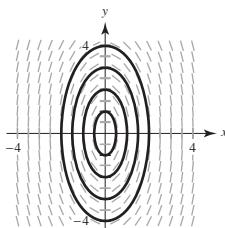
23. $y \approx \frac{3}{4}e^{0.379t}$ 25. $y = \frac{9}{20}e^{(1/2)\ln(10/3)t}$

27. About 7.79 in. 29. About 37.5 yr

31. (a) $S \approx 30e^{-1.7918/t}$ (b) 20,965 units

33. $y^2 = 5x^2 + C$ 35. $y = Ce^{8x^2}$

37. $y^4 = 6x^2 - 8$ 39. $y^4 = 2x^4 + 1$

41.

Graphs will vary.
 $4x^2 + y^2 = C$

- 43.** (a) 0.55 (b) 5250 (c) 150 (d) 6.41 yr

$$(e) \frac{dP}{dt} = 0.55P\left(1 - \frac{P}{5250}\right)$$

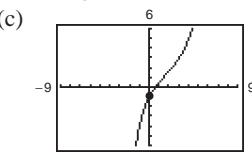
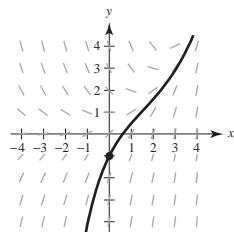
$$45. y = \frac{80}{1 + 9e^{-t}}$$

$$47. (a) P(t) = \frac{20,400}{1 + 16e^{-0.553t}} \quad (b) 17,118 \text{ trout} \quad (c) 4.94 \text{ yr}$$

$$49. dS/dt = k(L - S); S = L(1 - e^{-kt})$$

$$51. dP/dn = kP(L - P); P = CL/(e^{-Lkn} + C)$$

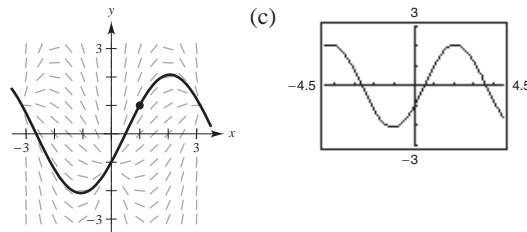
- 53.** (a) Answers will vary. (b) $y = \frac{1}{3}(2e^{x/2} - 5e^{-x})$



- 55.** (a) Answers will vary.

$$(b) y = -\cos x + 1.8305 \sin x$$

- (c)



$$57. y = -10 + Ce^x \quad 59. y = e^{x/4}\left(\frac{1}{4}x + C\right)$$

$$61. y = (x + C)/(x - 2) \quad 63. y = e^{5x}/10 + Ce^{-5x}$$

$$65. y = \frac{1}{10}e^{5x} + \frac{29}{10}e^{-5x}$$

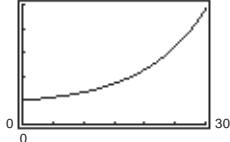
67. Answers will vary. Sample answer:

$$(x^2 + 3y^2)dx - 2xydy = 0; x^3 = C(x^2 + y^2)$$

69. Answers will vary. Sample answer:

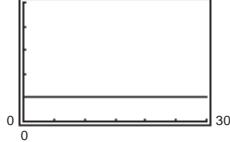
$$x^3y' + 2x^2y = 1; x^2y = \ln|x| + C$$

- 71.** (a)

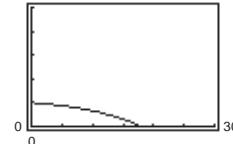


The balance continues to increase.

- (b)



The balance remains at \$500,000.

(c)

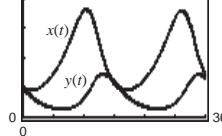
The balance decreases and is depleted in about 17.9 years.

- 73.** (a) Prey: $dx/dt = 0.3x - 0.02xy$

$$\text{Predator: } dy/dt = -0.4y + 0.01xy$$

- (b) (0, 0) and (40, 14)

- (c)



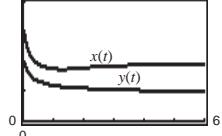
As t increases, both x and y oscillate.

- 75.** (a) Species 1: $dx/dt = 3x - x^2 - xy$

$$\text{Species 2: } dy/dt = 2y - y^2 - 0.5xy$$

- (b) (0, 0), (0, 2), (3, 0), and (2, 1)

- (c)



As t increases, x remains constant at approximately 2 and y remains constant at approximately 1.

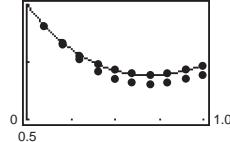
P.S. Problem Solving (page 433)

$$1. (a) y = 1/(1 - 0.01t)^{100}; \quad T = 100$$

$$(b) y = 1/\left[\left(\frac{1}{y_0}\right)^e - ket\right]^{1/e}; \quad \text{Explanations will vary.}$$

$$3. (a) (0, 1), (0.1, 0.91), (0.2, 0.83805), (0.3, 0.78244), (0.4, 0.74160), (0.5, 0.71415), (0.6, 0.69881), (0.7, 0.69442), (0.8, 0.69995), (0.9, 0.71446), (1, 0.73708)$$

- (b)

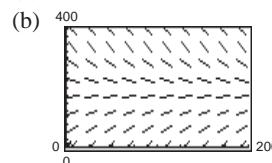


The modified Euler Method is more accurate.

$$5. 1481.45 \text{ sec} \approx 24 \text{ min}, 41 \text{ sec}$$

$$7. 2575.95 \text{ sec} \approx 42 \text{ min}, 56 \text{ sec}$$

$$9. (a) s = 184.21 - Ce^{-0.019t}$$



(c) As $t \rightarrow \infty$, $Ce^{-0.019t} \rightarrow 0$ and $s \rightarrow 184.21$.

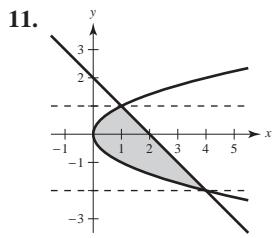
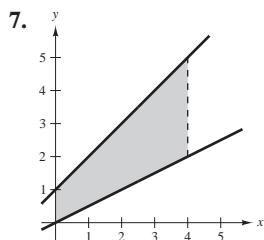
$$11. (a) C = C_0 e^{-Rt/V} \quad (b) 0$$

$$13. (a) C = \frac{Q}{R}(1 - e^{-Rt/V}) \quad (b) \frac{Q}{R}$$

Chapter 7**Section 7.1 (page 442)**

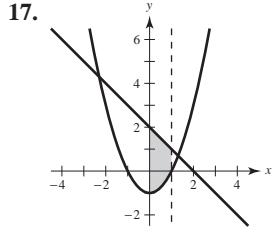
1.
$$-\int_0^6 (x^2 - 6x) dx$$

5.
$$-6 \int_0^1 (x^3 - x) dx$$

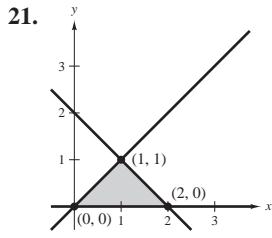


13. d

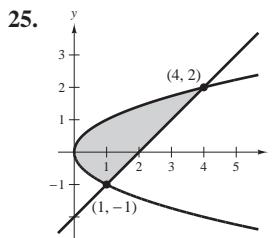
15. (a) $\frac{125}{6}$ (b) $\frac{125}{6}$

(c) Integrating with respect to y ; Answers will vary.

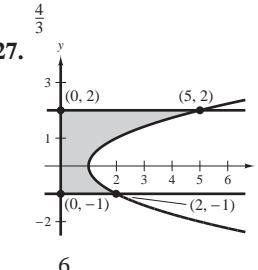
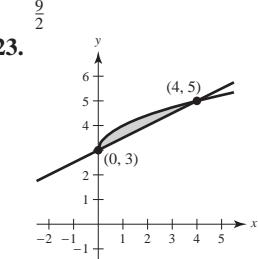
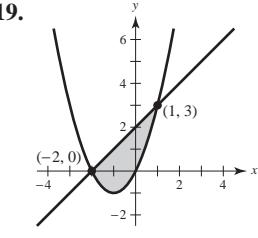
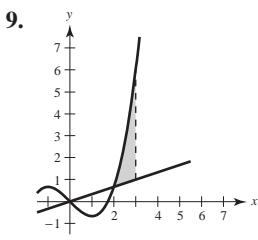
$\frac{32}{3}$



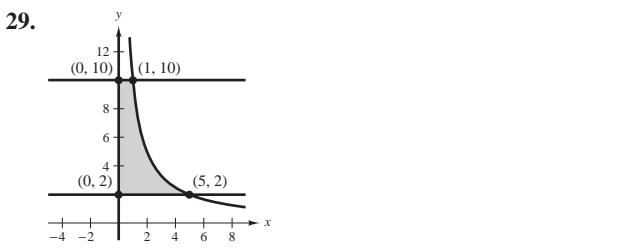
1



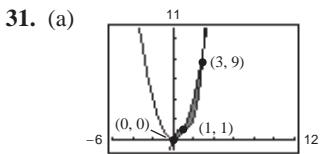
$\frac{9}{2}$



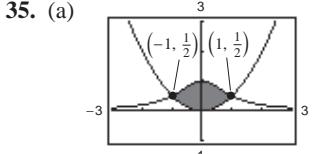
6



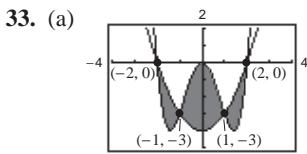
$10 \ln 5 \approx 16.094$



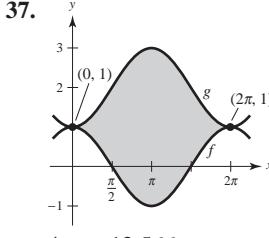
$\frac{37}{12}$



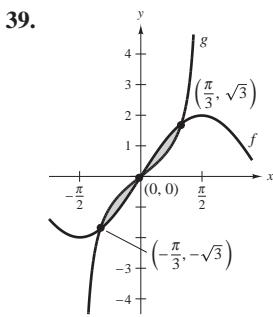
(b) $\pi/2 - 1/3 \approx 1.237$



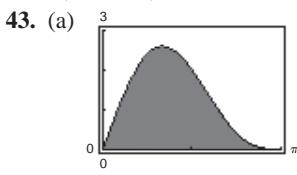
8



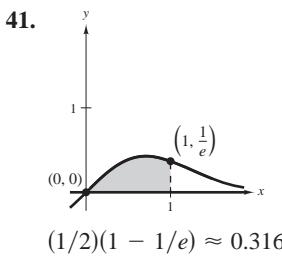
$4\pi \approx 12.566$



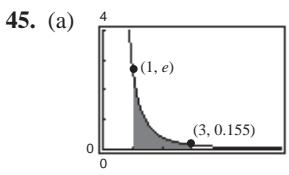
$2(1 - \ln 2) \approx 0.614$



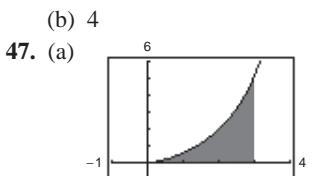
4



$(1/2)(1 - 1/e) \approx 0.316$

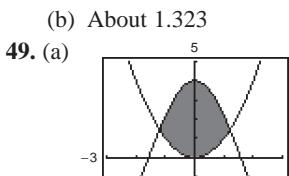


(b) About 1.323



(b) The function is difficult to integrate.

(c) About 4.7721

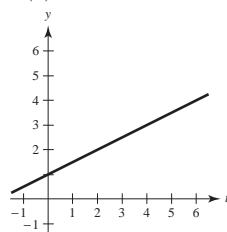


(b) The intersections are difficult to find.

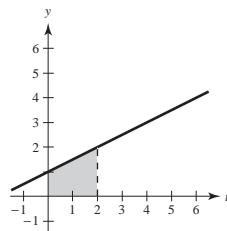
(c) About 6.3043

51. $F(x) = \frac{1}{4}x^2 + x$

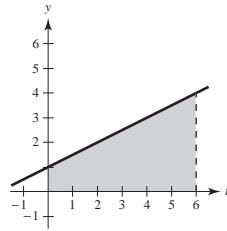
(a) $F(0) = 0$



(b) $F(2) = 3$

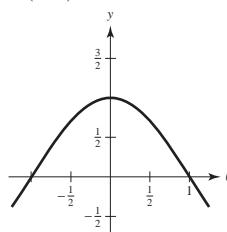


(c) $F(6) = 15$

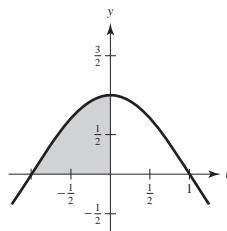


53. $F(\alpha) = (2/\pi)[\sin(\pi\alpha/2) + 1]$

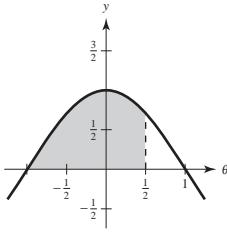
(a) $F(-1) = 0$



(b) $F(0) = 2/\pi \approx 0.6366$



(c) $F(1/2) = (\sqrt{2} + 2)/\pi \approx 1.0868$



55. 14 57. 16

59. Answers will vary. Sample answers:

(a) About 966 ft² (b) About 1004 ft²

61. $\int_{-2}^1 [x^3 - (3x - 2)] dx = \frac{27}{4}$

63. $\int_0^1 \left[\frac{1}{x^2 + 1} - \left(-\frac{1}{2}x + 1 \right) \right] dx \approx 0.0354$

65. Answers will vary.

Example: $x^4 - 2x^2 + 1 \leq 1 - x^2$ on $[-1, 1]$

$\int_{-1}^1 [(1 - x^2) - (x^4 - 2x^2 + 1)] dx = \frac{4}{15}$

67. (a) The integral $\int_0^5 [v_1(t) - v_2(t)] dt = 10$ means that the first car traveled 10 more meters than the second car between 0 and 5 seconds.

The integral $\int_0^{10} [v_1(t) - v_2(t)] dt = 30$ means that the first car traveled 30 more meters than the second car between 0 and 10 seconds.

The integral $\int_{20}^{30} [v_1(t) - v_2(t)] dt = -5$ means that the second car traveled 5 more meters than the first car between 20 and 30 seconds.

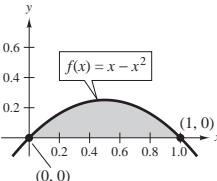
(b) No. You do not know when both cars started or the initial distance between the cars.

(c) The car with velocity v_1 is ahead by 30 meters.

(d) Car 1 is ahead by 8 meters.

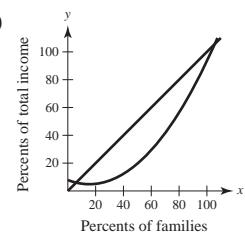
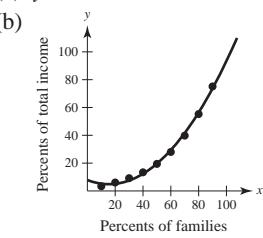
69. $b = 9(1 - 1/\sqrt[3]{4}) \approx 3.330$ 71. $a = 4 - 2\sqrt{2} \approx 1.172$

73. Answers will vary. Sample answer: $\frac{1}{6}$



75. R_1 ; \$11.375 billion

77. (a) $y = 0.0124x^2 - 0.385x + 7.85$



(b) About 2006.7

79. (a) About 6.031 m² (b) About 12.062 m³ (c) 60,310 lb

81. $\sqrt{3}/2 + 7\pi/24 + 1 \approx 2.7823$ 83. True

85. False. Let $f(x) = x$ and $g(x) = 2x - x^2$. f and g intersect at $(1, 1)$, the midpoint of $[0, 2]$, but

$$\int_a^b [f(x) - g(x)] dx = \int_0^2 [x - (2x - x^2)] dx = \frac{2}{3} \neq 0.$$

87. Putnam Problem A1, 1993

Section 7.2 (page 453)

1. $\pi \int_0^1 (-x + 1)^2 dx = \frac{\pi}{3}$ 3. $\pi \int_1^4 (\sqrt{x})^2 dx = \frac{15\pi}{2}$

5. $\pi \int_0^1 [(x^2)^2 - (x^5)^2] dx = \frac{6\pi}{55}$ 7. $\pi \int_0^4 (\sqrt{y})^2 dy = 8\pi$

9. $\pi \int_0^1 (y^{3/2})^2 dy = \frac{\pi}{4}$

11. (a) $9\pi/2$ (b) $(36\pi\sqrt{3})/5$ (c) $(24\pi\sqrt{3})/5$
(d) $(84\pi\sqrt{3})/5$

13. (a) $32\pi/3$ (b) $64\pi/3$ 15. 18π

17. $\pi(48 \ln 2 - \frac{27}{4}) \approx 83.318$ 19. $124\pi/3$

21. $832\pi/15$ 23. $\pi \ln 5$ 25. $2\pi/3$

27. $(\pi/2)(1 - 1/e^2) \approx 1.358$ 29. $277\pi/3$ 31. 8π

33. $\pi^2/2 \approx 4.935$ 35. $(\pi/2)(e^2 - 1) \approx 10.036$

37. 1.969 39. 15.4115 41. $\pi/3$ 43. $2\pi/15$

45. $\pi/2$ 47. $\pi/6$

49. A sine curve on $[0, \pi/2]$ revolved about the x -axis

51. The parabola $y = 4x - x^2$ is a horizontal translation of the parabola $y = 4 - x^2$. Therefore, their volumes are equal.

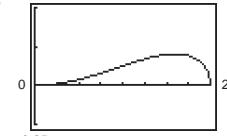
53. (a) This statement is true. Explanations will vary.

(b) This statement is false. Explanations will vary.

55. $2\sqrt{2}$ 57. $V = \frac{4}{3}\pi(R^2 - r^2)^{3/2}$ 59. Proof

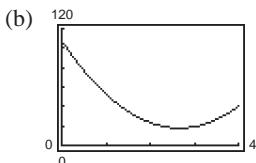
61. $\pi r^2 h [1 - (h/H) + h^2/(3H^2)]$

63. 0.5 65. (a) 60π (b) 50π



$\pi/30$

67. (a) $V = \pi(4b^2 - \frac{64}{3}b + \frac{512}{15})$



(c) $b = \frac{8}{3} \approx 2.67$

$b \approx 2.67$

69. (a) ii; right circular cylinder of radius r and height h
 (b) iv; ellipsoid whose underlying ellipse has the equation $(x/b)^2 + (y/a)^2 = 1$
 (c) iii, sphere of radius r
 (d) i; right circular cone of radius r and height h
 (e) v; torus of cross-sectional radius r and other radius R

71. (a) $\frac{81}{10}$ (b) $\frac{9}{2}$ 73. $\frac{16}{3}r^3$

75. (a) $\frac{2}{3}r^3$ (b) $\frac{2}{3}r^3 \tan \theta$; As $\theta \rightarrow 90^\circ$, $V \rightarrow \infty$.

Section 7.3 (page 462)

1. $2\pi \int_0^2 x^2 dx = \frac{16\pi}{3}$ 3. $2\pi \int_0^4 x\sqrt{x} dx = \frac{128\pi}{5}$

5. $2\pi \int_0^4 \frac{1}{4}x^3 dx = 32\pi$ 7. $2\pi \int_0^2 x(4x - 2x^2) dx = \frac{16\pi}{3}$

9. $2\pi \int_0^2 x(x^2 - 4x + 4) dx = \frac{8\pi}{3}$

11. $2\pi \int_2^4 x\sqrt{x-2} dx = \frac{128\pi}{15}\sqrt{2}$

13. $2\pi \int_0^1 x \left(\frac{1}{\sqrt{2\pi}} e^{-x^2/2} \right) dx = \sqrt{2\pi} \left(1 - \frac{1}{\sqrt{e}} \right) \approx 0.986$

15. $2\pi \int_0^2 y(2-y) dy = \frac{8\pi}{3}$

17. $2\pi \left[\int_0^{1/2} y dy + \int_{1/2}^1 y \left(\frac{1}{y} - 1 \right) dy \right] = \frac{\pi}{2}$

19. $2\pi \int_0^8 y^{4/3} dy = \frac{768\pi}{7}$ 21. $2\pi \int_0^2 y(4-2y) dy = 16\pi/3$

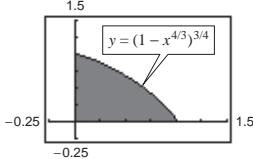
23. 8π 25. 16π

27. Shell method; it is much easier to put x in terms of y rather than vice versa.

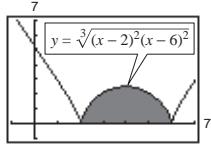
29. (a) $128\pi/7$ (b) $64\pi/5$ (c) $96\pi/5$

31. (a) $\pi a^3/15$ (b) $\pi a^3/15$ (c) $4\pi a^3/15$

33. (a)



(b) 1.506



(b) 187.25

37. (a) The rectangles would be vertical.
 (b) The rectangles would be horizontal.

39. Both integrals yield the volume of the solid generated by revolving the region bounded by the graphs of $y = \sqrt{x-1}$, $y = 0$, and $x = 5$ about the x -axis.

41. a, c, b

43. (a) Region bounded by $y = x^2$, $y = 0$, $x = 0$, $x = 2$

(b) Revolved about the y -axis

45. (a) Region bounded by $x = \sqrt{6-y}$, $y = 0$, $x = 0$

(b) Revolved about $y = -2$

47. Diameter = $2\sqrt{4 - 2\sqrt{3}} \approx 1.464$ 49. $4\pi^2$

51. (a) Proof (b) (i) $V = 2\pi$ (ii) $V = 6\pi^2$ 53. Proof

55. (a) $R_1(n) = n/(n+1)$ (b) $\lim_{n \rightarrow \infty} R_1(n) = 1$

(c) $V = \pi ab^{n+2}[n/(n+2)]$; $R_2(n) = n/(n+2)$

(d) $\lim_{n \rightarrow \infty} R_2(n) = 1$

(e) As $n \rightarrow \infty$, the graph approaches the line $x = b$.

57. (a) and (b) About $121,475 \text{ ft}^3$ 59. $c = 2$

61. (a) $64\pi/3$ (b) $2048\pi/35$ (c) $8192\pi/105$

Section 7.4 (page 473)

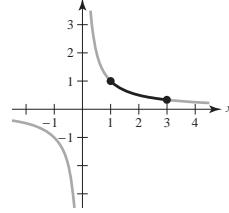
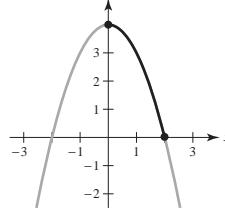
1. (a) and (b) 17 3. $\frac{5}{3}$ 5. $\frac{2}{3}(2\sqrt{2} - 1) \approx 1.219$

7. $5\sqrt{5} - 2\sqrt{2} \approx 8.352$ 9. 309.3195

11. $\ln[(\sqrt{2} + 1)/(\sqrt{2} - 1)] \approx 1.763$

13. $\frac{1}{2}(e^2 - 1/e^2) \approx 3.627$ 15. $\frac{76}{3}$

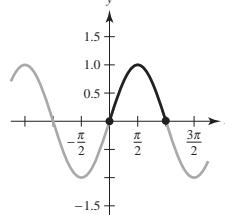
17. (a) 19. (a)



(b) $\int_0^2 \sqrt{1 + 4x^2} dx$

(c) About 4.647

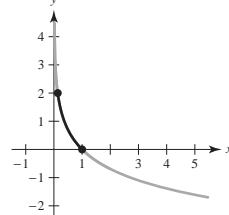
21. (a)



(b) $\int_0^\pi \sqrt{1 + \cos^2 x} dx$

(c) About 3.820

23. (a)

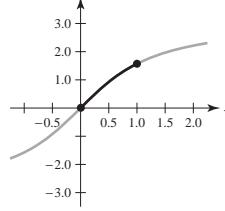


(b) $\int_0^2 \sqrt{1 + e^{-2y}} dy$

$$= \int_{e^{-2}}^1 \sqrt{1 + \frac{1}{x^2}} dx$$

(c) About 2.221

25. (a)



(b) $\int_0^1 \sqrt{1 + \left(\frac{2}{1+x^2} \right)^2} dx$

(c) About 1.871

27. b

29. (a) 64.125 (b) 64.525 (c) 64.666 (d) 64.672

31. $20[\sinh 1 - \sinh(-1)] \approx 47.0 \text{ m}$ 33. About 1480

35. $3 \arcsin \frac{2}{3} \approx 2.1892$

37. $2\pi \int_0^3 \frac{1}{3}x^3 \sqrt{1+x^4} dx = \frac{\pi}{9}(82\sqrt{82} - 1) \approx 258.85$

39. $2\pi \int_1^2 \left(\frac{x^3}{6} + \frac{1}{2x}\right) \left(\frac{x^2}{2} + \frac{1}{2x^2}\right) dx = \frac{47\pi}{16} \approx 9.23$

41. $2\pi \int_{-1}^1 2 dx = 8\pi \approx 25.13$

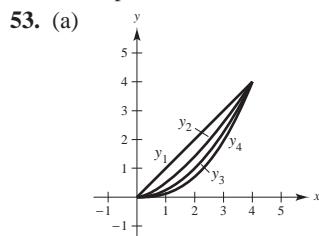
43. $2\pi \int_1^8 x \sqrt{1+\frac{1}{9x^{4/3}}} dx = \frac{\pi}{27}(145\sqrt{145} - 10\sqrt{10}) \approx 199.48$

45. $2\pi \int_0^2 x \sqrt{1+\frac{x^2}{4}} dx = \frac{\pi}{3}(16\sqrt{2} - 8) \approx 15.318$

47. 14.424

49. A rectifiable curve is a curve with a finite arc length.

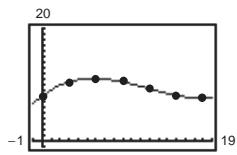
51. The integral formula for the area of a surface of revolution is derived from the formula for the lateral surface area of the frustum of a right circular cone. The formula is $S = 2\pi rL$, where $r = \frac{1}{2}(r_1 + r_2)$, which is the average radius of the frustum, and L is the length of a line segment on the frustum. The representative element is $2\pi f(d_i)\sqrt{1 + (\Delta y_i/\Delta x_i)^2} \Delta x_i$.



- (b) y_1, y_2, y_3, y_4
 (c) $s_1 \approx 5.657; s_2 \approx 5.759;$
 $s_3 \approx 5.916; s_4 \approx 6.063$

55. 20π 57. $6\pi(3 - \sqrt{5}) \approx 14.40$

59. (a) Answers will vary. Sample answer: 5207.62 in.³
 (b) Answers will vary. Sample answer: 1168.64 in.²
 (c) $r = 0.0040y^3 - 0.142y^2 + 1.23y + 7.9$



(d) 5279.64 in.³; 1179.5 in.²

61. (a) $\pi(1 - 1/b)$ (b) $2\pi \int_1^b \sqrt{x^4 + 1}/x^3 dx$

(c) $\lim_{b \rightarrow \infty} V = \lim_{b \rightarrow \infty} \pi(1 - 1/b) = \pi$

(d) Because $\frac{\sqrt{x^4 + 1}}{x^3} > \frac{\sqrt{x^4}}{x^3} = \frac{1}{x} > 0$ on $[1, b]$,

you have $\int_1^b \frac{\sqrt{x^4 + 1}}{x^3} dx > \int_1^b \frac{1}{x} dx = \left[\ln x\right]_1^b = \ln b$

and $\lim_{b \rightarrow \infty} \ln b \rightarrow \infty$. So, $\lim_{b \rightarrow \infty} 2\pi \int_1^b \frac{\sqrt{x^4 + 1}}{x^3} dx = \infty$.

63. Fleeing object: $\frac{2}{3}$ unit

Pursuer: $\frac{1}{2} \int_0^1 \frac{x+1}{\sqrt{x}} dx = \frac{4}{3} = 2\left(\frac{2}{3}\right)$

65. $384\pi/5$ 67–69. Proofs

Section 7.5 (page 483)

1. 48,000 ft-lb 3. 896 N-m 5. 40.833 in.-lb ≈ 3.403 ft-lb

7. 160 in.-lb ≈ 13.3 ft-lb 9. 37.125 ft-lb

11. (a) 487.805 mile-tons $\approx 5.151 \times 10^9$ ft-lb
 (b) 1395.349 mile-tons $\approx 1.473 \times 10^{10}$ ft-lb

13. (a) 2.93×10^4 mile-tons $\approx 3.10 \times 10^{11}$ ft-lb
 (b) 3.38×10^4 mile-tons $\approx 3.57 \times 10^{11}$ ft-lb

15. (a) 2496 ft-lb (b) 9984 ft-lb 17. $470,400\pi$ N-m

19. 2995.2π ft-lb 21. $20,217.6\pi$ ft-lb 23. 2457π ft-lb

25. 600 ft-lb 27. 450 ft-lb 29. 168.75 ft-lb

31. If an object is moved a distance D in the direction of an applied constant force F , then the work W done by the force is defined as $W = FD$.

33. The situation in part (a) requires more work. There is no work required for part (b) because the distance is 0.

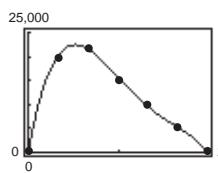
35. (a) 54 ft-lb (b) 160 ft-lb (c) 9 ft-lb (d) 18 ft-lb

37. $2000 \ln(3/2) \approx 810.93$ ft-lb 39. 3249.4 ft-lb

41. 10,330.3 ft-lb

43. (a) $16,000\pi$ ft-lb (b) 24,888.889 ft-lb

$$(c) F(x) = -16,261.36x^4 + 85,295.45x^3 - 157,738.64x^2 + 104,386.36x - 32,4675$$



- (d) 0.524 ft (e) 25,180.5 ft-lb

Section 7.6 (page 494)

1. $\bar{x} = -\frac{4}{3}$ 3. $\bar{x} = 4$ 5. (a) $\bar{x} = 8$ (b) $\bar{x} = -\frac{3}{4}$

7. $x = 6$ ft 9. $(\bar{x}, \bar{y}) = \left(\frac{10}{9}, -\frac{1}{9}\right)$ 11. $(\bar{x}, \bar{y}) = \left(2, \frac{48}{25}\right)$

13. $M_x = \rho/3, M_y = 4\rho/3, (\bar{x}, \bar{y}) = (4/3, 1/3)$

15. $M_x = 4\rho, M_y = 64\rho/5, (\bar{x}, \bar{y}) = (12/5, 3/4)$

17. $M_x = \rho/35, M_y = \rho/20, (\bar{x}, \bar{y}) = (3/5, 12/35)$

19. $M_x = 99\rho/5, M_y = 27\rho/4, (\bar{x}, \bar{y}) = (3/2, 22/5)$

21. $M_x = 192\rho/7, M_y = 96\rho, (\bar{x}, \bar{y}) = (5, 10/7)$

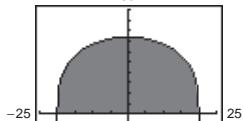
23. $M_x = 0, M_y = 256\rho/15, (\bar{x}, \bar{y}) = (8/5, 0)$

25. $M_x = 27\rho/4, M_y = -27\rho/10, (\bar{x}, \bar{y}) = (-3/5, 3/2)$

27.

$(\bar{x}, \bar{y}) = (3.0, 126.0)$

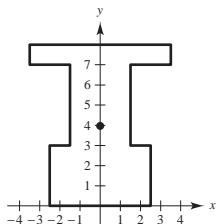
29.



$(\bar{x}, \bar{y}) = (0, 16.2)$

31.

33.



$(\bar{x}, \bar{y}) = \left(\frac{4+3\pi}{4+\pi}, 0\right)$

$(\bar{x}, \bar{y}) = \left(0, \frac{135}{34}\right)$

35. $(\bar{x}, \bar{y}) = \left(\frac{2+3\pi}{2+\pi}, 0\right)$ 37. $160\pi^2 \approx 1579.14$

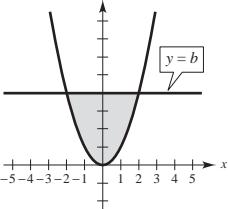
39. $128\pi/3 \approx 134.04$

41. The center of mass (\bar{x}, \bar{y}) is $\bar{x} = M_y/m$ and $\bar{y} = M_x/m$, where:
 1. $m = m_1 + m_2 + \dots + m_n$ is the total mass of the system.
 2. $M_y = m_1x_1 + m_2x_2 + \dots + m_nx_n$ is the moment about the y -axis.
 3. $M_x = m_1y_1 + m_2y_2 + \dots + m_ny_n$ is the moment about the x -axis.

43. See Theorem 7.1 on page 493. 45. $(\bar{x}, \bar{y}) = (b/3, c/3)$

47. $(\bar{x}, \bar{y}) = \left(\frac{(a+2b)c}{3(a+b)}, \frac{a^2+ab+b^2}{3(a+b)} \right)$

49. $(\bar{x}, \bar{y}) = (0, 4b/(3\pi))$

51. (a) 
- (b) $\bar{x} = 0$ by symmetry

(c) $M_y = \int_{-\sqrt{b}}^{\sqrt{b}} x(b-x^2) dx = 0$ because $x(b-x^2)$ is an odd function.

(d) $\bar{y} > b/2$ because the area is greater for $y > b/2$.

(e) $\bar{y} = (3/5)b$

53. (a) $(\bar{x}, \bar{y}) = (0, 12.98)$

(b) $y = (-1.02 \times 10^{-5})x^4 - 0.0019x^2 + 29.28$

(c) $(\bar{x}, \bar{y}) = (0, 12.85)$

55. $(\bar{x}, \bar{y}) = (0, 2r/\pi)$

57. $(\bar{x}, \bar{y}) = \left(\frac{n+1}{n+2}, \frac{n+1}{4n+2} \right)$; As $n \rightarrow \infty$, the region shrinks toward the line segments $y = 0$ for $0 \leq x \leq 1$ and $x = 1$ for $0 \leq y \leq 1$; $(\bar{x}, \bar{y}) \rightarrow \left(1, \frac{1}{4} \right)$.

Section 7.7 (page 501)

1. 1497.6 lb 3. 4992 lb 5. 748.8 lb 7. 1123.2 lb

9. 748.8 lb 11. 1064.96 lb 13. 117,600 N

15. 2,381,400 N 17. 2814 lb 19. 6753.6 lb

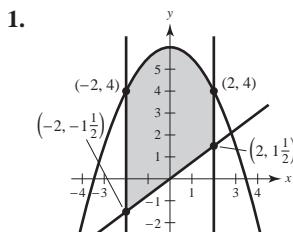
21. 94.5 lb 23–25. Proofs 27. 960 lb

29. Answers will vary. Sample answer (using Simpson's Rule): 3010.8 lb

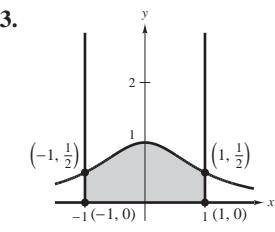
31. $3\sqrt{2}/2 \approx 2.12$ ft; The pressure increases with increasing depth.

33. Because you are measuring total force against a region between two depths

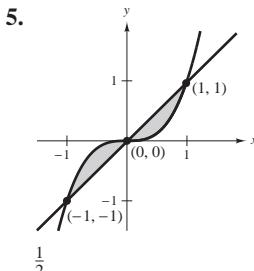
Review Exercises for Chapter 7 (page 503)



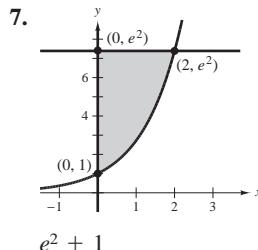
$64/3$



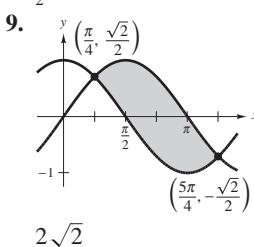
$\pi/2$



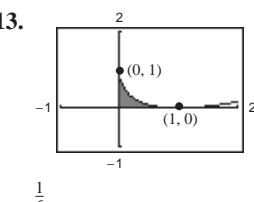
$\frac{1}{2}$



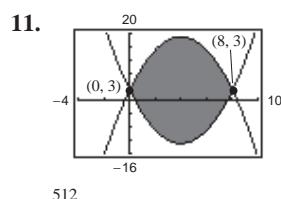
$e^2 + 1$



$2\sqrt{2}$



$\frac{1}{6}$



$\frac{512}{3}$

15. (a) 9920 ft^2

(b) $10,413\frac{1}{3} \text{ ft}^2$

17. (a) 9π (b) 18π

(c) 9π (d) 36π

19. $\pi^2/4$

21. $2\pi \ln 2.5 \approx 5.757$

23. 1.958 ft 25. $\frac{8}{15}(1 + 6\sqrt{3}) \approx 6.076$

27. 4018.2 ft

29. 15π 31. $62.5 \text{ in.-lb} \approx 5.208 \text{ ft-lb}$

33. $122,980\pi \text{ ft-lb} \approx 193.2 \text{ foot-tons}$ 35. 200 ft-lb

37. $a = 15/4$

39. 3.6 41. $(\bar{x}, \bar{y}) = \left(1, \frac{17}{5} \right)$

43. $(\bar{x}, \bar{y}) = \left(\frac{2(9\pi + 49)}{3(\pi + 9)}, 0 \right)$

45. 3072 lb

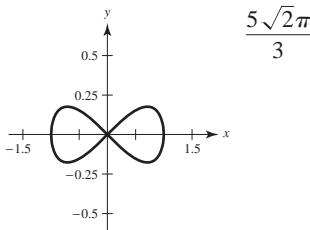
47. Wall at shallow end: 15,600 lb

Wall at deep end: 62,400 lb

Side wall: 72,800 lb

P.S. Problem Solving (page 505)

1. 3 3. $y = 0.2063x$

5. 

$\frac{5\sqrt{2}\pi}{3}$

7. $V = 2\pi \left[d + \frac{1}{2}\sqrt{w^2 + l^2} \right] lw$

9. $f(x) = 2e^{x/2} - 2$

11. 89.3%

13.

(a) $(\bar{x}, \bar{y}) = \left(\frac{63}{43}, 0 \right)$

(b) $(\bar{x}, \bar{y}) = \left(\frac{3b(b+1)}{2(b^2+b+1)}, 0 \right)$

(c) $\left(\frac{3}{2}, 0 \right)$

